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A comment on the role of prices for excludable public goods

Gilbert E. Metcalf · Jongsang Park

Abstract Blomquist and Christensen [(2005). The role of prices for excludable public goods, *International Tax and Public Finance*, *12*, 61–79] argue that welfare is initially decreasing in the price of an excludable public good and that the case for a positive price for an excludable public good price is weak. We argue that this result follows from their particular characterization of the public good and that an alternative and equally reasonable characterization overturns their result. Hence, the policy case for a positive price on the public good is stronger than Blomquist and Christiansen suggest.

Keywords Public goods · Optimal second-best taxation

JEL Classification H21 · H41

1 Introduction

In a recent article, Blomquist and Christiansen (2005) argue that a necessary condition to charge a price for an excludable public good to achieve Pareto efficiency in the presence of an optimal nonlinear income tax is that the marginal valuation of the public good be increasing in leisure. Furthermore, they argue that given this condition, welfare is initially nonincreasing in the price of the public good (and may decrease before possibly increasing). Thus, the authors argue that if it is uncertain what the

G. E. Metcalf (✉)
Department of Economics, Tufts University, Medford, MA 02155, USA;
National Bureau of Economic Research, 1050 Massachusetts Avenue, Cambridge, MA 02138, USA
e-mail: gmetcalf@tufts.edu

J. Park
Department of Economics, University of Michigan, 611 Tappan St., Ann Arbor, MI 48109-1220, USA

optimal second-best price of the public good is, it may be better to set a zero price than to set a low price in hopes of avoiding overshooting the optimal price.

We argue that this result follows from their particular characterization of the public good and that with an alternative and equally reasonable assumption, the condition that the marginal valuation of the public good is increasing in leisure is both necessary and sufficient to set a positive price for the excludable public good. Hence, the policy case for a positive price on the public good is stronger than Blomquist and Christiansen suggest.

In the next section, we set up and solve the model. In the following section, we explain the difference in results between our model and that of Blomquist and Christiansen and discuss different ways to characterize excludable public good access. In Section 4, we discuss the form of the Samuelson rule in our model. We conclude in Section 5.

Blomquist and Christiansen (henceforth B&C) model the government as providing G units of an excludable public good to which individuals can obtain access by paying a per unit price q. Upon payment of qg, they may consume g units of the public good with the constraint that $g \leq G$. They motivate this characterization of excludable public goods with such examples as weather forecasts with g measuring the amount of information the consumer purchases up to the maximal amount available; an art gallery with g measuring the number of rooms visited and G the total number of rooms available; and TV broadcasting where g measures the number of channels purchased and G the total number available.

Implicit in B&C's model is the restriction that consumers can purchase an excludable public good only once. Alternatively, we assume that consumers may enjoy the public good repeatedly, thereby allowing the government to charge a fee per use of the public good. An online weather service could charge a fee each time an individual wished to access up-to-date weather information; an art gallery could charge an entrance fee per visit; and a TV network could offer on-demand movies. This strikes us as a reasonable characterization of many excludable public goods (public parks, uncongested highways, museums, for example).

We shall see that the capacity constraint plays a key role in B&C's model. Excludable public goods can be charged on a per use basis or with all-or-nothing pricing (access pricing). B&C's model appears to be an example of per-unit pricing since different consumers may pay different amounts for the public good (depending on demand). But, in fact, their model is better understood as a model of multiple public goods with access pricing once the capacity constraint is binding.

B&C's modeling of public goods may well be appropriate in certain cases. Since B&G conclude that only a weak case can be made for charging for excludable public goods, however, it is important to determine if their conclusion is robust to the modeling of the public good. We turn now to that question.

2 Optimal pricing of excludable public goods

Following B&C, we assume two types of consumers (high ability and low ability) and that the government utilizes a nonlinear income tax to finance the public good. Type i consumers obtain utility over a consumption good, c^i, the public good, g^i, and leisure, L^i. They face a time endowment $Z = H^i + L^i$ where H^i is the number of hours worked. Their income is $Y^i = H^i \cdot w^i$, where w^i is their wage rate ($w^2 > w^1$). The government observes Y^i but not hours worked or the wage rate separately. Also, as in B&C, the government is decentralized in that the tax agency cannot share income information with the agency providing the public good.

While those assumptions follow B&G, we now deviate by assuming that the public good consumption by type i consumers is a function of the number of visits to (or utilizations of) the public good, v^i, and the amount of the public good provided, G:

$$g^i = g^i(v^i, G), \tag{1}$$

where the function g^i has the following properties:

$$g^i(0, G) = g^i(v^i, 0) = 0 \tag{2a}$$

$$\frac{\partial g^i}{\partial G} > 0 \tag{2b}$$

$$\frac{\partial g^i}{\partial v^i} > 0 \tag{2c}$$

$$\frac{\partial^2 g^i}{\partial G \partial v^i} > 0. \tag{2d}$$

Equation (2a) says that no public good consumption can occur unless the government provides some positive amount of the good (G) and the consumer uses the public good (v^i). Equations (2b) and (2c) state that public good consumption increases with use and the amount of the good provided by the government. The last equation states that the marginal consumption value of visits to the public good increases with the amount of the public good provided. Without loss of generality, we make the further simplifying assumption that public good consumption, g^i, is linear in visits (or utilization).[1] With that assumption, the function $g^i(v^i, G)$ takes the form $v^i \cdot h(G)$ with $h(0) = 0$ and $h'(G) > 0$. One simple characterization of public good consumption is $g^i = v^i \cdot G$.

