Introduction to System Optimization: Part 1

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Subsystem optimization results $\Rightarrow$ optimal system?

Objective: provide tools for developing system optimization strategy
System Optimization

- Subsystem optimization results $\Rightarrow$ optimal system?
- Objective: provide tools for developing system optimization strategy
Overview

1. Systems and Interactions
   - Definitions
   - Interaction Examples

2. System Consistency and Optimality

3. System Optimization Methods
   - AiO
   - IDF
   - ATC
What makes something a system?
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- Comprised of several components (or subsystems)
System Definition

What makes something a system?

- Comprised of several components (or subsystems)
- Interactions exist between the components
What makes something a system?

- Comprised of several components (or subsystems)
- Interactions exist between the components
  - Some aspect of one component influences the effect of changes in another component
Explicit Analysis Interactions

**Analysis:** evaluate $f$, $g$, and $h$ in design problem for a given $\mathbf{x}$

$y_{ij}$: analysis output passed from subsystem $j$ to $i$
(coupling variable)

**Example:** deflection and pressures in *aeroelastic* analysis
Interaction Example (First Type)

System objective function:

\[ f(y_{12}, y_{13}) = c_1(y_{12} - c_2)^2 + c_3(y_{13} - c_4)^2 + c_5 y_{12} y_{13} \]

Inputs to subsystem 1:
- \( y_{12} \): output (response) of subsystem 2
- \( y_{13} \): output (response) of subsystem 3

Three objective function terms:
1. Depends only on SS2 response
2. Depends only on SS3 response
3. Depends on a combination of the responses (interaction)
Interaction Example (First Type)

Case I: No Interaction $\mathbf{c} = (1, 1, 1, 1, 0)^T$
Interaction Example (First Type)

Case II: Interaction Present $c = (1, 1, 1, 1, -1)^T$
Interaction Example (Second Type)

Governing Equations of Motion:

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2
\end{bmatrix}
\begin{bmatrix}
  \ddot{z}_1 \\
  \ddot{z}_2
\end{bmatrix}
+
\begin{bmatrix}
  k_1 + k_2 & -k_2 \\
  -k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
  z_1 \\
  z_2
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  0
\end{bmatrix}
\]

**Coupled in stiffness:** Motion of \( m_1 \) (i.e. \( x_1(t) \)) is dependent of the value of \( k_2 \) (\( \therefore z_1(t) \) depends on the state of \( z_2 \)), and visa-versa.
Example system:

\[ a_1(x_1, x_2, x_4) \]
\[ a_2(x_6, y_{21}) \]
\[ a_3(x_2, x_3, x_4, y_{31}, y_{34}) \]
\[ a_4(x_4, x_5, y_{41}) \]
Interaction Identification

**Example:** Interaction between automotive subsystems

- structure
- powertrain
- suspension
- steering
- braking
- cabin (interior geometry, HVAC, etc.)
What might result if interactions are ignored in system design?
What might result if interactions are ignored in system design?

- Missed opportunity to improve performance (not system optimal)
- Incompatibility between subsystems (inconsistent system)
Analysis Interactions

- What are some interactions between your subsystems?
- Specific effects of ignoring interactions?
A system has coupling variable consistency if for every coupling variable,

\[ y_{ij} - a_j(x_j, y_j) = 0 \]

is satisfied.
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is satisfied.

**System Analysis:** task of solving system analysis equations for \( y_p \), given \( x \).
Subsystem Optimality $\neq$ System Optimality
From Subsystem to System Formulation

Subsystem $i$ optimization formulation:

$$\min_{x_i} f_i(x_i, y_p)$$

subject to

$$g_i(x_i, y_p) \leq 0$$
$$h_i(x_i, y_p) = 0$$

System optimization formulation:

$$\min_{x} f(x, y_p)$$

subject to

$$g(x) = [g_1, g_2, \ldots, g_N] \leq 0$$
$$h(x) = [h_1, h_2, \ldots, h_N] = 0$$

where:

$$f = \begin{cases} 
  f_k & \text{select one subsystem objective} \\
  \sum_{i=1}^{N} w_i f_i(x, y_p) & \text{weighted sum of subsystem objectives} \\
  f(x, y_p) & \text{define new function}
\end{cases}$$
All-in-One (AiO) Design Approach

