Topology Optimization of Compliant Suspension Mechanisms

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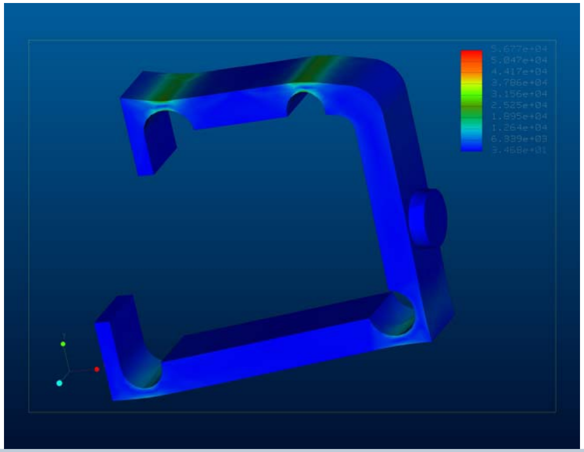


CSTO Project

- Objective:
  - Develop a generalized method for the optimization of a Compliant Suspension Topology.
- Contributors:
  - James Allison (Optimal Design Laboratory)
  - Michael Cherry (Compliant System Design Laboratory)
  - Zachary Kreiner (Compliant System Design Laboratory)

CSTO Project

- Overview:
  - Compliant Mechanisms
  - Compliant Suspensions
  - Topology Optimization
  - Case Study
  - Optimization Model
    - Multiple Objective Functions
    - Genetic Algorithms
  - Results

Compliant Mechanisms

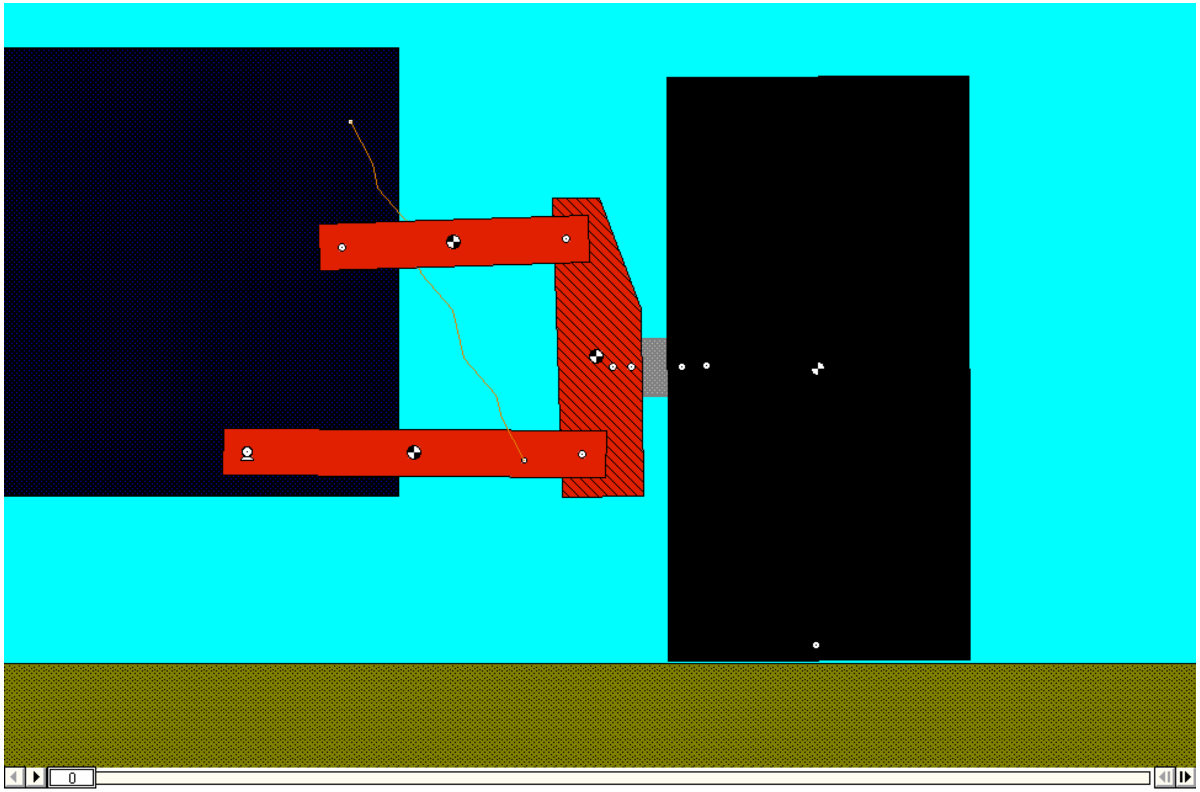
- What is a compliant mechanism?
  - Device that utilizes elastic deformation of material to produce a desired motion or force transmission instead of rigid linkages and joints.
- Why use compliant mechanisms?
  - Fewer parts (easy manufacture/assembly/ good reliability)
  - Stiffness and damping can be built into the mechanism. Energy is stored as strain energy in the material.
  - Larger design space: enables the design of novel mechanisms.

Compliant Suspensions

- Suspensions provide stiffness and usually damping for a motion along a prescribed path. Examples include:
    - Automotive suspensions
    - Industrial machinery
    - Mountain bike suspensions
- Compliant suspensions rely on elastic deformation of the system to provide a desired motion when a particular force is applied.