Individuals of type i maximize the utility function

$$U^i(c^i, g^i, H^i) \tag{3}$$

subject to the budget constraint

$$c^i + p_v \cdot v^i = w^i \cdot H^i - T(Y^i) \equiv B^i, \tag{4}$$

[1] This does not imply that utility is linear in visits.

where p_v is the price per use for the public good set by the government, $T(Y^i)$ is a nonlinear income tax, and B^i is after-tax income.[2]

It will be convenient to define the price of the public good, p_g, as the total payment per unit of the public good, g^i:

$$p_\mathrm{g} \equiv \frac{p_v v^i}{g^i} = \frac{p_v}{h(G)}. \tag{5}$$

The price of g^i rises with p_v and falls with G. While the government's public good instruments are p_v and G, we can equivalently characterize them in terms of p_g and G.[3] Lastly, we find it convenient to work with the indirect utility function (conditional on labor income) in terms of observables,

$$V^i(B^i, Y^i, p_\mathrm{g}, G) = \max_{v^i} U^i\left(B^i - p_\mathrm{g} v^i h(G), v^i h(G), \frac{Y^i}{w^i}\right),$$

where we have substituted in the individual's budget constraint to eliminate private consumption.[4]

As in B&C, we wish to characterize the information-constrained Pareto efficient tax and public good pricing policy. Specifically, the Pareto efficient allocation is described as the solution to the following problem:

$$\underset{B^1, Y^1, B^2, Y^2, p_\mathrm{g}, G}{\mathrm{Max}} \; V^1(B^1, Y^1, p_\mathrm{g}, G) \tag{6}$$

subject to

$$V^2(B^2, Y^2, p_\mathrm{g}, G) \geq \bar{V}^2 \tag{7}$$

$$V^2(B^2, Y^2, p_\mathrm{g}, G) \geq \hat{V}^2(B^1, Y^1, p_\mathrm{g}, G) \tag{8}$$

$$N^2(Y^2 - B^2) + N^1(Y^1 - B^1) + p_\mathrm{g}(N^1 \cdot g^1 + N^2 \cdot g^2) - m \cdot G \geq 0, \tag{9}$$

where m is the marginal cost of producing the public good and $\hat{V}^2$ refers to the utility of a high-ability type choosing the income—public good bundle intended for a low-ability type.[5] Equation (7) ensures the solution is Pareto efficient, Eq. (8) is a self-selection constraint to distinguish high and low ability types, and Eq. (9) is the government's budget constraint (assuming N^i type i people). The Lagrangian and

[2] We ignore the problem of infinite demand for visits (v^i) when the price of the public good is set equal to zero. It is straightforward to include a private cost for the public good (for example, it costs time and/or money to travel to a park or a museum) to insure an interior solution. Adding a private cost does not change our results.

[3] Following B&C, we assume that p_v and p_g are nonnegative.

[4] Equivalently, we could have defined the conditional indirect utility function V^i in terms of before-tax income and tax payments rather than before-tax and after-tax income.

[5] As in B&C, we assume the usual single crossing property for utility of the two types.

first-order conditions for this problem are

$$\begin{aligned}\Lambda &= V^1(B^1, Y^1, p_g, G) + \beta[V^2(B^2, Y^2, p_g, G) - \bar{V}^2] \\ &\quad + \rho[V^2(B^2, Y^2, p_g, G) - \hat{V}^2(B^1, Y^1, p_g, G)] \\ &\quad + \mu[N^2(Y^2 - B^2) + N^1(Y^1 - B^1) + p_g(N^1 \cdot g^1 + N^2 \cdot g^2) - m \cdot G]\end{aligned} \tag{10}$$

$$\frac{\partial \Lambda}{\partial B^1} = \frac{\partial V^1}{\partial B^1} - \rho \cdot \frac{\partial \hat{V}^2}{\partial B^1} - \mu \cdot N^1 + \mu \cdot p_g \cdot N^1 \cdot \frac{\partial g^1}{\partial B^1} = 0 \tag{11}$$

$$\frac{\partial \Lambda}{\partial Y^1} = \frac{\partial V^1}{\partial Y^1} - \rho \cdot \frac{\partial \hat{V}^2}{\partial Y^1} + \mu \cdot N^1 + \mu \cdot p_g \cdot N^1 \cdot \frac{\partial g^1}{\partial Y^1} = 0 \tag{12}$$

$$\frac{\partial \Lambda}{\partial B^2} = \beta \cdot \frac{\partial V^2}{\partial B^2} + \rho \cdot \frac{\partial V^2}{\partial B^2} - \mu \cdot N^2 + \mu \cdot p_g \cdot N^2 \cdot \frac{\partial g^2}{\partial B^2} = 0 \tag{13}$$

$$\frac{\partial \Lambda}{\partial Y^2} = \beta \cdot \frac{\partial V^2}{\partial Y^2} + \rho \cdot \frac{\partial V^2}{\partial Y^2} + \mu \cdot N^2 + \mu \cdot p_g \cdot N^2 \cdot \frac{\partial g^2}{\partial Y^2} = 0 \tag{14}$$

