Introduction to Design Optimization

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June 23, 2005

Value of Analysis and Prediction Tools

- Enables testing in the virtual world

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What else?

Without them, full-scale prototypes are required, which may:

- be to expensive to build more than one
- require substantial time for each realization
- risk human safety during testing

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⇒ Either case requires a *design* change

Proposing Design Changes

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Trial and error methods:

- Use intuition or experience-based knowledge to propose a new design
- Change one aspect of the product at a time and see what happens

Proposing Design Changes

How can we generate new designs that will improve upon previous designs?

Problems with trial and error methods:

- May require excessive number of tests (a problem even in the virtual world)
- Still never know if we found the 'best' design
- Many products are too complex to design based on intuition

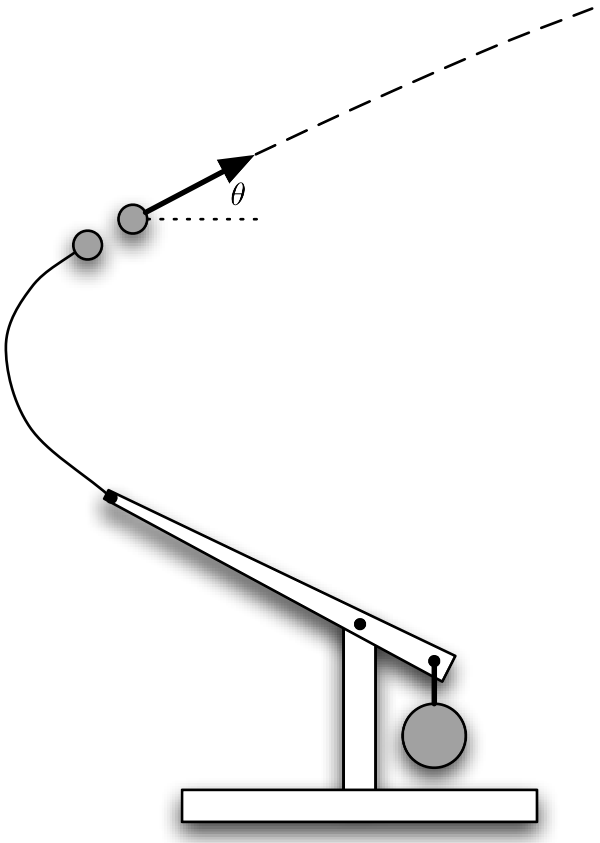
Proposing Design Changes

How can we generate new designs that will improve upon previous designs?

Scientific approach:

Optimization, a branch of mathematics, provides the necessary tools to move efficiently toward a design that is superior to all other alternatives.

Example 1: Trebuchet Design



Projectile Motion

Horizontal Position $x = v_0 \cos\theta t$

Horizontal Velocity $v_x = v_0 \cos\theta$

Vertical Position $y = v_0 \sin\theta t - \frac{1}{2} g t^2$

Vertical Velocity $v_y = v_0 \sin\theta - gt$

Trebuchet Design Objective

Distance traveled:

$$x_{max} = \frac{2}{g} v_0^2 \sin\theta \cos\theta$$

Optimization problem:

$$\max_{\theta} \quad \frac{2}{g} v_0^2 \sin\theta \cos\theta$$

Optimal Trebuchet Design

Using calculus-based optimization techniques, it is possible to show that the maximum launch distance is obtained when:

$$\sin\theta = \cos\theta$$

which is satisfied when:

$$\theta = 45^\circ$$

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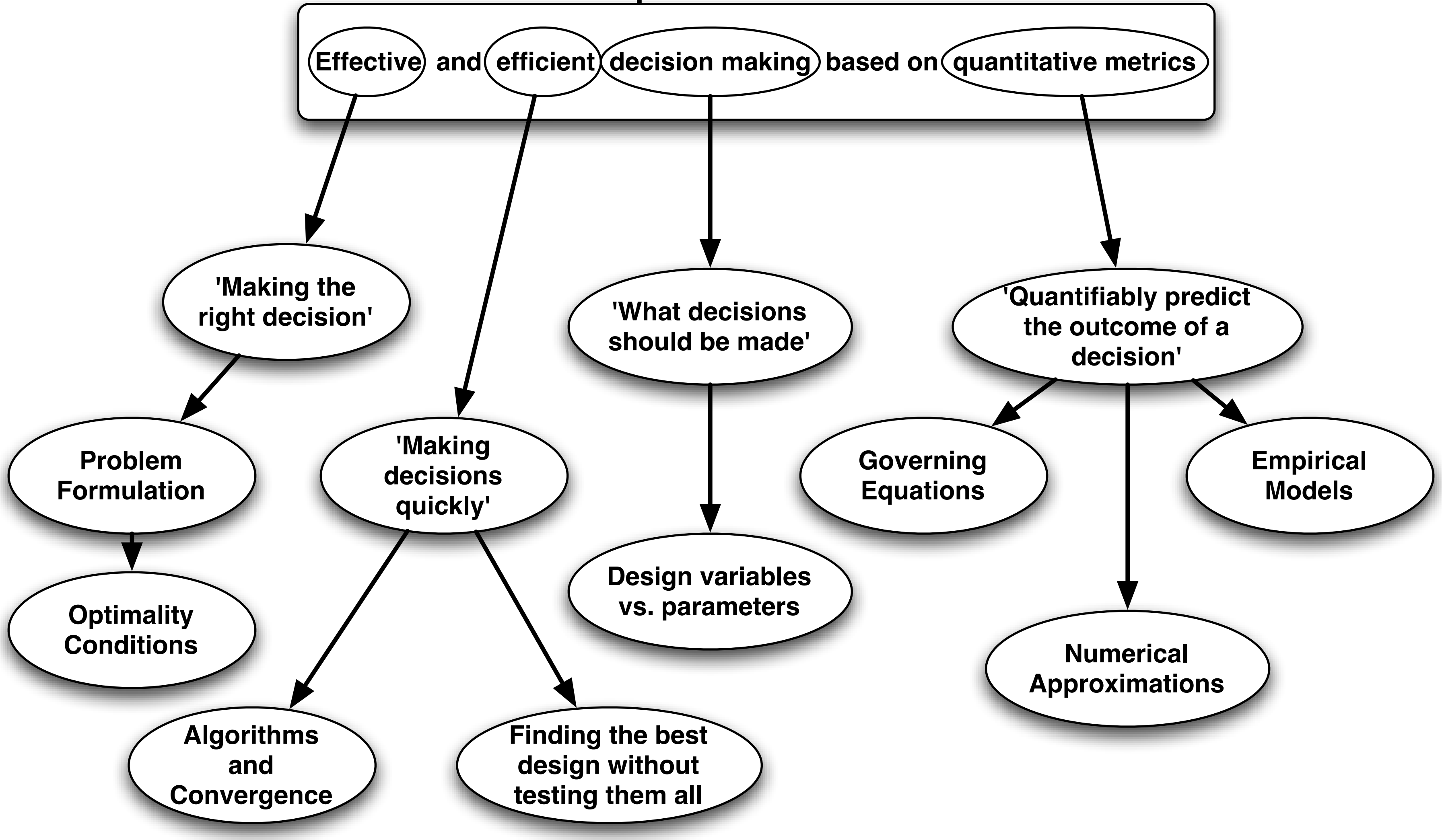
- ▷ What if more than one or two decisions need to be made?
- ▷ What if each prediction takes hours, or even days?

Computational Exercise:

1. Use MS Excel$^{\text{TM}}$to calculate $x_{max} = \frac{2}{g} v_0^2 \sin\theta \cos\theta$, and use trial and error to find the optimal θ (radians). Use $g = 9.81$, $v_0 = 15$.
2. Create a plot of the trajectory that depends on the angle θ from part 1. Use $y = x \tan\theta - gx^2/2v_0^2 \cos^2\theta$.
3. Plot x_{max} as a function of θ from 0 to $\pi/2$ radians.
4. Use MS Excel$^{\text{TM}}$Solver (may need to include the add-in) to maximize x_{max} by varying θ.

Definition of Design Optimization?

Optimization:



Effective and efficient decision making based on quantitative metrics

'Making the right decision'

Problem Formulation

Optimality Conditions

'Making decisions quickly'

Algorithms and Convergence

Finding the best design without testing them all

'What decisions should be made'

Design variables vs. parameters

'Quantifiably predict the outcome of a decision'

Governing Equations

Numerical Approximations

Empirical Models

The Best Design Decisions

To look for the best design, we need to formally define what we mean by *best*.

- How do we obtain quantitative metrics for comparison (modeling)?
- How do we formulate the optimization problem?

Standard Negative Null Form

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & g(\mathbf{x}) \leq \mathbf{0} \\ & h(\mathbf{x}) = \mathbf{0} \end{aligned}$$

Formulation Choices

We choose:

- what is the objective and what are constraints
- what items we make decisions about (design variables) and what items are held fixed (design parameters)

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These choices will affect the solution to the design optimization problem.