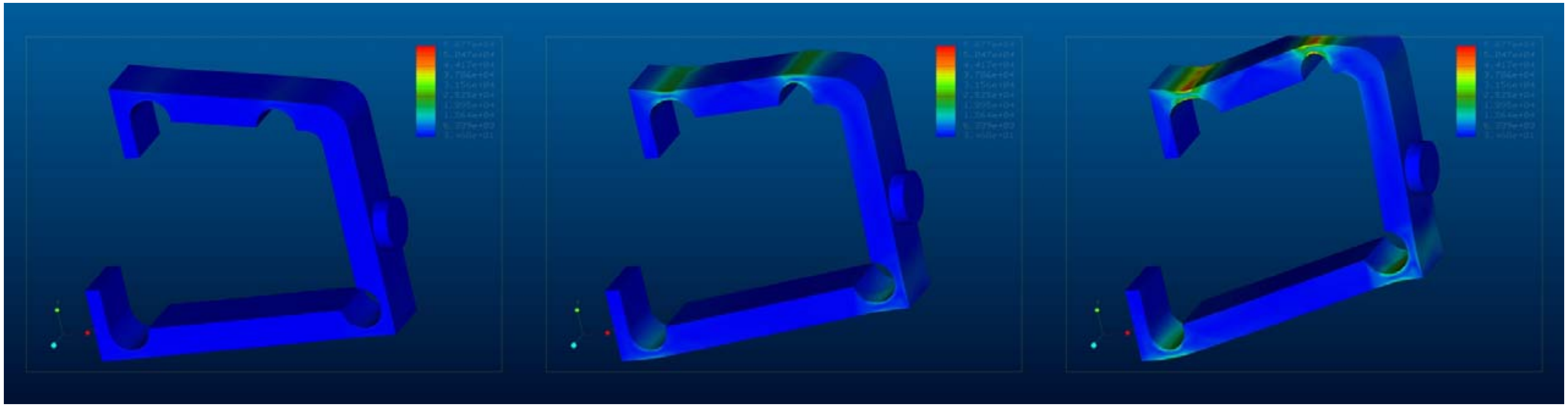
Compliant Mechanisms

- A traditional suspension uses rigid linkages and pin or slider joints to produce the desired motion.



Compliant Mechanisms

- A compliant suspension flexes instead.



Undeformed

Partially deformed

Fully deformed

FEA analysis of a compliant suspension that approximates the motion of a four-bar linkage. Von Mises stress (related to strain energy) is indicated by the color code.

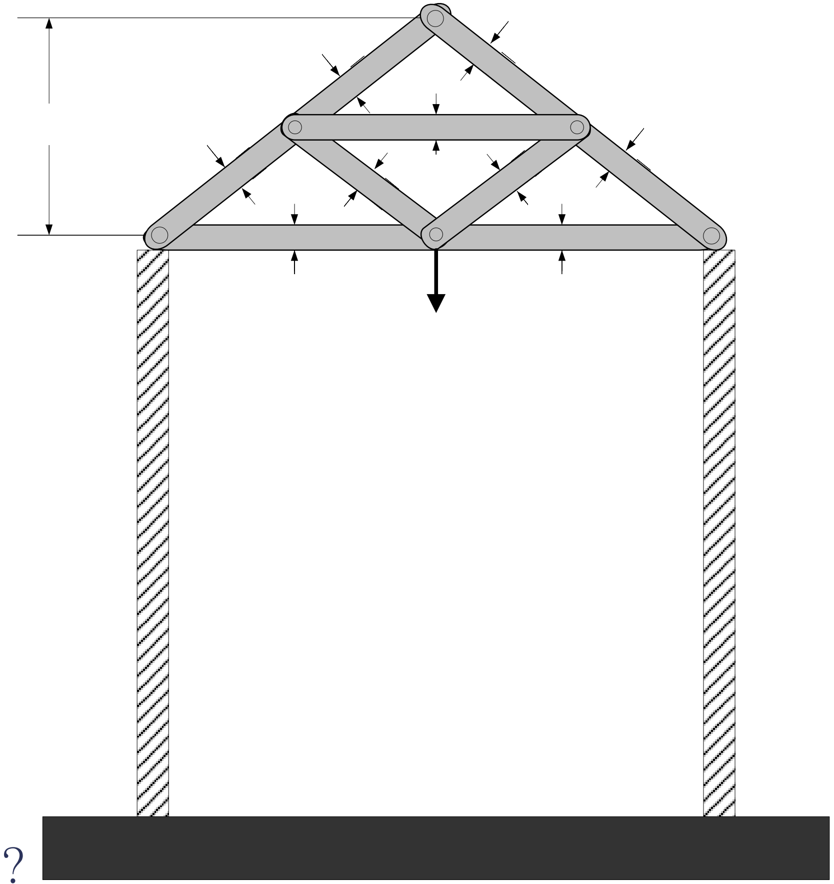
Topology Optimization

- A parametric structure design is easily optimized if the topology is known.

$$\begin{aligned} &\text{minimize} && f(\bar{x}) = \delta_c \text{ (vertical deflection of truss center)} \\ &\text{subject to} && g_i(\bar{x}) \le 0 \\ &\text{where} && g_1(\bar{x}) = m_{actual} - m_{\max} \\ & && g_2(\bar{x}) = \sigma_{actual} - \sigma_{yield} \end{aligned}$$

$$(\bar{x}) = \begin{Bmatrix} h \\ t_1 \\ t_2 \\ \vdots \\ t_p \end{Bmatrix} \quad \begin{aligned} p &= \#\text{ of truss members} \\ t_i &= \text{thickness of i}^{\text{th}} \text{ member} \end{aligned}$$

$$\bar{x} \in R^+$$

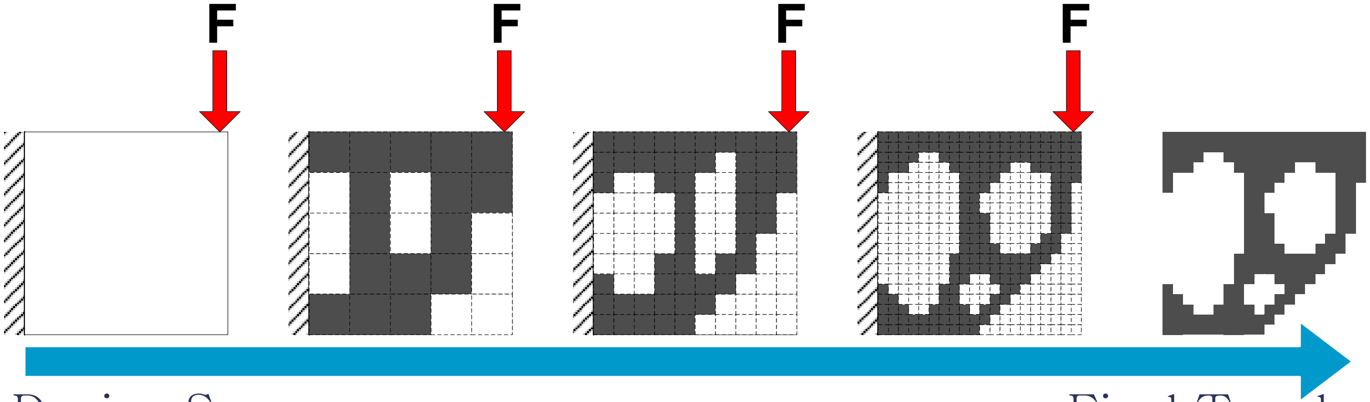


But how do we know the structure configuration, or ‘topology’ is optimal?