$$\begin{aligned}\frac{\partial \Lambda}{\partial p_g} &= \frac{\partial V^1}{\partial p_g} + \beta \cdot \frac{\partial V^2}{\partial p_g} + \rho \cdot \frac{\partial V^2}{\partial p_g} - \rho \cdot \frac{\partial \hat{V}^2}{\partial p_g} + \mu(N^1 \cdot g^1 + N^2 \cdot g^2) \\ &\quad + \mu \cdot p_g\left[N^1 \cdot \frac{\partial g^1}{\partial p_g} + N^2 \cdot \frac{\partial g^2}{\partial p_g}\right] \le 0, \quad \text{and} \quad p_g\left(\frac{\partial \Lambda}{\partial p_g}\right) = 0\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\partial \Lambda}{\partial G} &= \frac{\partial V^1}{\partial G} + \beta \cdot \frac{\partial V^2}{\partial G} + \rho \cdot \frac{\partial V^2}{\partial G} - \rho \cdot \frac{\partial \hat{V}^2}{\partial G} \\ &\quad + \mu \frac{\partial p_g}{\partial G}(N^1 \cdot g^1 + N^2 \cdot g^2) + \mu \cdot p_g\left[N^1 \cdot \frac{\partial g^1}{\partial G} + N^2 \cdot \frac{\partial g^2}{\partial G}\right] - \mu \cdot m = 0,\end{aligned} \tag{16}$$

where β is the shadow price for the constraint on the high-ability type's utility, ρ is the shadow price for the self-selection constraint, and μ is the shadow price on the government revenue constraint.

We show in the Appendix that Eq. (15) can be written as

$$\frac{\partial \Lambda}{\partial p_g} = \rho \frac{\partial \hat{V}^2}{\partial B^1}[\hat{g}^2 - g^1] + \mu \cdot p_g \sum S^i_{gg} \le 0, \quad \text{and} \quad p_g\left(\frac{\partial \Lambda}{\partial p_g}\right) = 0, \tag{15$'$}$$

where $S^i_{gg} < 0$ is the ith-type individual's own-price Slutsky term and $\hat{g}^2$ is the consumption of the public good by a high-ability type mimicking a low-ability type. As B&C note, the first term captures the social benefit from relaxing (or the social cost from tightening) the self-selection constraint, while the second term describes the distortionary effect due to the wedge between the government price of the public good (p_g) and the marginal social cost of access (zero, given the assumption of nonrivalness in consumption). Note that, from Eq. (15′), if the optimal price is positive, it is given by the formula

$$p_g^* = \frac{\partial \hat{V}^2}{\partial B^1} \frac{\rho[g^1 - \hat{g}^2]}{\mu \sum S_{gg}^i}. \tag{17}$$

The analysis so far is analogous to that of Blomquist and Christiansen. Let us now consider the demand for the public good for the Type 1 consumer relative to a Type 2 consumer who mimics a Type 1 consumer. Blomquist and Christiansen's Proposition 2 rules out the possibility of a positive price in the case that $\hat{g}^2 \leq g^1$. This also follows directly from inspection of Eq. (17).[6]

B&C's result that $\hat{g}^2 > g^1$ is a necessary but not sufficient condition for a positive price on the public good depends crucially on their modeling assumption for the public good. Once we replace rationing with our assumptions embodied in Eqs. (1) and (2), we can show

Proposition 1. *The condition $\hat{g}^2 > g^1$ is a sufficient condition for a positive price on the excludable public good to be Pareto improving from an initial position with p_g equal to zero.*

Proof: At $p_g = 0$, the first-order condition for the access price is $\frac{\partial \Lambda}{\partial p_g}|_{p_g=0} = \rho \frac{\partial \hat{V}^2}{\partial B^1}[\hat{g}^2 - g^1]$. Since utility is increasing in after-tax income and the constraint on minimal utility for the high-ability type must be binding, $\frac{\partial \Lambda}{\partial p_g}|_{p_g=0} > 0$. □

3 Discussion

The condition $\hat{g}^2 > g^1$ is central to both B&C's result and our Proposition 1. This condition means that public good consumption is a complement to leisure (since a high-ability type earning the same income as a low-ability type will consume more leisure).[7] As intuition for this condition, recall that if a nonlinear income tax is employed and utility is weakly separable between leisure and consumption goods, then the optimal tax rate on all commodities is zero (see, for example, Deaton (1979)). Since a price for the public good in excess of the private cost is analogous to a commodity tax, the optimal commodity tax result suggests that p_g should equal zero when utility is weakly separable in this fashion. That intuition is correct. A mimicking high-ability type earns the same income as a low-ability type but consumes more leisure (since his wage rate is higher). But the separability assumption means that these two consumers will consume the same amount of the public good ($\hat{g}^2 = g^1$). Thus, raising the price of the public good does nothing to help distinguish between high and low-ability types and so provides no welfare gain from distorting the price of the public good.[8]

[6] The shadow prices ρ and μ are nonnegative by construction. The partial derivative of the mimicker's utility function with respect to B is positive by nonsatiation in the private consumption good.

[7] We show in the Appendix that the condition $\hat{g}^2 > g^1$ is true if and only if $M\hat{R}S_{GB}^2 > MRS_{GB}^1$. This relationship will be used to analyze optimal provision rules in the next section.

[8] Edwards et al. (1994) find an analogous result for nonexcludable public goods.

Where utility is nonseparable and $\hat{g}^2 > g^1$, raising the price of the public good can help to distinguish between high and low-ability types. Starting at p_{g} equal to zero, the benefit from being able to distinguish high and low-ability types more than offsets the distortion arising from setting p_{g} greater than zero. This discussion emphasizes the importance of labor distortions in the optimal pricing of the public good.