Mathematical techniques will help find the answer, but we define the question

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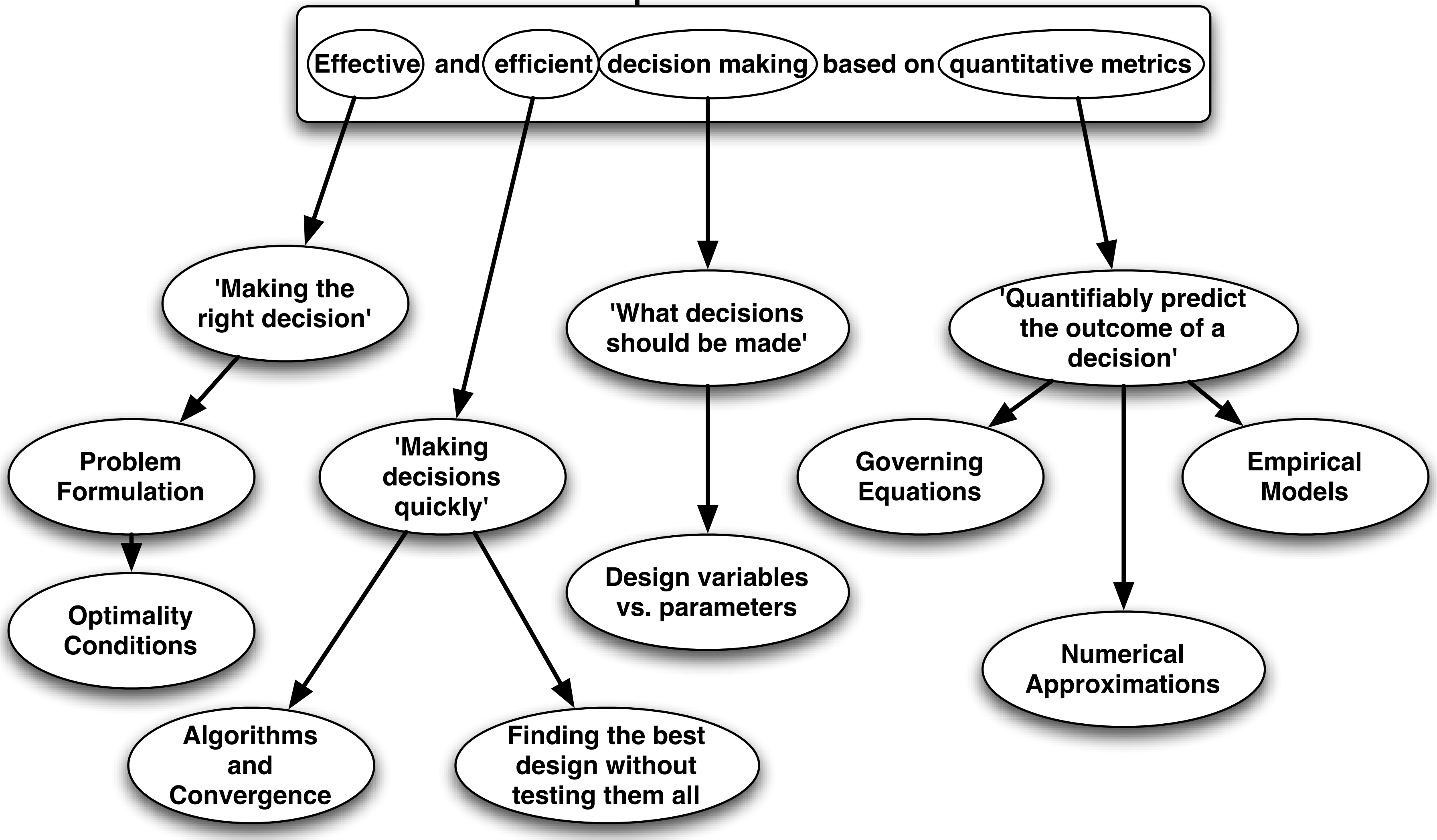
$\Rightarrow$ A necessary condition for optimality is a zero (horizontal) slope or gradient.

$\Rightarrow$ Many optimization algorithms are based on this principle.

Optimization Examples in Nature

- Gravitational Potential Energy
  - Objects seek position of minimum gravitational potential energy: $V = mgh$
- Surface Energy
  - Bubbles seek to minimize surface area $\Rightarrow$ spherical shape/fewer,larger bubbles
  - Crystallization/grain growth
- Atomic Spacing
- Survival of the fittest: optimization of organisms or entire ecosystems (genetic algorithms)

Optimization:



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Fast Design Decisions

Optimization Analogy: finding the low point of a curve

While in a fishing boat, find the deepest point of a pond.