Topology Optimization

- Determination of the ideal design configuration.
- Performance analyzed with FEA or other tools.

Bracket Example:



Design Space

Final Topology

- Purely binary problem: no continuous counterpart.
- The binary design variables describe the existence of material in an element of the design space mesh.

Case Study

- Mountain Bike Rear Suspension

Objective: develop a generalized compliant suspension topology optimization method.

- Case Study will facilitate the development of the optimization method

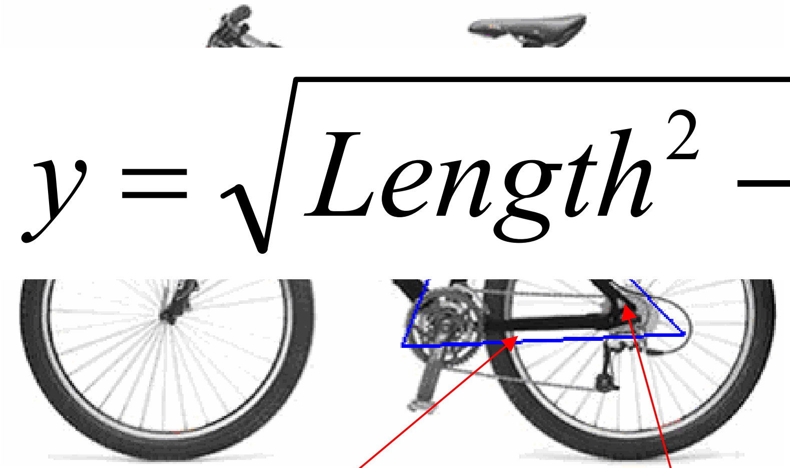
Case Study

- Primarily rigid link designs
  - Complicated assembly, multiple joints, expensive, and heavy
- Air shock provides stiffness and damping

Case Study

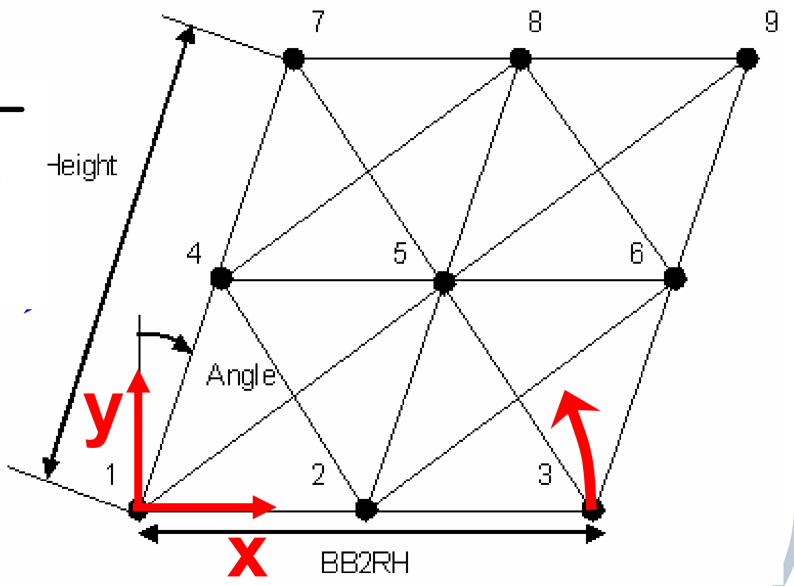
- Replace rear triangle with a monolithic compliant system
- Follow circular deflection path

$$y = \sqrt{Length^2 - x^2}$$



Region Modeled

Rear Wheel Hub



Case Study

- Simplifying assumptions made initially to develop basic method:
  - 2D model (torsion and out-of-plane forces ignored)
  - Rough mesh (must be scalable)
  - Neglect stress initially.
  - Ignore buckling and fatigue.
- Concentrate on *path accuracy*, *system mass*, and *longitudinal rigidity*.

Optimization Model

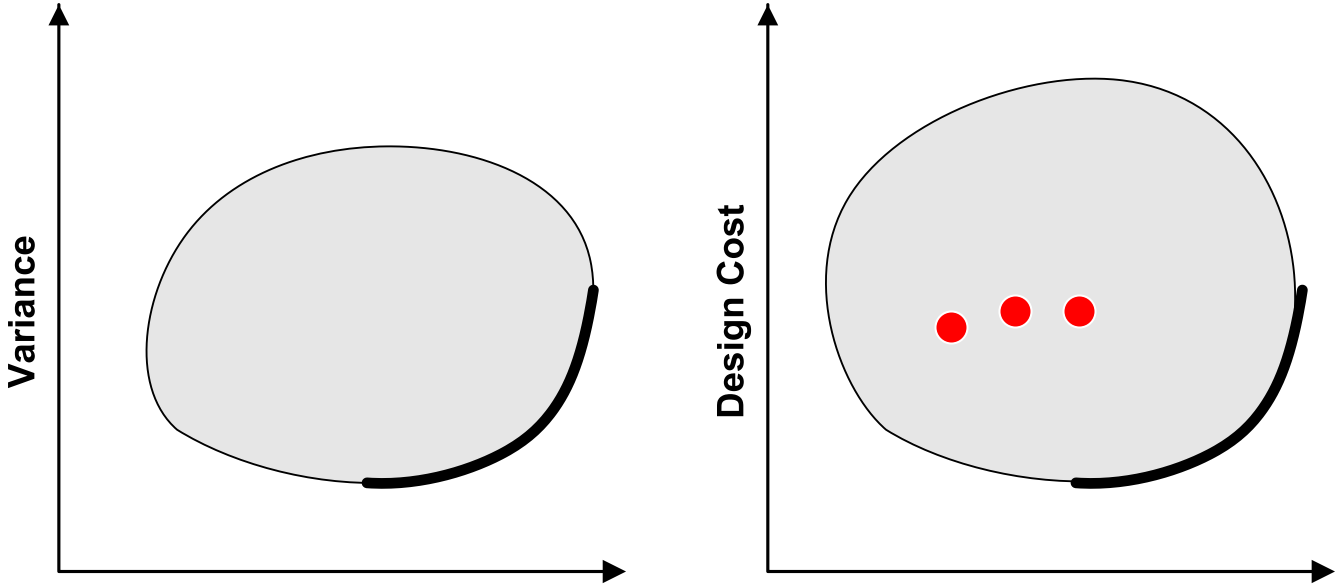
- Dealing with multiple objective functions
  - Create a composite objective function

$$f\left(\overline{x}\right)=f_1\left(\overline{x}\right)w_1+f_2\left(\overline{x}\right)w_2+f_3\left(\overline{x}\right)w_3$$