The assumption of rationing in B&C's model is key to understanding the difference in results between their model and ours. In their model, both types of consumers are rationed over an initial range of prices for the public good between zero and $\bar{p}_{\mathrm{g}}$, and altering the public good price has no impact on utility.[9] At $\bar{p}_{\mathrm{g}}$, however, consumption of the public good begins to fall below G, the capacity constraint, and a marginal price increase must lower utility. This is because a mimicking high-ability consumer consumes the same amount of the public good as the low-ability consumer (the rationed amount) and the marginal price increase does not differentially impact the mimicking consumer relative to the low-ability type (and so discourage mimicking). But since $\bar{p}_{\mathrm{g}}$ is strictly positive, a first-order welfare loss arises from the distortion to public good pricing. Put differently, the benefit from relaxing the self-selection constraint in B&C's model is second-order at $\bar{p}_{\mathrm{g}}$, while the distortion from pricing the public good greater than social marginal cost is first-order. With our modeling of the public good, precisely the opposite occurs. A marginal increase in the public good price, p_{g}, from zero has a first-order benefit in relaxing the self-selection constraint and a second-order impact on the public good pricing distortion.

Summing up, the condition $\hat{g}^2 > g^1$ is a necessary and sufficient condition for a positive price on the public good in our model. B&C's conclusion that "[t]he policy case for a price may thus appear rather weak" (p. 61) depends on consumption of the public good initially being constrained as its price is raised from zero. With an alternative and equally reasonable characterization of excludable public goods, we find that utility unambiguously increases as the price is increased starting at zero. We thus find a stronger policy case to be made for pricing excludable public goods.

We have emphasized that the optimal pricing of the public good is sensitive to different modeling of the public good. We next provide a few examples to guide thinking on the appropriate modeling of the public good. To understand the importance of our distinction, consider the following examples. Two drivers are using an uncongested road. One uses the entire road once per day; the other uses it twice each day. In one sense, both drivers are rationed if they would prefer a longer road (B&C's model). In another sense, neither is rationed since they can use the road as many times as they would like (our model).

As another example, consider an art museum with five rooms. A museum visitor chooses to view three of the five rooms. In our model, a decision by the museum owner to spend money to improve all five rooms increases the amount of the public good as well as the museum visitor's utility since the rooms she does visit have been improved. In B&C's model, an increase in G is the addition of a sixth room to the art museum. The increase in G, however, does not raise utility for the museum visitor since she is not rationed. B&C argue that consumers get no benefit from this sixth room if they

[9] In this price range, raising the public good price is a lump-sum charge to the consumer which is rebated lump-sum through lower taxes.

only wish to visit three rooms. Their modeling really amounts to an offer of multiple public goods (where each room is a public good and accessible at a set price).[10]

In fact, the examples that B&C offer to motivate their model can be viewed as bundles of multiple public goods. Cable television service with 10 channels can be viewed as a bundle of 10 public goods. The cable company could offer service on a per-channel basis.[11] A weather forecast can be a bundle of multiple public goods. One public good could be a basic service, and a second public good a service with more detail. And in both these cases, consumers may visit and use these public goods repeatedly and could be charged on a pay-per-use basis in accord with our model.

In closing, we note an historic example that fits our model framework well—that of British lighthouses. According to Coase (1974), ships using the lighthouse paid fees that were "so much per net ton payable per voyage for all vessels arriving at, or departing from, ports in Britain" (p. 361). Because only ships would come near the British lighthouses if they planned to enter or leave British ports, it was possible to levy a usage charge. Presumably if ships evaded the payment, they would be barred from future entry into (or exit from) these ports, thereby denying them the use of the public good.

4 The Samuelson rule revisited

B&C discuss how the Samuelson rule is modified in their model based on the valuation of the public good by a low-ability type consumer and a high-ability type consumer mimicking a low-ability type. They note that as in other models of public goods in second-best frameworks (Atkinson and Stern, 1974; King, 1986; Boadway and Keen, 1993; Edwards et al., 1994, among others), the rule must be modified to the extent that leisure and the public good are related. We make this relationship more precise in this section for our characterization of public goods.

The analysis in the previous section shows that introducing p_g as the price of the public good allows us to draw on the literature on optimal commodity taxation with a nonlinear income tax (e.g., Christiansen, 1984; Edwards et al., 1994). Analogous to Edwards et al.'s Proposition 3 is

Proposition 2. *Pareto efficiency in the provision of the public good, G, in the presence of self-selection constraints requires*

$$\sum \mathrm{MRS}^i_{GB} = m + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}^2}{\partial B^1} \cdot \left[M\hat{R}S^2_{GB} - MRS^1_{GB}\right] - p_v \sum \frac{\partial v^{i^c}}{\partial G}, \tag{18}$$

where MRS^i_{GB} is the marginal rate of substitution between the public and private good for a type i consumer and $M\hat{R}S^2_{GB}$ is the marginal rate of substitution between the

[10] Note too the strong assumption that consumers receive no benefit from the expansion of G that might arise from the option value to visit the sixth room in the future. We do not pursue this idea further in this paper.

[11] In fact, the chairman of the Federal Communications Commission recently proposed just this pricing scheme for satellite and cable television companies. See Labaton (2005) for details.

public and private good for a high-ability type choosing the income—public good bundle intended for the low-ability type.