- Nest system analysis within system optimization
- Can identify system optimum
  
  *but:*
  - Can be very computationally expensive
  - Requires a complete system analysis for every design iteration
  - May not converge
Example Problem—Single Element Aeroelasticity

- **Aeroelasticity**: Requires both aerodynamic and structural analysis
- **Application**: Air-flow sensor design
Air-flow Sensor Analysis

▷ Structural Analysis:

\[ M = k\theta = \frac{1}{2} F \ell \cos \theta \]

Given a design \((\ell, w)\) and a drag force \(F\), solve for the corresponding deflection \(\theta\).

▷ Aerodynamic Analysis:

\[ F = C A f v^2 = C \ell w \cos \theta v^2 \]

Given a design \((\ell, w)\) and a deflection \(\theta\), find the drag force \(F\).

▷ System Analysis:

Given a design \((\ell, w)\), find the equilibrium values \(F\) and \(\theta\).
Air-flow Sensor Design

(Sensor Calibration Problem)

AiO Formulation:

\[
\begin{align*}
\min_{\ell, w} & \quad (\theta - \hat{\theta})^2 \\
\text{subject to} & \quad F - F_{max} \leq 0 \\
& \quad \ell w - A = 0
\end{align*}
\]

Design Parameters: \( k, A, F_{max}, C, v \)
Solution by Monotonicity Analysis

For cases where meeting the deflection target requires a drag force greater than $F_{max}$, the target cannot be met, and the inequality constraint will be active.
Solution by Monotonicity Analysis

For cases where meeting the deflection target requires a drag force greater than $F_{\text{max}}$, the target cannot be met, and the inequality constraint will be active.

\[ F_{\text{max}} = C\ell w \cos \theta v^2 \Rightarrow \theta = \cos^{-1} \left( \frac{F_{\text{max}}}{CAv^2} \right) \]

\[ k\theta - \frac{1}{2} F_{\text{max}} \ell \cos \theta = 0 \Rightarrow \ell^* = \frac{2k \cos^{-1} \left( \frac{F_{\text{max}}}{CAv^2} \right) CAv^2}{F_{\text{max}}^2} \]
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$$F_{\text{max}} = C \ell w \cos \theta v^2 \implies \theta = \cos^{-1} \left( \frac{F_{\text{max}}}{CAv^2} \right)$$

$$k \theta - \frac{1}{2} F_{\text{max}} \ell \cos \theta = 0 \implies \ell^* = \frac{2k \cos^{-1} \left( \frac{F_{\text{max}}}{CAv^2} \right) CAv^2}{F_{\text{max}}^2}$$

- $F$, $\theta$ and $\ell^*$ known $\implies$ solve for $w^*$ using $w^* = A/\ell^*$
Shared variable: $\ell$
Coupling variables: $\theta$ and $F$
System analysis requires finding $\theta$ and $F$ such that:

\[
\begin{align*}
\theta &= \theta(F) \\
F &= F(\theta)
\end{align*}
\]
System Analysis Options

- Solve the system analysis equations at each optimization iteration (AiO)
- Could we use the optimization algorithm to solve these equations?
System Analysis Options

- Solve the system analysis equations at each optimization iteration (AiO)
- Could we use the optimization algorithm to solve these equations?
  - Yes!
    - Make the system analysis equations constraints
    - Let the optimizer choose values for the coupling variables, in addition to design variables
Why Use the Optimizer for Analysis?