  - Select one objective function, and convert the remaining functions to constraints

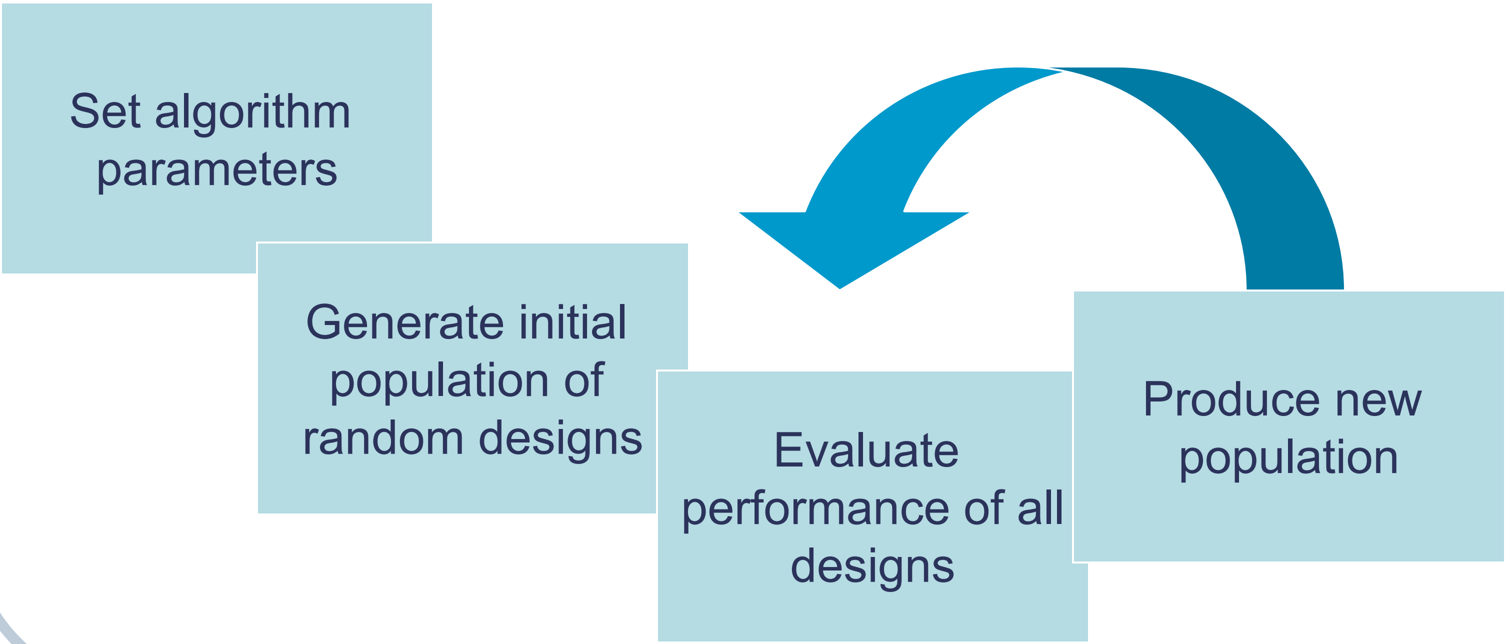
Optimization Model

- Exploring Tradeoffs: Pareto Surfaces
- Selecting points on the efficient surface.



Optimization Model

- Genetic Algorithms
  - Heuristic (random)
  - Well suited for binary problems
  - Based on survival of the fittest: evolves into best design



Set algorithm parameters

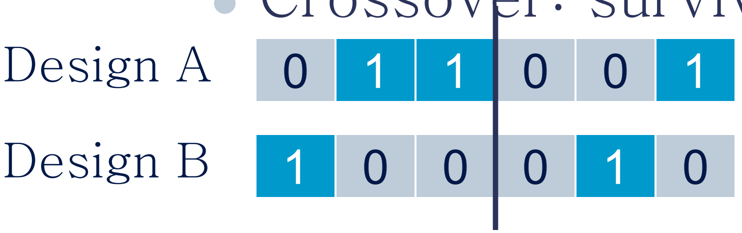
Generate initial population of random designs

Evaluate performance of all designs

Produce new population

Optimization Model

- Genetic Operators: Producing the next generation
  - Crossover: survival of the fittest