Proof: see Appendix. □

This condition provides two modifications to the Samuleson (1954) rule that the sum of the marginal rates of substitution ($\sum MRS^i_{GB}$) should equal the marginal rate of transformation (m). First, it adds a term that depends on the social benefit from deterring mimicking by high-ability types. Second, it adds a revenue term. Focusing on this latter term first, the marginal cost of the public good is lowered to the extent that increased provision of the public good (G) increases utilization (v^i) and therefore entry fee revenue ($p_v \cdot v^i$) which in turn allows a lower optimal value of p_v. The modification of the Samuelson rule for revenue effects of increased public good provision is well known (see Atkinson and Stern, 1974, for example).

The term including the difference in the *MRS* between the mimicking and low-ability consumer reflects the benefits arising from the public good's ability to help distinguish between high and low-ability consumers. To see why this is so, assume that visits (v^i) are unaffected by changes in the amount of the public good (G) so we can ignore the revenue term in Eq. (18). Further assume that the mimicking high-ability type places a higher value on the public good than does the low-ability type $M\hat{R}S^2_{GB} > MRS^1_{GB}$). Then $\sum MRS^i_{GB} > m$ at the second-best optimum. If $\sum MRS^i_{GB}$ is monotonically decreasing in G, then applying the Samuelson rule would lead to an overprovision of the public good.[12] Consider the allocation at the Samuelson rule, and now decrease G marginally. At the same time, lower the taxes of all individuals choosing the high-ability bundle by the amount MRS^2_{GB} and reduce the taxes of all individuals choosing the low-ability bundle by MRS^1_{GB}. Utility for any individual choosing the bundle intended for their ability type will be unchanged, and the government's budget remains balanced. But the mimicker has a loss of utility since the value of the loss of the public good ($M\hat{R}S^2_{GB}$) exceeds the gain from lower taxes (MRS^1_{GB}). This makes mimicking less attractive and allows for a relaxation of the self-selection constraint, which in turn allows for an increase in utility for the low-ability types without adversely affecting the high-ability types.[13]

As Boadway and Keen (1993) point out in the context of nonexcludable public goods, the term $\frac{\rho}{\mu} \cdot \frac{\partial \hat{V}^2}{\partial B^1} \cdot [M\hat{R}S^2_{GB} - MRS^1_{GB}]$ in Eq. (18) equals zero if utility is weakly separable between the consumption good and the public good on the one hand and leisure on the other hand. Weak separability of this kind implies that the *MRS* between the private and the public good is independent of labor supply. Since the only difference between a mimicking high-ability consumer and a low-ability consumer is their labor supply, the *MRS* for both must be the same. And as discussed in the last section, weak separability also implies that the planner should not charge a positive price for public good usage, so that both the second term and the third term vanish with this form of weak separability. We formalize this result as

[12] Atkinson and Stern (1974) note that the aggregate *MRS* may not have a monotone relationship to G. We will assume it here to develop intuition.

[13] Boadway and Keen (1993) use this same intuition in the case of a nonexcludable public good with no entry fee.

Proposition 3. *If utility for both types of consumers is of the form* $U^i(F^i(c^i, g^i), H^i)$, *then the Samuelson rule holds:*

$$\sum MRS^i_{GB} = m. \tag{19}$$

5 Conclusion

Blomquist and Christiansen show in their model of constrained public good consumption that utility decreases in entry price for an excludable public good before (possibly) increasing. We have argued that this result depends importantly on their characterization of the public good. With an alternative and equally reasonable characterization of excludable public goods, we find that utility rises if an incremental entry fee is applied to a public good (starting from zero), so long as utility is not weakly separable between private and public good consumption on the one hand and leisure on the other. We find, therefore, that their conclusion that a weak case (at best) can be made for charging for excludable public goods is not robust to the modeling of the public good. Hence, the policy case for a positive price on the public good is stronger than Blomquist and Christiansen suggest.

We also characterize the modified Samuelson rule for the provision of an excludable public good with an entry fee and find that the original Samuelson rule must be modified for a revenue impact as well as a distortion term arising from the desire to distinguish high-ability from low-ability consumers. This discussion emphasizes the importance of the relation between the demand for leisure and the demand for the public good in setting optimal public good prices as well as in determining the efficient level of public good provision.

Appendix

A.1 Derivation of Eq. (15′)

From $V^i(B^i, Y^i, p_g, G) = \max_{\nu^i} U(B^i - p_g\nu^i G, \nu^i G, \frac{Y^i}{w^i})$, the envelope theorem implies

$$\frac{\partial V^i}{\partial p_g} = -\frac{\partial V^i}{\partial B^i} g^i. \tag{A.1}$$

Using the Slutsky equation for g^i, that is,

$$S^i_{gg} = \frac{\partial g^i}{\partial p_g} + g^i \frac{\partial g^i}{\partial B^i}, \tag{A.2}$$

Equation (15) becomes:

$$\begin{aligned}
\frac{\partial \Lambda}{\partial p_{\mathrm{g}}} &= \frac{\partial V^1}{\partial p_{\mathrm{g}}} + \beta \cdot \frac{\partial V^2}{\partial p_{\mathrm{g}}} + \rho \cdot \frac{\partial V^2}{\partial p_{\mathrm{g}}} - \rho \cdot \frac{\partial \hat{V}^2}{\partial p_{\mathrm{g}}} + \mu(N^1 \cdot g^1 + N^2 \cdot g^2) \\
&\quad + \mu \cdot p_{\mathrm{g}} \left[N_1 \cdot \frac{\partial g^1}{\partial p_{\mathrm{g}}} + N_2 \cdot \frac{\partial g^2}{\partial p_{\mathrm{g}}} \right] \\
&= -\frac{\partial V^1}{\partial B^1} g^1 - \beta \cdot \frac{\partial V^2}{\partial B^1} g^2 - \rho \cdot \frac{\partial V^2}{\partial B^2} g^2 + \rho \cdot \frac{\partial \hat{V}^2}{\partial B^1} \hat{g}^2 + \mu(N^1 \cdot g^1 + N^2 \cdot g^2) \\
&\quad + \mu \cdot p_{\mathrm{g}} \left[N^1 \cdot \left(S^1_{gg} - g^1 \frac{\partial g^1}{\partial B^1} \right) + N^2 \cdot \left(S^2_{gg} - g^2 \frac{\partial g^2}{\partial B^2} \right) \right]. \qquad \text{(A.3)}
\end{aligned}$$