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1. Grid search (take a long time, don't know if you found it)

2. Steepest descent
   - Take a few extra measurements around a point to get a sense of 'downhill'
   - Move in the downhill direction until the bottom starts going back uphill
   - Find a new direction, and repeat until there is no more downhill

Algorithm Convergence

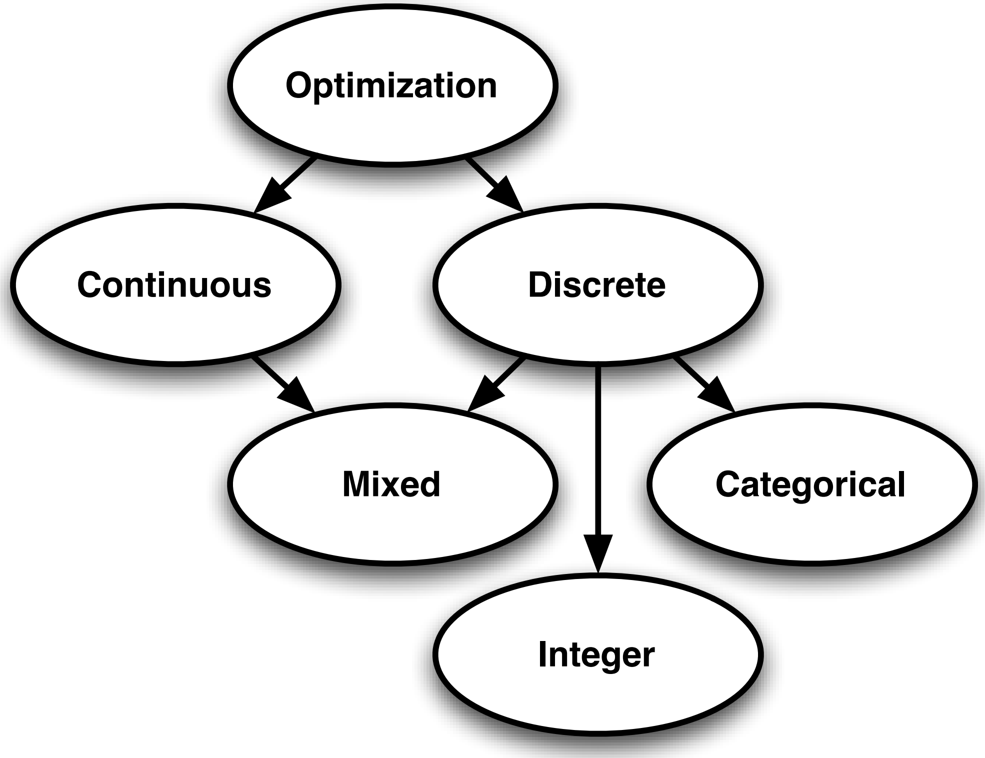
Convergence Existence The algorithm will converge to a design

Convergence Rate How many iterations are required for convergence

Algorithm Effectiveness

An effective optimization algorithm finds the best design without having to test all alternatives.

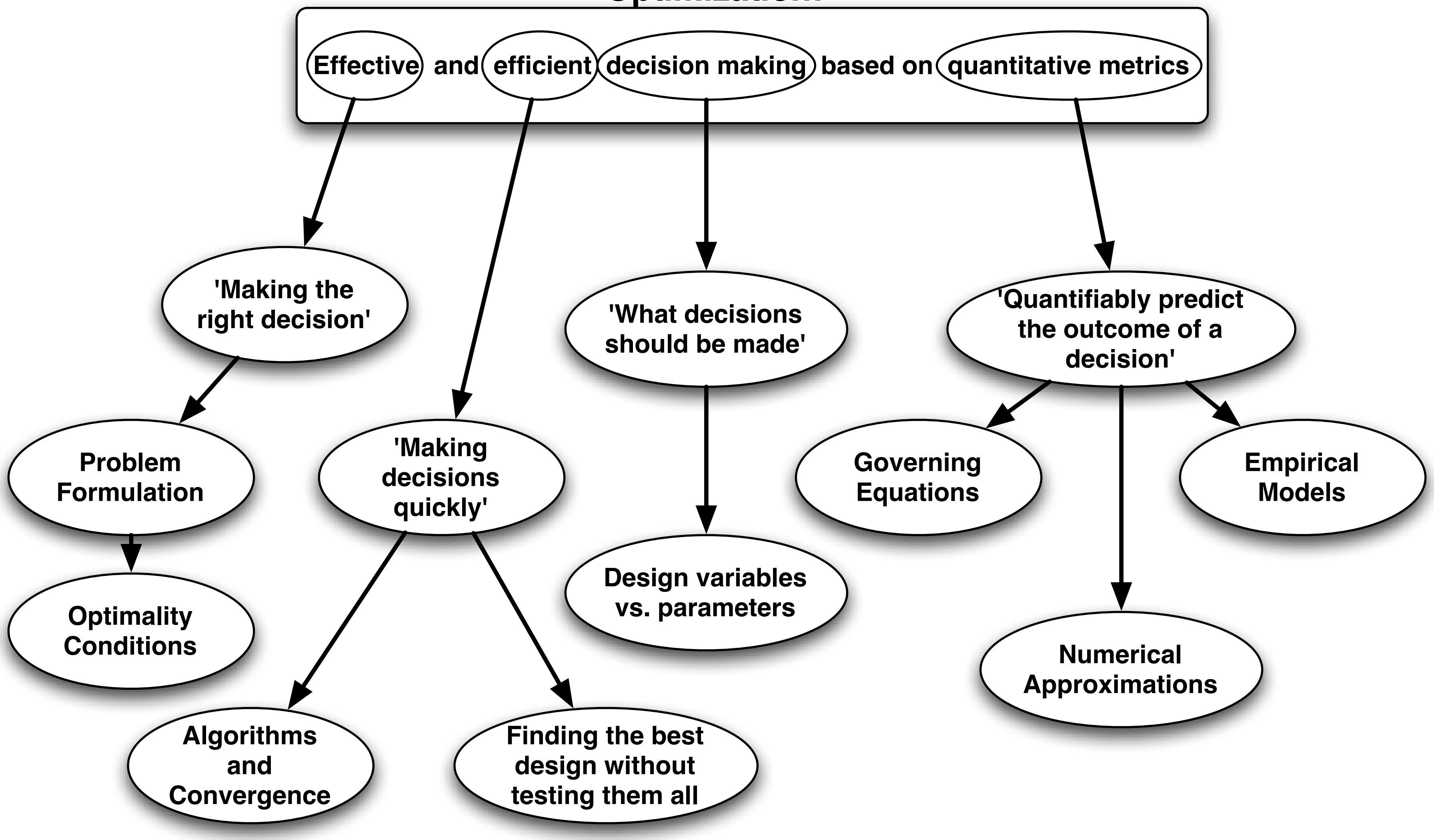
Algorithm Categorization: by Variable Type