Design A	0	1	1	0	0	1
Design B	1	0	0	0	1	0

Generation 2

  - Mutation: maintaining diversity

1	0	0	1	1	0

Mathematical Model

$$f(\bar{x},t) = f_1(\bar{x},t)w_1 + f_2(\bar{x},t)w_2 + f_3(\bar{x},t)w_3$$

$f_2(\bar{x},t) =$ system volume $(length^3, \propto mass)$

$$g(\bar{x},t) = \left|\delta_{\max} - \delta_{\text{desired}}\right| = 0$$

subject to: $g(\bar{x},t) = 0$

where $x_i \in \{0,1\}, i = 1,2\ldots n$

ub node

f hub node

$$t \in \mathbf{R}^+$$

$x_i \in \{0$

$t \in \mathbb{R}^+$

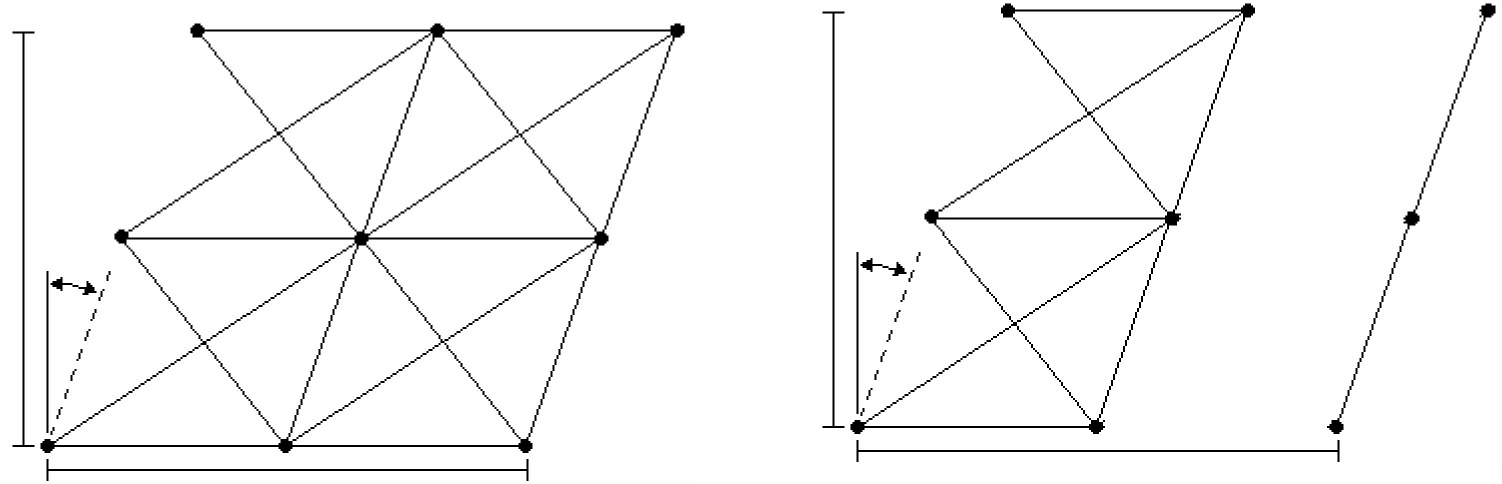
$w_i \in \mathbb{R}$

n)

$$w_i \in \mathbf{R}^+, i = 1,2,3$$

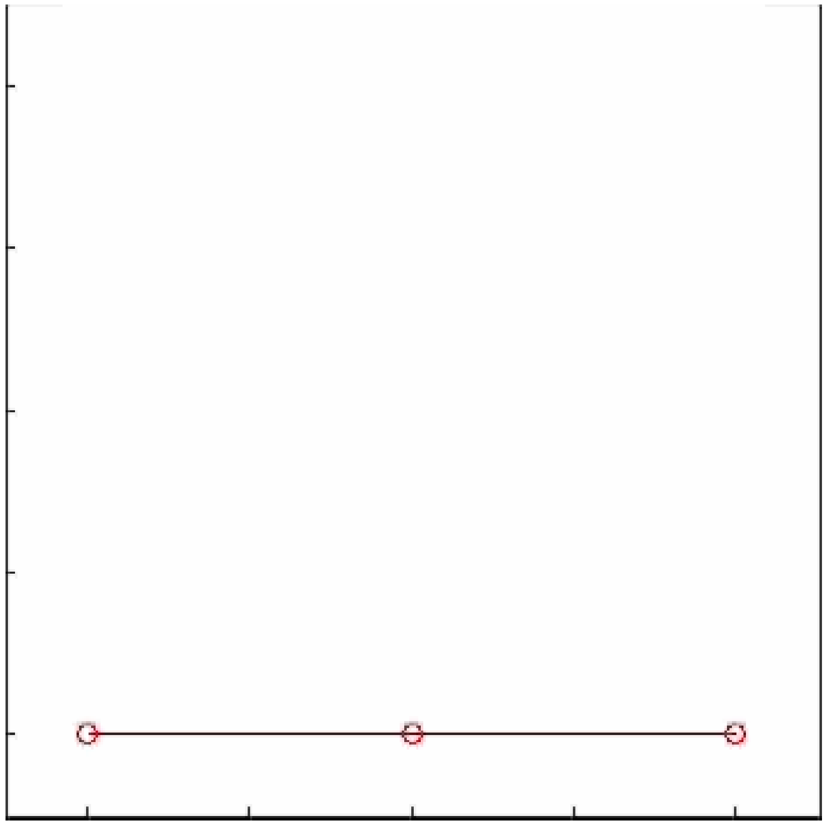
Optimization Model

- Connectivity between rear hub and seat tube
  - Path method guarantees connectivity
  - Element method allows disconnected designs



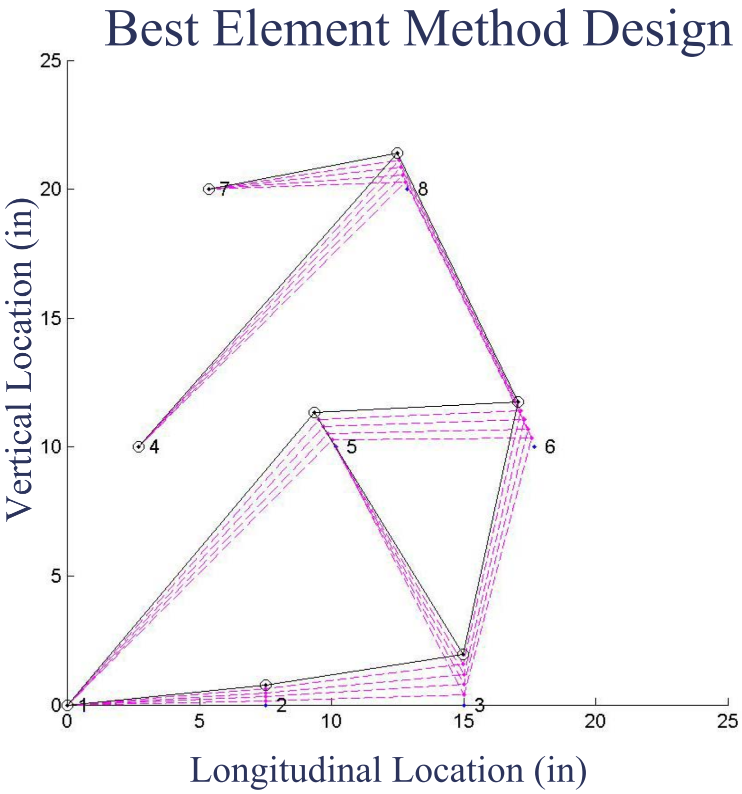
Optimization: Path Method

- 3 X 3 matrix
- 113 paths
  - Guaranteed Connectivity
- Tendency towards dense meshes
  - Difficult to maintain a diverse population
  - Mutation and crossover furthered the problem of densification