Adding and subtracting $\rho \cdot g^1 \frac{\partial \hat{V}^2}{\partial B^1}$, (A.3) becomes:

$$\begin{aligned}
\frac{\partial \Lambda}{\partial p_{\mathrm{g}}} &= -\frac{\partial V^1}{\partial B^1} g^1 - \beta \cdot \frac{\partial V^2}{\partial B^1} g^2 - \rho \cdot \frac{\partial V^2}{\partial B^2} g^2 + \rho \cdot \frac{\partial \hat{V}^2}{\partial B^1} \hat{g}^2 + \rho \cdot g^1 \frac{\partial \hat{V}^2}{\partial B^1} - \rho \cdot g^1 \frac{\partial \hat{V}^2}{\partial B^1} \\
&\quad + \mu(N^1 \cdot g^1 + N^2 \cdot g^2) + \mu \cdot p_{\mathrm{g}} \left[N^1 \cdot \left(S^1_{gg} - g^1 \frac{\partial g^1}{\partial B^1} \right) + N^2 \cdot \left(S^2_{gg} - g^2 \frac{\partial g^2}{\partial B^2} \right) \right] \\
&= \rho \cdot \frac{\partial \hat{V}^2}{\partial B^1} \hat{g}^2 - \rho \cdot g^1 \frac{\partial \hat{V}^2}{\partial B^1} + \mu \cdot p_{\mathrm{g}} \left[N^1 \cdot \left(S^1_{gg} \right) + N^2 \cdot \left(S^2_{gg} \right) \right]. \qquad \text{(A.4)}
\end{aligned}$$

The last equality follows directly from substituting Eqs. (11) and (13) into the expression. Therefore, $\frac{\partial \Lambda}{\partial p_{\mathrm{g}}} = \rho \frac{\partial \hat{V}^2}{\partial B^1} [\hat{g}^2 - g^1] + \mu \cdot p_{\mathrm{g}} \sum S^i_{gg}$. □

A.2 Proof of Proposition 2

First, we have another expression of the indirect utility function V^{*i} with p_v.

$$V^i(B^i, Y^i, p_{\mathrm{g}}, G) = V^i\left(B^i, Y^i, \frac{p_v}{G}, G\right) \equiv V^{*i}(B^i, Y^i, p_v, G) \qquad \text{(A.5)}$$

Note that except for Y^i, both V^i and V^{*i} are all expressed in terms of the instruments that the government can control.

Second, from the alternative indirect utility of $V^{*i}(B^i, Y^i, p_v, G)$ of Eq. (A.5), *MRS* is defined as

$$MRS^i_{GB} = \frac{\partial V^{*i}}{\partial G} \Bigg/ \frac{\partial V^{*i}}{\partial B^i} \qquad \text{(A.6)}$$

with B being the numeraire.

Next, let us verify the Slutsky equation for v (the access to the public good) with change in G. Consider the identity,

$$v^{i^c}(\bar{u}_i, p_v, G) = v^i(e^i(p_v, G, w^i, \bar{u}^i), p_v, G). \tag{A.7}$$

Taking derivatives with respect to G, we get

$$\frac{\partial v^{i^c}}{\partial G} = \frac{\partial v^i}{\partial G} + \frac{\partial v^i}{\partial B^i} \cdot \frac{\partial e^i}{\partial G}. \tag{A.8}$$

Since $\frac{\partial e^i}{\partial G}$ describes changes in how much income is needed to keep the utility remaining unchanged when G gets higher, that is,

$$\frac{\partial e^i}{\partial G} = -\frac{\partial V^{*i}}{\partial G} \Big/ \frac{\partial V^{*i}}{\partial B^i} = -MRS^i_{GB}, \tag{A.9}$$

Equation (A.8) becomes:

$$\frac{\partial v^{i^c}}{\partial G} = \frac{\partial v^i}{\partial G} - \frac{\partial v^i}{\partial B^i} \cdot \frac{\partial V^{*i}}{\partial G} \Big/ \frac{\partial V^{*i}}{\partial B^i} = \frac{\partial v^i}{\partial G} - \frac{\partial v^i}{\partial B^i} \cdot MRS^i_{GB}. \tag{A.8$'$}$$

