Topology Optimization of Compliant Suspension Mechanisms

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CSTO Project

- **Objective:**
  - Develop a generalized method for the optimization of a Compliant Suspension Topology.

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CSTO Project

- Overview:
  - Compliant Mechanisms
  - Compliant Suspensions
  - Topology Optimization
  - Case Study
  - Optimization Model
    - Multiple Objective Functions
    - Genetic Algorithms
  - Results
Compliant Mechanisms

- **What is a compliant mechanism?**
  - Device that utilizes elastic deformation of material to produce a desired motion or force transmission instead of rigid linkages and joints.

- **Why use compliant mechanisms?**
  - Fewer parts (easy manufacture/assembly/ good reliability)
  - Stiffness and damping can be built into the mechanism. Energy is stored as strain energy in the material.
  - Larger design space: enables the design of novel mechanisms.
Compliant Suspensions

- Suspensions provide stiffness and usually damping for a motion along a prescribed path. Examples include:
  - Automotive suspensions
  - Industrial machinery
  - Mountain bike suspensions
- Compliant suspensions rely on elastic deformation of the system to provide a desired motion when a particular force is applied.
A traditional suspension uses rigid linkages and pin or slider joints to produce the desired motion.
Compliant Mechanisms

- A compliant suspension flexes instead.

FEA analysis of a compliant suspension that approximates the motion of a four-bar linkage. Von Mises stress (related to strain energy) is indicated by the color code.
A parametric structure design is easily optimized if the topology is known.

minimize \( f(\vec{x}) = \delta_v \) (vertical deflection of truss center)
subject to \( g_i(\vec{x}) \leq 0 \)
where
\[
\begin{align*}
g_1(\vec{x}) &= m_{\text{actual}} - m_{\text{max}} \\
g_2(\vec{x}) &= \sigma_{\text{actual}} - \sigma_{\text{yield}}
\end{align*}
\]

\[
(\vec{x}) = \begin{bmatrix} h \\ t_1 \\ t_2 \\ \vdots \\ t_p \end{bmatrix}, \quad p = \# \text{ of truss members} \\
t_i = \text{thickness of } i^{\text{th}} \text{ member}
\]

\( \vec{x} \in R^+ \)

But how do we know the structure configuration, or ‘topology’ is optimal?
• Determination of the ideal design configuration.
• Performance analyzed with FEA or other tools.

Bracket Example:

• Purely binary problem: no continuous counterpart.
• The binary design variables describe the existence of material in an element of the design space mesh.
Case Study

• Mountain Bike Rear Suspension

Objective: develop a generalized compliant suspension topology optimization method.

• Case Study will facilitate the development of the optimization method.
Case Study

- Primarily rigid link designs
  - Complicated assembly, multiple joints, expensive, and heavy
- Air shock provides stiffness and damping
Case Study

- Replace rear triangle with a monolithic compliant system
- Follow circular deflection path

$$y = \sqrt{Length^2 - x^2}$$

Region Modeled Rear Wheel Hub
Case Study

- Simplifying assumptions made initially to develop basic method:
  - 2D model (torsion and out-of-plane forces ignored)
  - Rough mesh (must be scalable)
  - Neglect stress initially.
  - Ignore buckling and fatigue.

- Concentrate on path accuracy, system mass, and longitudinal rigidity.
Dealing with multiple objective functions

- Create a composite objective function
  \[ f(\bar{x}) = f_1(\bar{x})w_1 + f_2(\bar{x})w_2 + f_3(\bar{x})w_3 \]

- Select one objective function, and convert the remaining functions to constraints
Optimization Model

- Exploring Tradeoffs: Pareto Surfaces
- Selecting points on the efficient surface.
Optimization Model

- Genetic Algorithms
  - Heuristic (random)
  - Well suited for binary problems
  - Based on survival of the fittest: evolves into best design

1. Set algorithm parameters
2. Generate initial population of random designs
3. Evaluate performance of all designs
4. Produce new population
Optimization Model

- Genetic Operators: Producing the next generation
  - Crossover: survival of the fittest
    Design A: 0 1 1 0 0 1
    Design B: 1 0 0 0 1 0
  - Mutation: maintaining diversity
    Generation 2: 1 0 0 1 1 0
Mathematical Model

\[ f(\bar{x}, t) = f_1(\bar{x}, t)w_1 + f_2(\bar{x}, t)w_2 + f_3(\bar{x}, t)w_3 \]

\[ f_2(\bar{x}, t) = \text{system volume (length}^3, \infty \text{ mass)} \]

\[ g(\bar{x}, t) = \left| \delta_{\text{max}} - \delta_{\text{desired}} \right| = 0 \]

subject to: \[ \sigma(\bar{x}, t) = 0 \]

where: \[ x_i \in \{0,1\}, i = 1,2...n \]

\[ t \in \mathbb{R}^+ \]

\[ x_i \in \mathbb{R} \]

\[ t \in \mathbb{R}^+ \]

\[ w_i \in \mathbb{R}^+, i = 1,2,3 \]
Connectivity between rear hub and seat tube:
- Path method guarantees connectivity
- Element method allows disconnected designs
Optimization: Path Method

- 3 X 3 matrix
- 113 paths
  - Guaranteed Connectivity
- Tendency towards dense meshes
  - Difficult to maintain a diverse population
  - Mutation and crossover furthered the problem of densification
Optimization: Element Method

- Enables mesh refinement
  - 18 Variables for a 3 X 3 mesh
  - Only 39 for a 4 X 4 mesh
    (20,596 for path method)
- Much higher probability of attaining a more sparse graph
- Comparable GA run-time
  - (for 3x3)
- This method chosen and used for topology optimization

Best Element Method Design

23% Better Performance
Utilized functions from Matlab GA Toolbox (www.shef.ac.uk/%7Egaipp/ga-toolbox)

Utilized functions from NLFET (www.nlfet.sourceforge.net)
Design Evolution
Tuning the GA

- Population Size: 60 – 80
- Generations: ≈50
- Cross-over Probability: ≈90%
- Mutation Probability: ≈1%

Convergence of a Particular GA Run

\[
\text{Generation} \quad \text{Log}_{10}(f(x))
\]

\[
\begin{array}{c}
0.0 \\
0.05 \\
0.1 \\
0.15 \\
0.2 \\
0.25 \\
0.3 \\
0.35 \\
0.4 \\
0.45 \\
0.5 \\
0.55 \\
0.6 \\
0.65 \\
0.7 \\
0.75 \\
0.8 \\
0.85 \\
0.9 \\
0.95 \\
1.0
\end{array}
\]

\[
\begin{array}{c}
10 \\
20 \\
30 \\
40 \\
50 \\
60 \\
70 \\
80 \\
90 \\
0
\end{array}
\]
Optimization Results: Pareto Surface

Pareto set generated by 97 different GA runs. (run time = 5 days)
Optimization Results:

Different Statistical Selection Methods

Design Found by $\min(\sum(\text{variance}))$ Method

Design Found by $\min(\max(\text{variance}))$ Method

This is the better design when considering aesthetics and ergonomics.
Discussion of Results

- **Confirmed design analysis with Ansys**
  - MATLAB code within ≈ 2.5%

- **Future work:**
  - Improve mesh resolution
  - Consider 3D problem
  - Consider stress, buckling, fatigue
  - Continuous optimization on beam thicknesses and node locations
  - Non-linear analysis
  - Damping and dynamic response

ANSYS Diagram of Axial Stress
Questions will now be addressed.

More information will soon be available at:

www.umich.edu/~jtalliso

Direct further questions to:

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