Optimization: Element Method

- Enables mesh refinement
  - 18 Variables for a 3 X 3 mesh
  - Only 39 for a 4 X 4 mesh
    (20,596 for path method)
- Much higher probability of attaining a more sparse graph
- Comparable GA run-time
  - (for 3x3)
- This method chosen and used for topology optimization



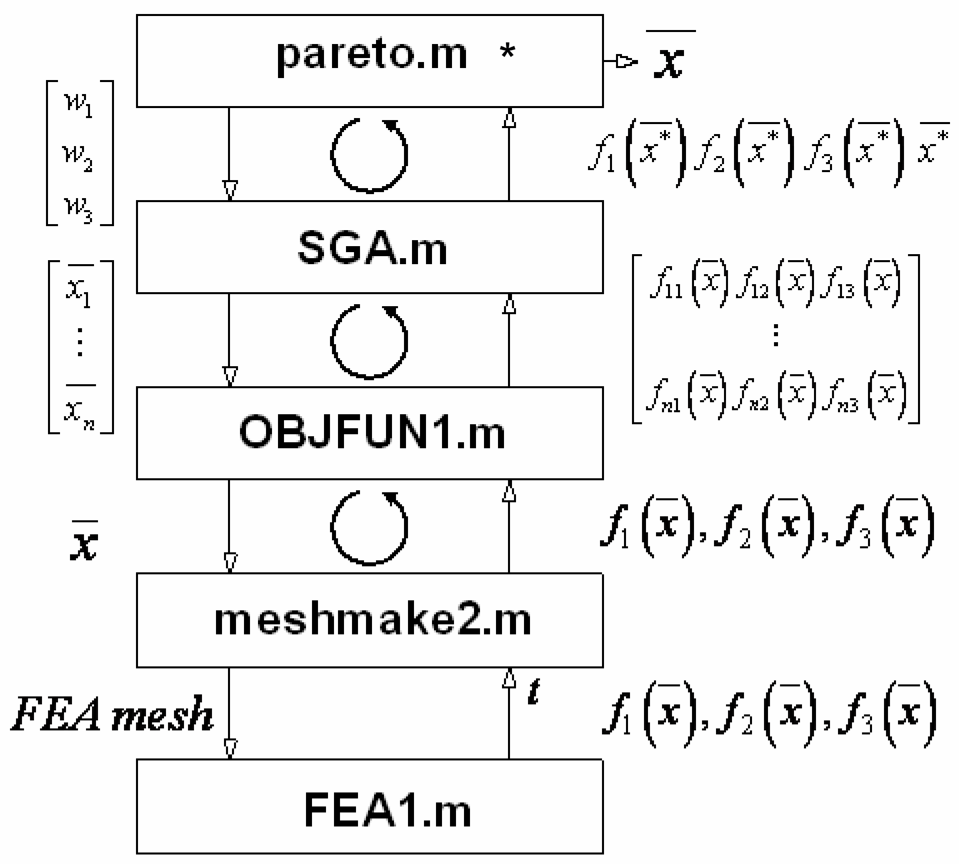
Best Element Method Design

23% Better Performance

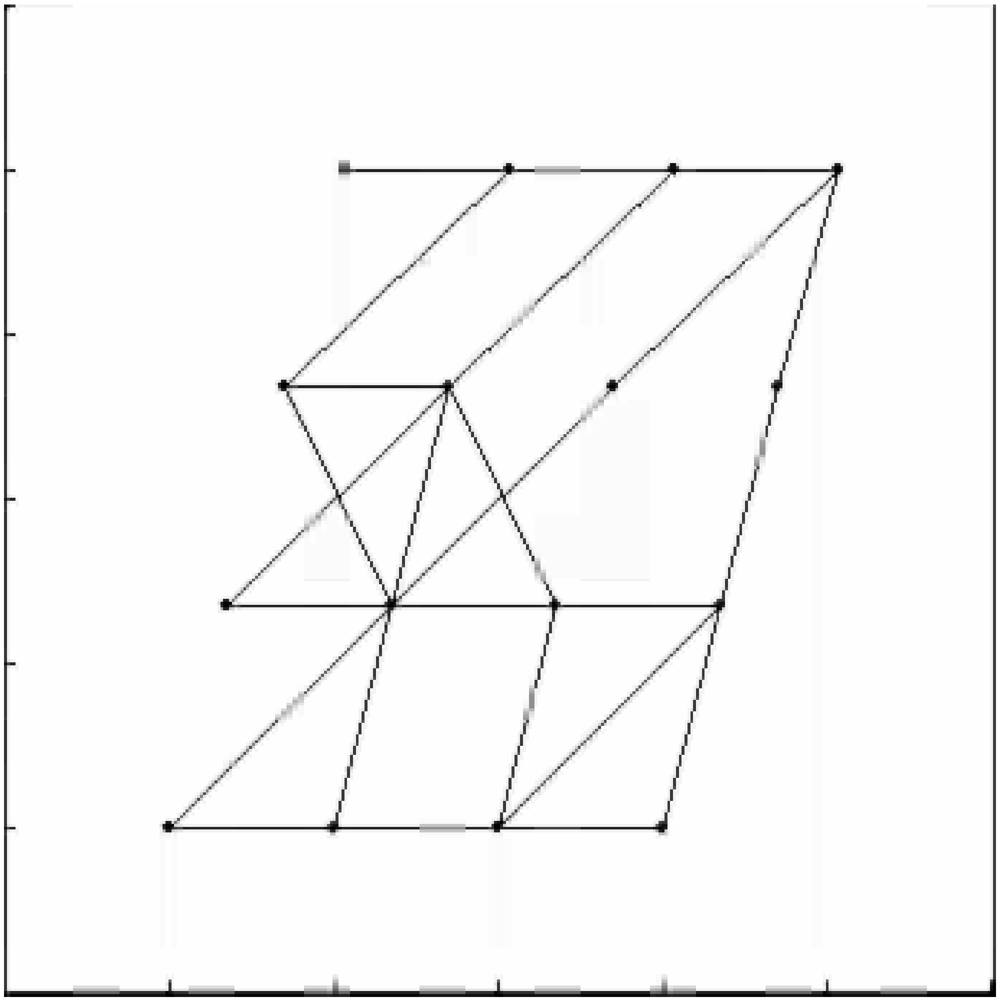
Optimization Model

Utilized functions from Matlab GA Toolbox (www.shef.ac.uk/%7Egaipp/ga-toolbox)