Now, social welfare maximization problem with this indirect utility function is:

$$\begin{aligned}\Lambda = {} & V^{*1}(B^1, Y^1, p_v, G) + \beta[V^{*2}(B^2, Y^2, p_v, G) - \bar{V}^{*2}] + \rho[V^{*2}(B^2, Y^2, p_v, G) \\ & - \hat{V}^{*2}(B^1, Y^1, p_v, G)] + \mu[N^2(Y^2 - B^2) + N^1(Y^1 - B^1) \\ & + p_v(N^1 \cdot v^1 + N^2 \cdot v^2) - m \cdot G],\end{aligned} \tag{10$'$}$$

and the first-order conditions are

$$\frac{\partial \Lambda}{\partial B^1} = \frac{\partial V^{*1}}{\partial B^1} - \rho \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} - \mu \cdot N^1 + \mu \cdot p_v \cdot N^1 \cdot \frac{\partial v^1}{\partial B^1} = 0 \tag{A.10}$$

$$\frac{\partial \Lambda}{\partial Y^1} = \frac{\partial V^{*1}}{\partial Y^1} - \rho \cdot \frac{\partial \hat{V}^{*2}}{\partial Y^1} + \mu \cdot N^1 + \mu \cdot p_v \cdot N^1 \cdot \frac{\partial v^1}{\partial Y^1} = 0 \tag{A.11}$$

$$\frac{\partial \Lambda}{\partial B^2} = \beta \cdot \frac{\partial V^{*2}}{\partial B^2} + \rho \cdot \frac{\partial V^{*2}}{\partial B^2} - \mu \cdot N^2 + \mu \cdot p_v \cdot N^2 \cdot \frac{\partial v^2}{\partial B^2} = 0 \tag{A.12}$$

$$\frac{\partial \Lambda}{\partial Y^2} = \beta \cdot \frac{\partial V^{*2}}{\partial Y} + \rho \cdot \frac{\partial V^{*2}}{\partial Y} + \mu \cdot N^2 + \mu \cdot p_v \cdot N^2 \cdot \frac{\partial v^2}{\partial Y} = 0 \tag{A.13}$$

$$\begin{aligned}\frac{\partial \Lambda}{\partial G} = {} & \frac{\partial V^{*1}}{\partial G} + \beta \frac{\partial V^{*2}}{\partial G} + \rho \frac{\partial V^{*2}}{\partial G} - \rho \frac{\partial \hat{V}^{*2}}{\partial G} \\ & - \mu \cdot m + \mu \cdot p_v \left[N^1 \cdot \frac{\partial v^1}{\partial G} + N^2 \cdot \frac{\partial v^2}{\partial G} \right] = 0.\end{aligned} \tag{A.14}$$

Adding and subtracting $\rho \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} \cdot \frac{\frac{\partial V^{*1}}{\partial G}}{\frac{\partial V^{*1}}{\partial B^1}}$ to Eq. (A.14), we obtain:

$$\begin{aligned}
\frac{\partial \Lambda}{\partial G} &= \frac{\partial V^{*1}}{\partial G} + \beta \frac{\partial V^{*2}}{\partial G} + \rho \frac{\partial V^{*2}}{\partial G} - \rho \frac{\partial \hat{V}^{*2}}{\partial G} - \mu \cdot m + \mu \cdot p_v \left[N^1 \cdot \frac{\partial v^1}{\partial G} + N^2 \cdot \frac{\partial v^2}{\partial G} \right] \\
&= \left[\frac{\partial V^{*1}}{\partial B^1} - \rho \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} \right] \cdot \frac{\frac{\partial V^{*1}}{\partial G}}{\frac{\partial V^{*1}}{\partial B^1}} + (\beta + \rho) \cdot \frac{\partial V^{*2}}{\partial B^2} \cdot \frac{\frac{\partial V^{*2}}{\partial G}}{\frac{\partial V^{*2}}{\partial B^2}} + \rho \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} \cdot \left[\frac{\frac{\partial V^{*1}}{\partial G}}{\frac{\partial V^{*1}}{\partial B^1}} - \frac{\frac{\partial \hat{V}^{*2}}{\partial G}}{\frac{\partial \hat{V}^{*2}}{\partial B^1}} \right] \\
&\quad - \mu \cdot m + \mu \cdot p_v \left[N^1 \cdot \frac{\partial v^1}{\partial G} + N^2 \cdot \frac{\partial v^2}{\partial G} \right] \\
&= \left[\frac{\partial V^{*1}}{\partial B^1} - \rho \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} \right] \cdot MRS^1_{GB} + (\beta + \rho) \cdot \frac{\partial V^{*2}}{\partial B^2} \cdot MRS^2_{GB} \\
&\quad + \rho \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} \cdot \left[MRS^1_{GB} - M\hat{R}S^2_{GB} \right] - \mu \cdot m + \mu \cdot p_v \left[N^1 \cdot \frac{\partial v^1}{\partial G} + N^2 \cdot \frac{\partial v^2}{\partial G} \right] \\
&= \left[\mu \cdot N^1 - \mu \cdot p_v \cdot N^1 \cdot \frac{\partial v^1}{\partial B^1} \right] \cdot MRS^1_{GB} + \left[\mu \cdot N^2 - \mu \cdot p_v \cdot N^2 \cdot \frac{\partial v^2}{\partial B^2} \right] \cdot MRS^2_{GB} \\
&\quad + \rho \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} \cdot \left[MRS^1_{GB} - M\hat{R}S^2_{GB} \right] - \mu \cdot m + \mu \cdot p_v \left[N^1 \cdot \frac{\partial v^1}{\partial G} + N^2 \cdot \frac{\partial v^2}{\partial G} \right] = 0.
\end{aligned} \tag{A.14$'$}$$

