1 Division of Focal-Plane Polarimeters

This modality of polarimetry has micro-optical polarization elements integrated directly into the focal plane array. One common configuration is to have several focal plane arrays imaging the same scene with each focal plane having a uniquely oriented wire grid polarizer placed in front of it. For example, one could have four focal planes imaging the same scene with wire grid polarizers oriented at $\{0°, 90°, 45°, 135°\}$ allowing the determination of $\{S_0, S_1, S_2\}$. These polarimeters are extremely sensitive to mis-registration as the Stokes vectors are sums and differences of the intensity images. Other sources of error include: polarization angle error, polarization leakage, retardance, and camera noise.

2 Stokes Vectors and Mueller Matrices

[1] In the Mueller matrix calculus, the Stokes vector $\mathbf{S}$ is used to describe the polarization state of a light beam, and the Mueller matrix $\mathbf{M}$ to describe the polarization altering characteristics of a device.

$$\mathbf{S}_{\text{out}} = \mathbf{M}\mathbf{S}_{\text{in}}$$

3 Polarization Angle Error

When a polarization element is rotated about the beam of light by an angle θ such that the angle of incidence is unchanged the Mueller matrix $\mathbf{M}(\theta)$ is given by

$$\mathbf{M}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \mathbf{M} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) & 0 \\ 0 & -\sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where $\mathbf{M}$ is the Mueller matrix before rotation. When building the DoFP polarimeter it is impossible to align the wire grid polarizers perfectly with respect to each other. To model this alignment error we can treat θ as a Gaussian random variable with known first and second moments.

4 Diattenuation (Polarization leakage)

Diattenuation is the property of an optical element whereby the intensity transmittance of the exiting beam depends on the polarization state of the incoming beam (polarizers are strongly diattenuating optical elements). The diattenuation of an optical element is defined as

$$D = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}} + T_{\text{min}}}$$

where T is intensity transmittance. The Mueller matrix corresponding to diattenuation of a linear polarizer (like a wire grid polarizer) oriented at 0° is (Intensity eigenstate transmittances q, r)

$$\mathbf{M} = \frac{1}{2} \begin{pmatrix} q+r & q-r & 0 & 0 \\ q-r & q+r & 0 & 0 \\ 0 & 0 & 2\sqrt{qr} & 0 \\ 0 & 0 & 0 & 2\sqrt{qr} \end{pmatrix}$$

The eigenstate transmittances may change over time and use of the polarimeter, to model this we can let q and r be random variables with known pdf and first and second moments.

The Mueller matrix describing both rotation and diattenuation is given by (let $\alpha = T_{\text{min}}/T_{\text{max}}$)

$$\mathbf{M} = \frac{1}{2} \begin{pmatrix} 1+\alpha & (1-\alpha)\cos(2\theta) & (1-\alpha)\sin(2\theta) & 0 \\ (1-\alpha)\cos(2\theta) & (1+\alpha)\cos^2(2\theta) + 2\sqrt{\alpha}\sin^2(2\theta) & \frac{1}{2}(1-\sqrt{\alpha})^2\sin(4\theta) & 0 \\ (1-\alpha)\sin(2\theta) & \frac{1}{2}(1-\sqrt{\alpha})^2\sin(4\theta) & 2\sqrt{\alpha}\cos^2(2\theta) + \frac{1}{2}(1+\alpha)\sin^2(2\theta) & 0 \\ 0 & 0 & 0 & \sqrt{\alpha} \end{pmatrix}$$

[1] Taken verbatim from the handbook of optics

5 Retardance

TBD

6 Data Reduction and Final Model

Ultimately the measurements that are made are of intensity and not Stokes vectors. The process of converting intensity measurements into Stokes vectors is known as data reduction. Let the data reduction matrix be denoted by $\mathbf{A}$, then the measurements $\mathbf{Y}$ are related to the Stokes vectors $\mathbf{S}$ via

$$\mathbf{Y} = \mathbf{AMS}$$

Previous methods used for determining optimal parameters for the imaging system invoked the necessity of numerical inversion of the matrix product $\mathbf{AM}$ and so focused on the condition number of the matrix product.

7 The Problem at Hand

We take the above model and consider the additive Gaussian noise case:

$$\mathbf{Y} = \mathbf{AMS} + \varepsilon$$

The cost function is then

$$\Psi(Y|S) = \frac{1}{2} \parallel Y - AMS \parallel^2$$

To calculate the gradient we remember that there are random parameters in the Mueller matrix. Let $t = [t_1, \ldots, t_J]$ denote the random vector that $\mathbf{M}$ depends on and let $\mathbf{P}$ be the matrix product $\mathbf{AM}$. Then,

$$\begin{aligned} \nabla\Psi(Y|S) &= \nabla\frac{1}{2} \parallel Y - PS \parallel^2 \\ &= [-P'(Y - PS); \langle (Y - PS), -\frac{\partial P}{\partial t_1} S\rangle, \ldots, \langle (Y - PS), -\frac{\partial P}{\partial t_J} S\rangle] \end{aligned}$$