Utilized functions from NLFET (www.nlfet.sourceforge.net)

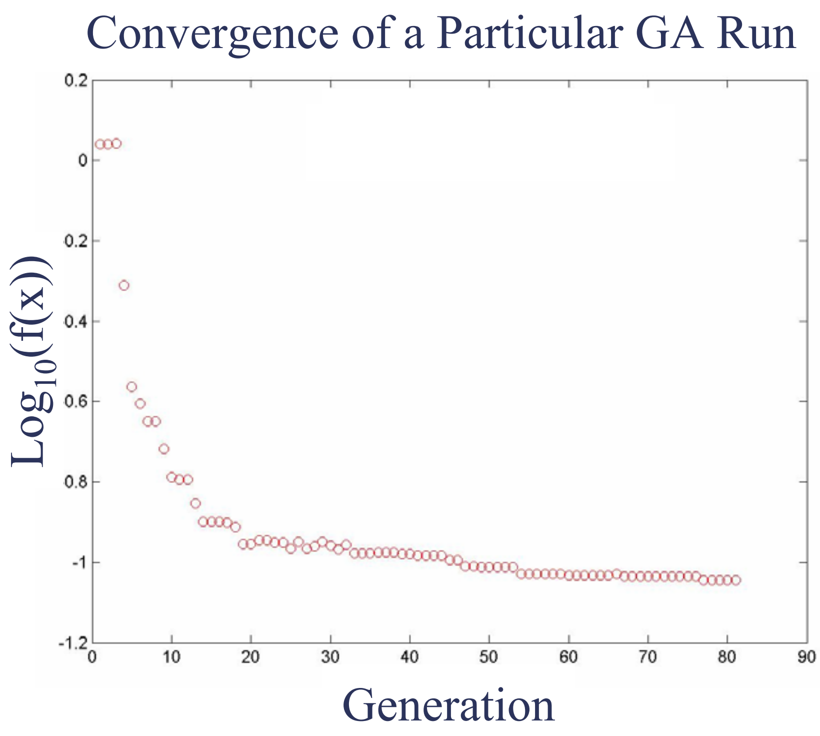


Design Evolution

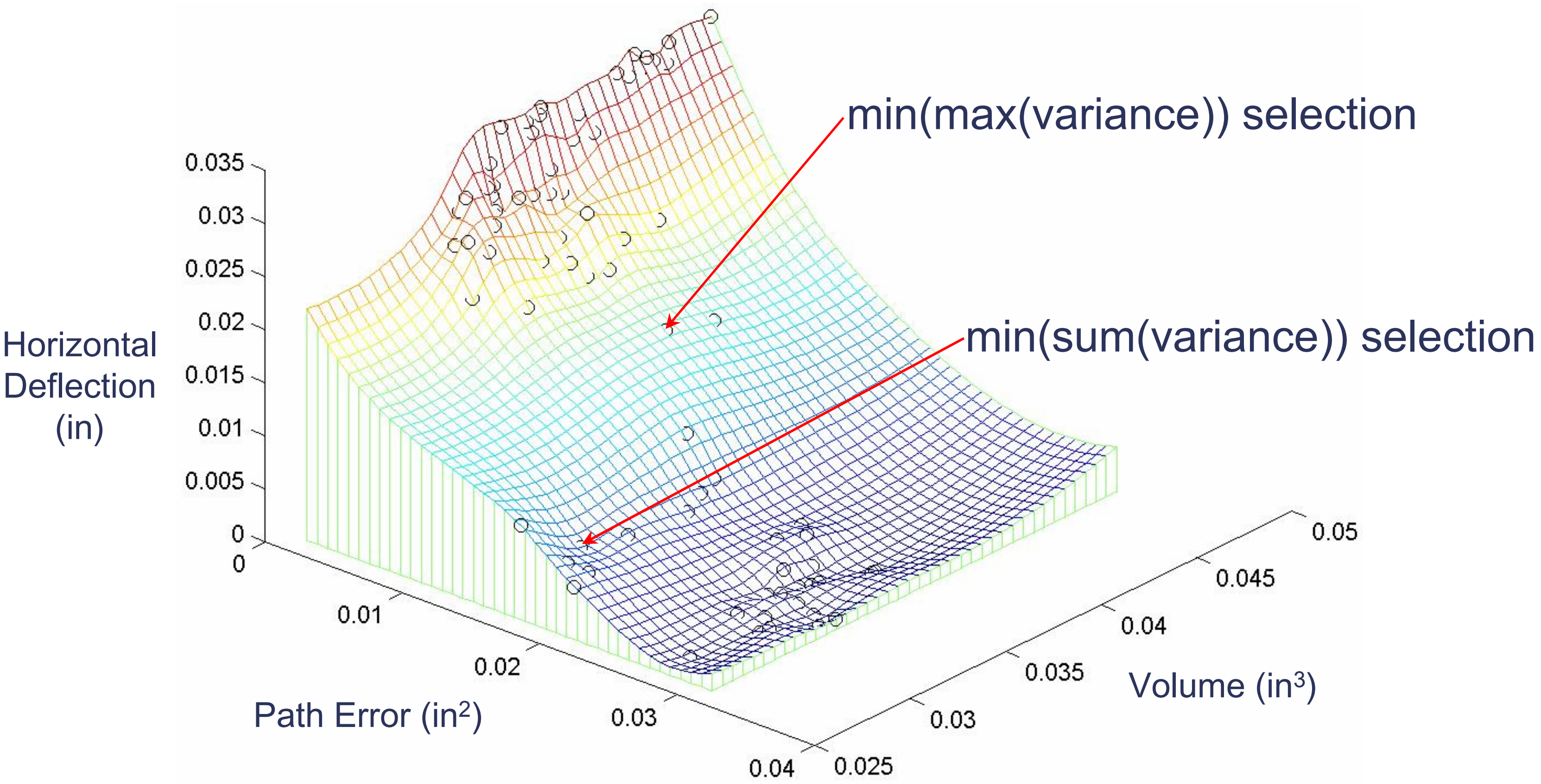


Tuning the GA

- Population Size: 60 – 80
- Generations: ≈50
- Cross-over Probability: ≈90%
- Mutation Probability: ≈1%