The last equality follows directly from applying Eqs. (A.10) and (A.12). By dividing Eq. (A.14$'$) by μ and rearranging, we get:

$$\begin{aligned}
N^1 \cdot MRS^1_{GB} &+ N^2 \cdot MRS^2_{GB} + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} \cdot \left[MRS^1_{GB} - M\hat{R}S^2{}_{GB} \right] \\
&= m - p_v \left[N^1 \cdot \left\{ \frac{\partial v^1}{\partial G} - \frac{\partial v^1}{\partial B^1} \cdot MRS_1 \right\} + N^2 \cdot \left\{ \frac{\partial v^2}{\partial G} - \frac{\partial v^2}{\partial B^2} \cdot MRS^2_{GB} \right\} \right] \\
&= m - p_v \left[N^1 \cdot \frac{\partial v^{1^c}}{\partial G} + N^2 \cdot \frac{\partial v^{2^c}}{\partial G} \right].
\end{aligned} \tag{A.14$''$}$$

The last equality follows from applying the Slutsky equation for v. Therefore,

$$\Sigma MRS^i_{GB} = m + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}^{*2}}{\partial B^1} \cdot \left[M\hat{R}S^2_{GB} - MRS^1_{GB} \right] - p_v \sum \frac{\partial v^{i^c}}{\partial G}.$$

From the identity $V^i(B^i, Y^i, p_g, G) = V^{*i}(B^i, Y^i, p_v, G)$, we get $\frac{\partial V^{*i}}{\partial B^i} = \frac{\partial V^i}{\partial B^i}$. Thus,

$$\Sigma MRS^i_{GB} = m + \frac{\rho}{\mu} \cdot \frac{\partial \hat{V}^2}{\partial B^1} \cdot \left[M\hat{R}S^2_{GB} - MRS^1_{GB} \right] - p_v \sum \frac{\partial v^{i^c}}{\partial G}. \qquad \square$$

A.3 Proof: $\hat{g}^2 > g^1$ if and only if $M\hat{R}S^2_{GB} > MRS^1_{GB}$

First recall that the condition that $\hat{g}^2 > g^1$ is identical to the condition that a mimicker values the public good consumption more than a type-1 individual. Since a mimicker and a type-1 individual have the same disposable income, it should be the case $\hat{c}^2 < c^1$. Therefore, $\hat{g}^2 > g^1$ if and only if $M\hat{R}S^2_{gc} > MRS^1_{gc}$ when evaluated at the same (g, c, B).

(i) $\hat{g}^2 > g^1$ implies $M\hat{R}S^2_{GB} > MRS^1_{GB}$. From the definition of MRS_{Gc} which implicitly assumes that other things are equal including v, we may fix v and normalize it at unity. Thus, $\frac{\partial\hat{U}^2}{\partial g}/\frac{\partial\hat{U}^2}{\partial c} > \frac{\partial U^1}{\partial g}/\frac{\partial U^1}{\partial c}$ implies $\frac{\partial\hat{U}^2}{\partial G}/\frac{\partial\hat{U}^2}{\partial c} > \frac{\partial U^1}{\partial G}/\frac{\partial U^1}{\partial c}$. By the envelope theorem, we get $\frac{\partial V}{\partial G} = \frac{\partial U}{\partial G}$. Also, it is easy to show that $\frac{\partial V}{\partial B} = \frac{\partial U}{\partial c}$ because the private good is the numeraire. Hence, $\frac{\partial\hat{U}^2}{\partial G}/\frac{\partial\hat{U}^2}{\partial c} > \frac{\partial U^1}{\partial G}/\frac{\partial U^1}{\partial c}$ is identical to $\frac{\partial\hat{V}^2}{\partial G}/\frac{\partial\hat{V}^2}{\partial B} > \frac{\partial V^1}{\partial G}/\frac{\partial V^1}{\partial B}$, or $M\hat{R}S^2_{GB} > MRS^1_{GB}$.

(ii) $M\hat{R}S^2_{GB} > MRS^1_{GB}$ implies $\hat{g}^2 > g^1$. By the same logic as earlier, $\hat{g}^2 < g^1$ implies $M\hat{R}S^2_{GB} < MRS^1_{GB}$; and $\hat{g}^2 = g^1$ implies $M\hat{R}S^2_{GB} = MRS^1_{GB}$. Thus, the contrapositive is true: $M\hat{R}S^2_{GB} > MRS^1_{GB}$ implies $\hat{g}^2 > g^1$. □

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References

Atkinson, A., & Stern, N. (1974). Pigou, taxation, and public goods. *Review of Economic Studies*, *41*, 119–128.

Blomquist, S., & Christiansen, V. (2005). The role of prices for excludable public goods. *International Tax and Public Finance*, *12*, 61–79.

Boadway, R., & Keen, M. (1993). Public goods, self-selection and optimal income taxation. *International Economic Review*, *34*(3), 463–478.

Christiansen, V. (1984). Which commodity taxes should supplement the income tax? *Journal of Public Economics*, *24*, 195–220.

Coase, R. H. (1974). The lighthouse in economics. *Journal of Law and Economics*, *17*(2), 357–376.

Deaton, A. (1979). The distance function in consumer behaviour with applications to index numbers and optimal taxation. *Review of Economic Studies*, *46*(3), 391–405.

Edwards, J., Keen, M., & Tuomala, M. (1994). Income tax, commodity taxes and public good provision: A brief guide. *FinanzArchiv*, *51*, 472–487.

King, M. A. (1986). A Pigovian rule for the optimum provision of public goods. *Journal of Public Economics*, *30*, 273–291.

Labaton, S. (2005). *New package deals urged for cable and satellite TV*. New York Times, New York.

Samuleson, P. (1954). The pure theory of public expenditure. *Review of Economics and Statistics*, *36*, 387–389.