Algorithm Categorization: by Function Type

- linear vs. nonlinear
- noisy vs. smooth
- convex vs. non-convex

Optimization:



What Decisions to Make?

- Parametric design vs. Conceptual design (CAD example)
- Design variables vs. design parameters

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Tradeoff:

- Fewer design variables ⇒ easier to solve
- More design variables ⇒ more design options, better results

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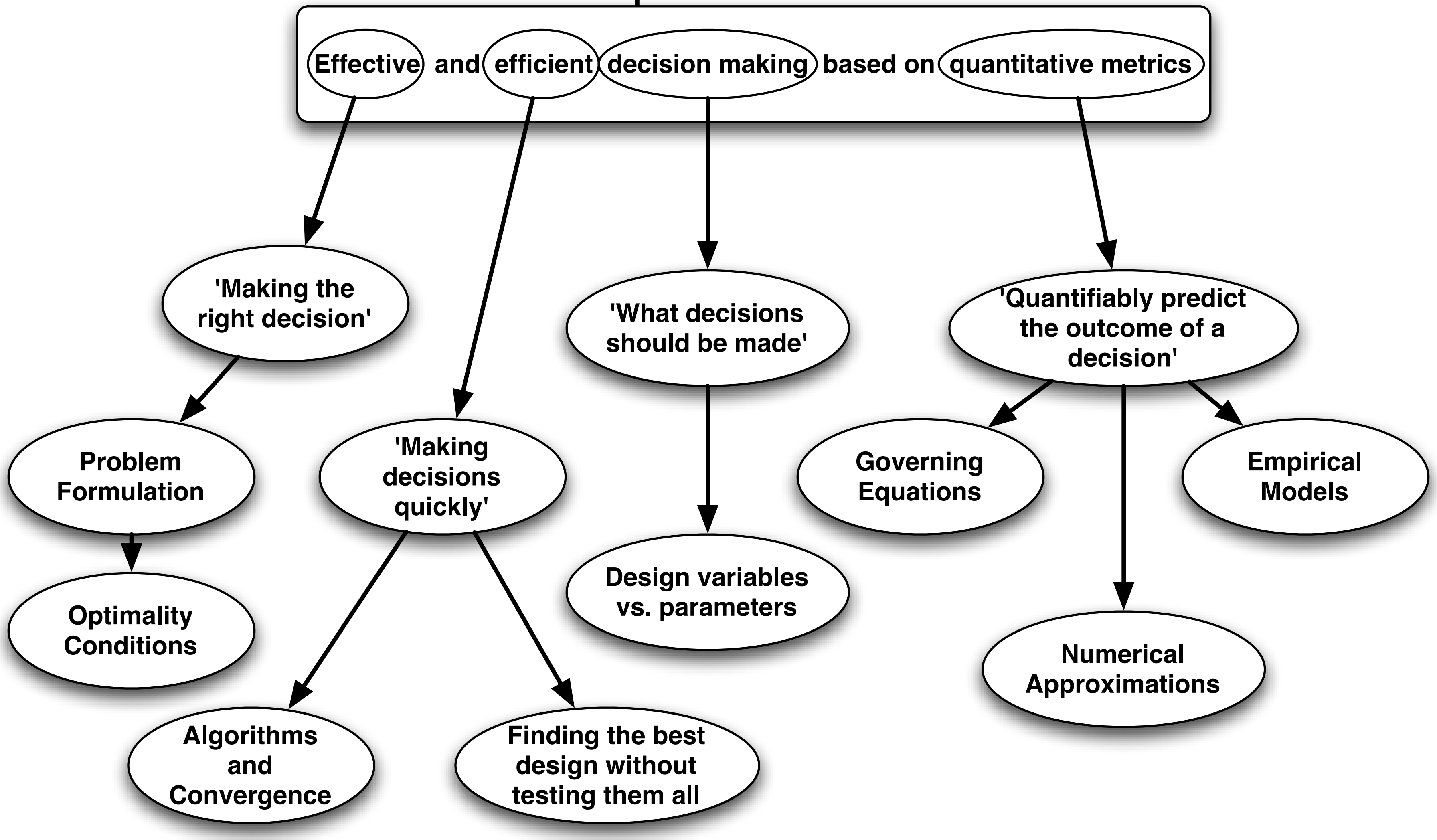