- No longer have to completely solve system analysis when you are far from the system optimum
- Breaks feedback loops (eliminating nested iterations)
- Enables coarse-grained parallel computation
- AiO can overlook global optimum in some cases, or even fail
Air Flow Sensor Reformulation

**AiO Formulation:**

$$\min_{\ell, w} \ (\theta - \hat{\theta})^2$$

subject to

$$F - F_{\text{max}} \leq 0$$

$$\ell w - A = 0$$

System analysis is implicitly required in the calculation of $F$ and $\theta$.

**New Formulation:**

$$\min_{\ell, w, \theta, F} \ (\theta - \hat{\theta})^2$$

subject to

$$F - F_{\text{max}} \leq 0$$

$$\ell w - A = 0$$

$$\theta - \theta(\ell, F) = 0$$

$$F - F(\ell, w, \theta) = 0$$
**Individual Disciplinary Feasible (IDF) Method**

**IDF**: simplest formal approach to combining analysis and optimization tasks.

**IDF General Formulation**:

$$\begin{align*}
\min_{x, y} & \quad f(x, y) \\
\text{subject to} & \quad g(x, y) = [g_1, g_2, \ldots, g_N] \leq 0 \\
& \quad h(x, y) = [h_1, h_2, \ldots, h_N] = 0 \\
& \quad h_{aux}(x, y) = Sa(x, y) - y = 0
\end{align*}$$
IDF Architecture

System Optimizer

SS1 Analysis

SS2 Analysis

$g_1, h_1, y_{i1}$

$g_2, h_2, y_{i2}$

$x_{\ell1}, x_{s1}, y_{1j}$

$x_{\ell2}, x_{s2}, y_{2j}$

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**IDF Formulation Example: Electric Water Pump**

\[
\min_x \quad P_e = VI
\]

subject to

\[
P \geq P_{\text{min}} = 100 \text{ kPa}
\]

\[
T \leq T_{\text{max}} = 428 \text{ K}
\]

\[
L + \ell_c \leq 0.2 \text{ m}
\]

\[
Q = 1.55 \cdot 10^{-3} \text{ m}^3/\text{sec}
\]

**Analysis Functions**

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T = a_1(I, \omega, d, d_2, d_3, L, \ell_c))</td>
<td>Mot. winding temp. (K)</td>
</tr>
<tr>
<td>(I = a_2(\tau, T, d, d_2, d_3, L))</td>
<td>Motor current (amps)</td>
</tr>
<tr>
<td>(\omega = a_3(I, T, d, d_2, d_3, L, \ell_c))</td>
<td>Motor speed (rad/sec)</td>
</tr>
<tr>
<td>(\tau = a_4(\omega, D_2, b, \beta_1, \beta_2, \beta_3))</td>
<td>Pump drive torque (Nm)</td>
</tr>
<tr>
<td>(P = a_5(\omega, D_2, b, \beta_1, \beta_2, \beta_3))</td>
<td>Pressure differential (kPa)</td>
</tr>
</tbody>
</table>
IDF seems helpful for many problems, but what if I have more design variables than my optimization algorithm can handle?
IDF seems helpful for many problems, but what if I have more design variables than my optimization algorithm can handle? **Multilevel Methods:**

- Use multiple optimization algorithms to share the load
- Distributed decision making reduces individual problem dimension
Multi-level Methods

- Optimization algorithm coupled with every element of the system
  - Local optimizers make local decisions (distributed decision making)
  - Can utilize specialized optimization algorithms

- Best for sparse problem structures
  - Many local decisions required, but relatively few subsystem connections
  - Possible to reduce individual problem dimension with multilevel methods
Analytical Target Cascading (ATC)

- Multi-level system design method
- Developed based on needs in the automotive industry
- Intended for hierarchical problems with object-based decomposition

(covered in detail in a later lecture)
Individual Disciplinary Feasible (IDF) Method

Levels $i$

$i = 1$

$j = A$

$i = 2$

$j = B$  $j = C$

$i = 3$

$j = D$  $j = E$  $j = F$  $j = G$