Optimization Results: Pareto Surface

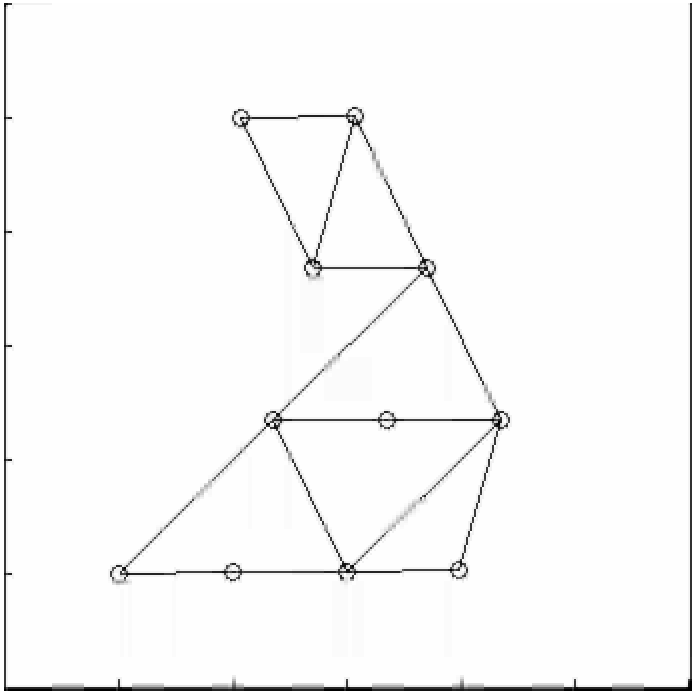


Pareto set generated by 97 different GA runs. (run time = 5 days)

Optimization Results:

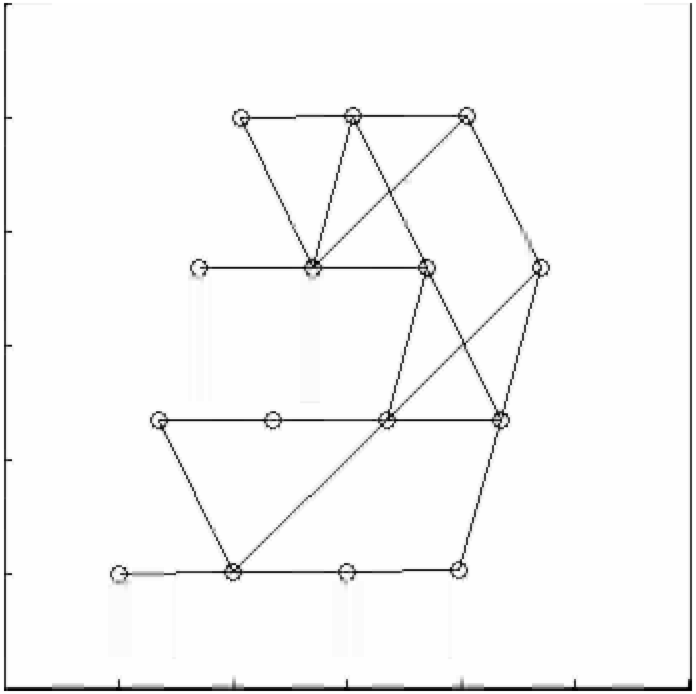
Different Statistical Selection Methods

Design Found by min(sum(variance)) Method



This is the better design when considering aesthetics and ergonomics.

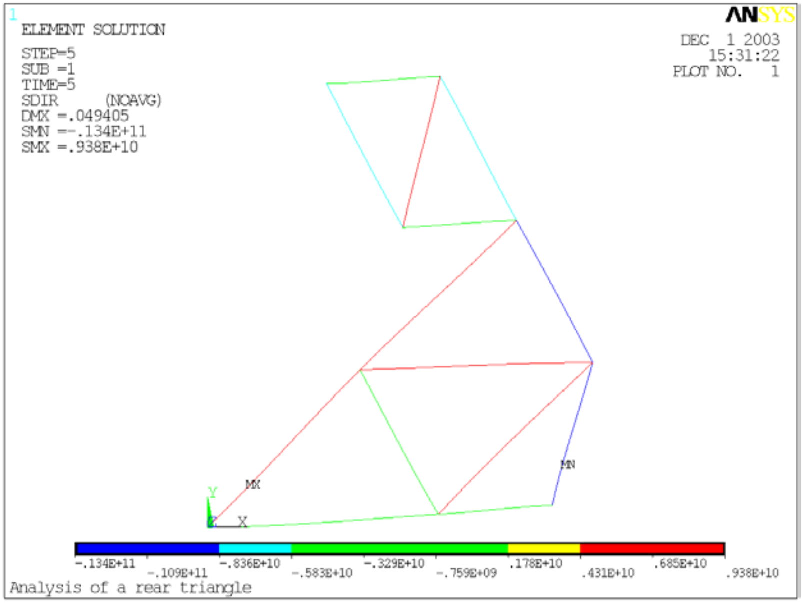
Design Found by min(max(variance)) Method



Discussion of Results

- Confirmed design analysis with Ansys
  - MATLAB code within ≈ 2.5%
- Future work:
  - Improve mesh resolution
  - Consider 3D problem
  - Consider stress, buckling, fatigue
  - Continuous optimization on beam thicknesses and node locations
  - Non-linear analysis
  - Damping and dynamic response

ANSYS Diagram of Axial Stress



Questions will now be addressed.

More information will soon be available at:

www.umich.edu/~jtalliso

Direct further questions to:

optimize@umich.edu