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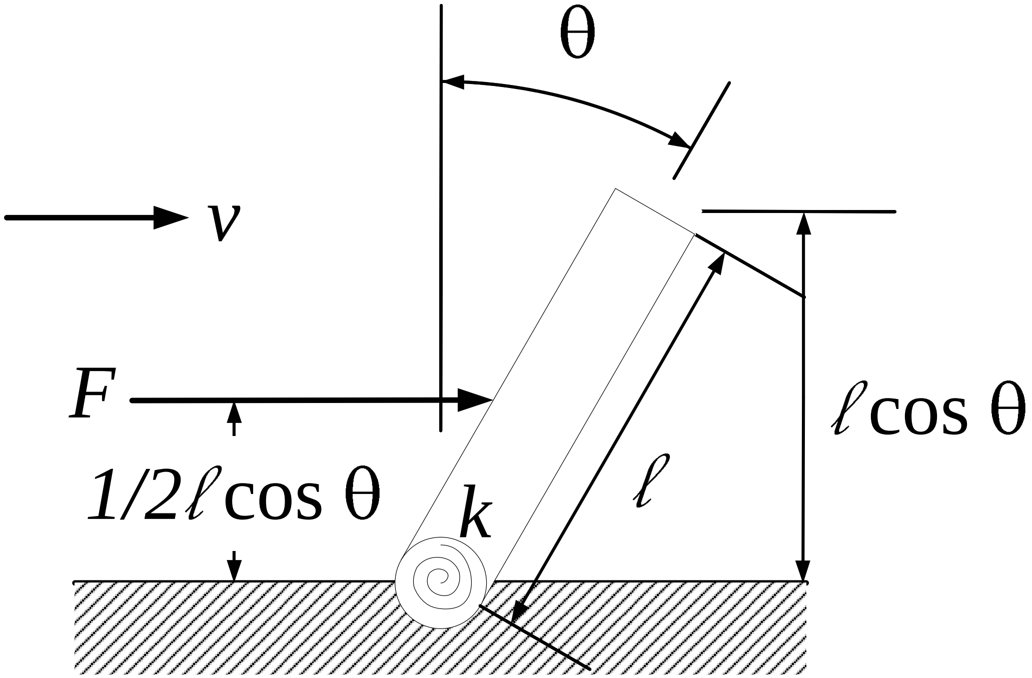
Modeling: Quantification of Designs

- Optimization results are only as good as the model's accuracy
- Must make a tradeoff between computation time and accuracy
- Rough models: preliminary optimization studies (find desirable configuration and reduced set of important design variables)
- Hi-Fidelity models: later stages of design optimization

Example 2: Air Flow Sensor Design



Sensor Calibration Problem

$$
\begin{aligned}
\min_{\ell,\mathbf{w}} \quad & (\theta - \theta^T)^2 \\
\text{subject to} \quad & F - F_{max} \leq 0 \\
& \ell w - A = 0
\end{aligned}
$$

Multidisciplinary Analysis

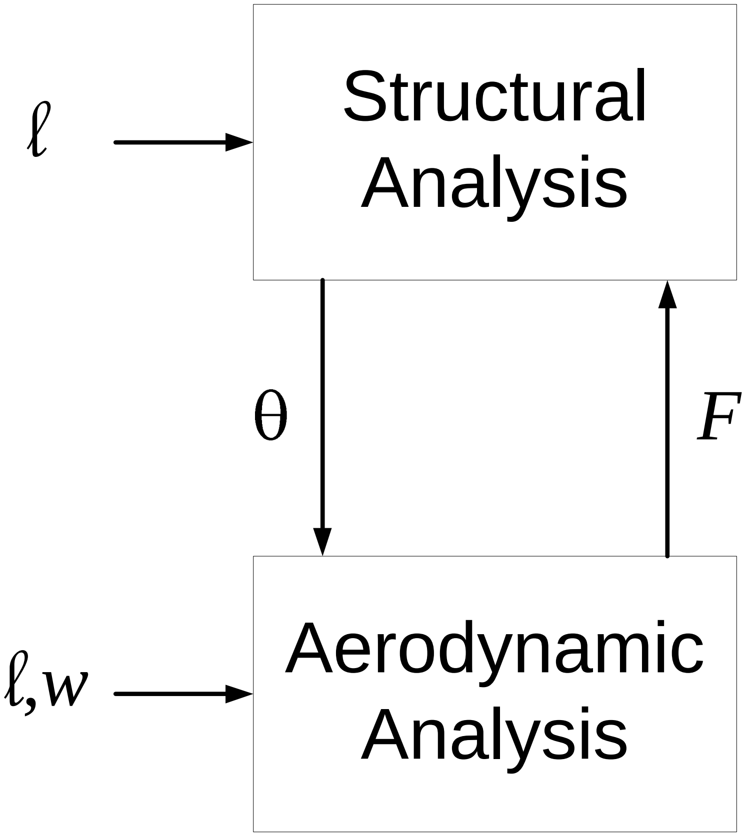
Aerodynamic Analysis:

$$F = CA_f v^2 = C\ell w \cos\theta v^2$$

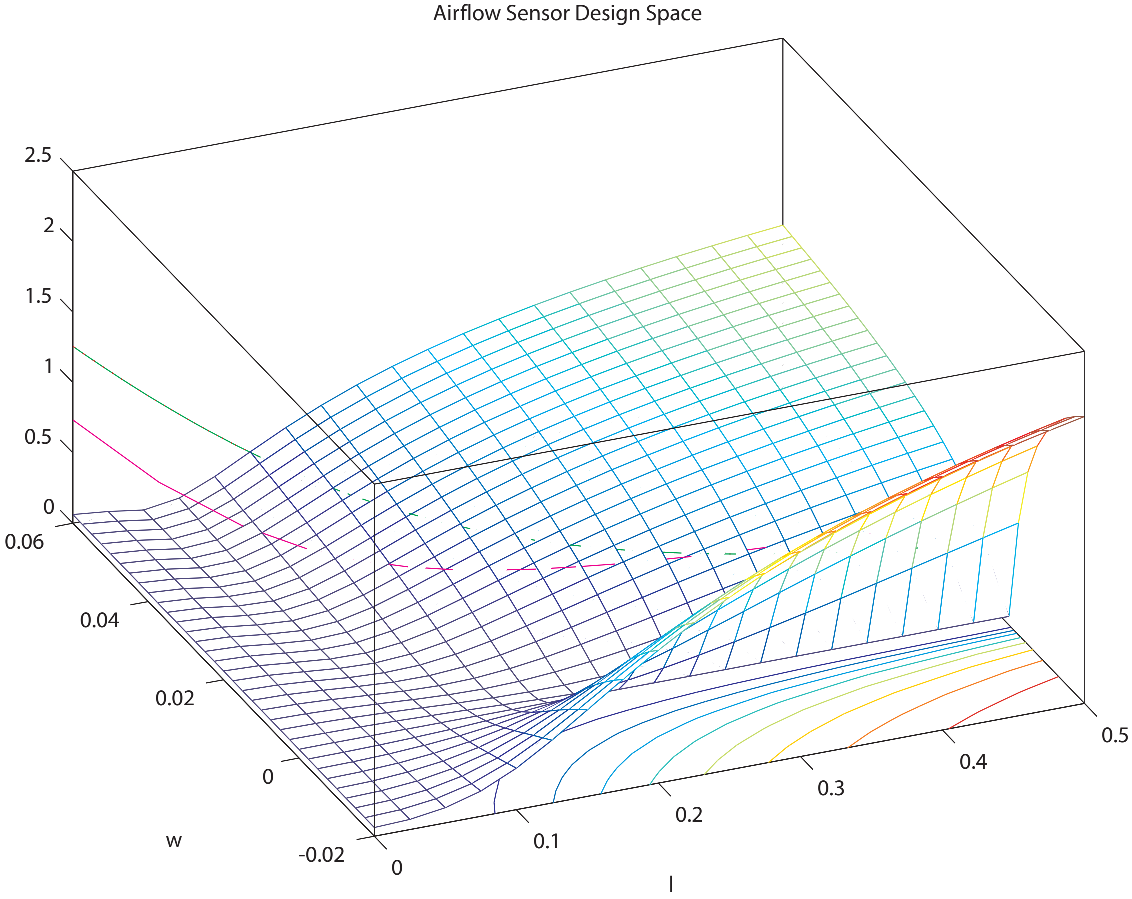
Structural Analysis:

$$k\theta = \frac{1}{2} F\ell \cos\theta$$

System Interdependency



Design Space Visualization



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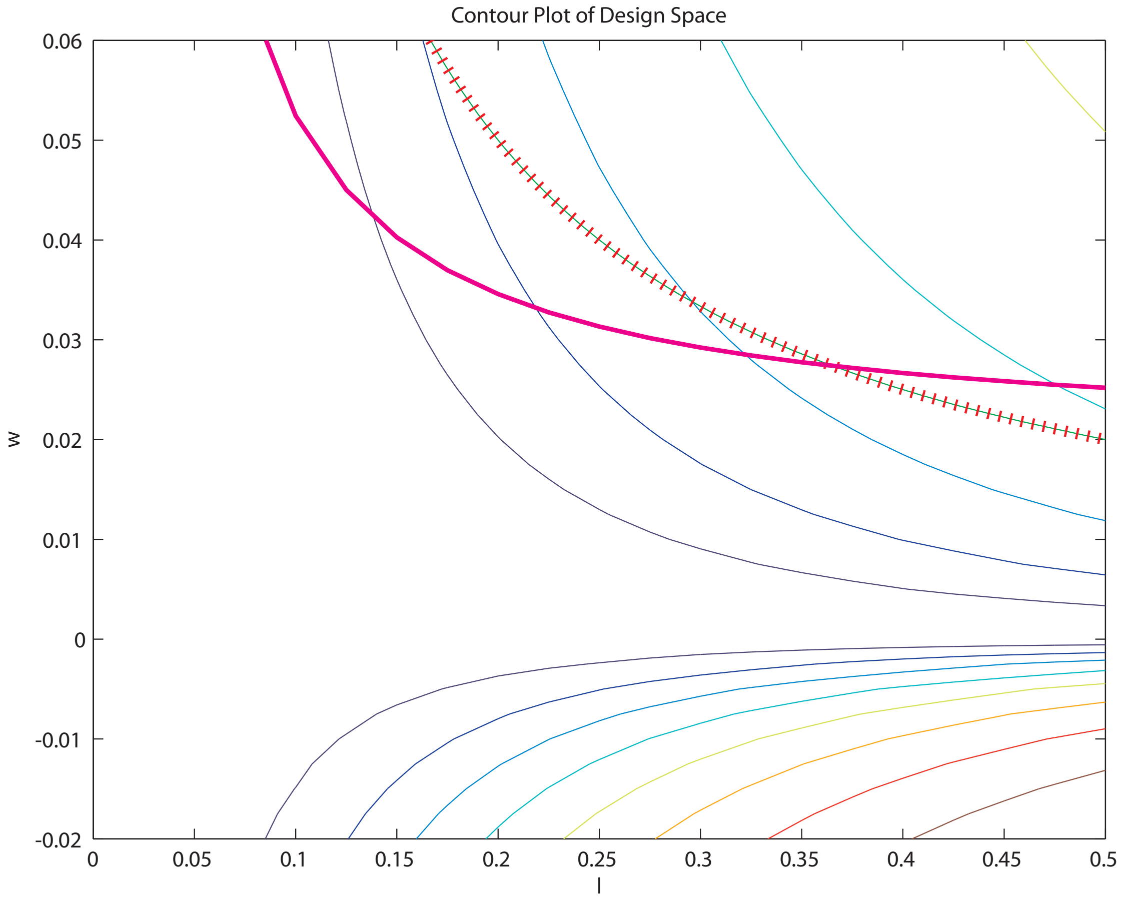

Summary of Basic Topics

Design Optimization has value in aiding us to find the best designs in a small amount of time.

Limitations: The ‘optimal design’ is only optimal with respect to how the problem was formulated, to what values were chosen as design variables, and to the accuracy of the modeling used.

Questions to Ask:

- Objective function accurately reflect what is really wanted?
- Hidden constraints or interactions?
- Is there uncertainty or variation in:
  - Manufacturing processes or supplies?
  - Product usage?
  - Environment the product exists in?
  - Modeling or analysis?

Additional Topics for Discussion

1. Organizations and Economies as optimization processes
2. Complex system optimization
   (a) Analytical Target Cascading: coordinating between system-level and component-level thinking
   (b) Multidisciplinary Design Optimization: integration of many different disciplinary analysis into an overall system optimization
   (c) Software for large-scale optimization: Optimus, iSIGHT

3. Additional optimization applications
    (a) Fitting models to experimental results
    (b) Design of experiments
    (c) Operations Research: military operations planning, airport problem, diet problem, project management
    (d) Manufacturing: optimization of CNC tool paths, tolerance allocation

4. Multi-objective optimization

5. Product family/product platform design

6. Local vs. global optima

7. Generalized reduced gradient method (algorithm used in MS Excel$^{\text{TM}}$)