Behavioral Model and Experimental Validation for a Spool-Packaged Shape Memory Alloy Actuator

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ABSTRACT

Shape memory alloy (SMA) based actuators have the potential to be lower mass, more compact, and more simplistic than conventional based actuators (electrical, hydraulic, etc); however, one of the key issues that plague their broad use is packaging since long lengths of wire are often necessary to achieve reasonable actuation strokes. Spooling the wire around pulleys or mandrels is one approach to package the wire more compactly and is useful in customizing the footprint of the actuator to the available application space. There is currently a lack of predictive models for actuator designs with spooled packaging that account for the variation of stress and strain along the wire's length and the losses due to friction. A spooling model is a critical step toward the application of this technique to overcome the packaging limitations on SMA actuators. This paper presents the derivation of an analytical predictive model for rotary spooled SMA actuators that accounts for general geometric parameters (mandrel diameter, wire length, wire diameter, and wrap angle), SMA material characteristics, loss parameters (friction), and the external loading profile. An experimental study validated the model with good correlation and provided insight into the effects of load and wrap angle. Based upon the model and experimental results, the main limitation to this approach, binding, is discussed. The analytical model and experimental study presented in this paper provide a foundation to design future actuators and insight into the behavioral impact of this packaging technique.

Keywords: Shape Memory Alloy, spooled packaging, actuator, GM/UM SMS Collaborative Research Laboratory

1. INTRODUCTION

Because of the reduction in cost and increase in quality control of shape memory alloys (SMA) in wire form, SMA based devices are being successfully implemented more and more in industry, space, defense and medical applications that are severely package constrained and have a need for high performance actuation at a greater level of simplicity than conventional actuator technologies such as electrical motors and fluidic power devices can provide. For SMA devices, several technical challenges are traditionally cited as disadvantages – speed, mechanical attachment, creep, and packaging – many of which have been resolved in the last few years. For example, since SMA is thermally actuated it is naturally lower in bandwidth than actuators with electric or magnetic field stimuli, but recently very quick response times (<5 ms) have been demonstrated that make it viable for applications such as crashworthiness or panel deployment that require extremely fast response times and have ample time to reset during another part of the actuation cycle. Additional techniques such as cooling the actuator in water have been shown to improve actuator frequency to near 20 Hz, while thin-film forms of SMA can operate at frequencies around 100 Hz. Another technical challenge has been mechanical attachment to SMA, but recently new methods for mechanical crimping and brazing have shown promise in overcoming these issues. A final example is the degradation of SMA wire performance due to creep over time which has made consistent operation difficult, but recent techniques in shakedown under load have generated wire with reduced creep and more repeatable performance. Thus, many of the hurdles limiting SMA are currently being addressed, but the need to package SMA compactly while maintaining its high energy density still remains an active issue.

SMA based actuators are often chosen for engineering applications that require a high performance actuator to be packaged in an existing space with tight form constraints. For example, SMA has been used for a variety of applications including morphing aircraft structures with internally packaged actuators, actuators packaged within an automobile’s swing panels (hoods, doors, hatches, etc.) and the engine compartment, low-cost latch actuators for consumer appliances, actuators for compact prosthetic and orthotic devices, and ingestible and implantable biomedical devices. SMA is a good candidate for these types of applications based on its unparalleled energy density, which makes for lightweight, low volume devices. The simplistic actuator forms allowed by SMA are also desirable from robustness, cost, and manufacturing perspectives.
Unfortunately, many approaches encounter packaging difficulties in creating high performance actuators with practical form factors. For example, in the SAMPSON Smart Inlet project, an antagonistic SMA actuator was designed and built to deform the cowl of an F-15 engine inlet. To achieve the required 6 inches of deflection, a very long actuator, 12.5 feet in length, was used, which made packaging the SMA actuator a major research issue. A need to package long wires is noted in SMA actuated prosthetics research, in the design of a leg muscle exerciser for paraplegics, and in the design of a swallowable digestive tract monitoring and clamping device. In Smart Attachment Mechanisms, linear SMA wires are used to manipulate an active surface; in this case, the volume of actuator portion of the active surface is directly related to the stroke requirements of the actuator. Engineering applications will often require the actuator to be packaged in an existing space such as within the structure of a jet engine inlet, airfoil, rotor blade, car door, car hood, handheld medical instrument, etc. Since linear forms generally require actuators 10 to 50 times longer than the stroke, more creative approaches are needed to fit actuators within the widely varying possibilities of form constraints.

To meet packaging constraints, different geometrical forms of SMA can be used such as torsion tubes and cylinders, helical springs, and wires. SMA torsion tubes can be used to package an actuator in a long narrow space such as rotor blades and airfoils. They can produce large torques and rotations, but the shear loading reduces the maximum energy density in half compared to an actuator in tension. Additionally, they are expensive to manufacture, bulky due to additional heating elements, slow to heat and cool, and limited to a narrow range of applications that require rotational motion and can accommodate long actuators. Several commercial applications from the 1980s and 1990s used SMA springs to provide large displacements, but they were typically limited to applications with low force requirements. While SMA springs can produce large motions, they sacrifice 95% of the output force and 60% of the work output with respect to an actuator loaded in pure tension with equal material volume. Compared to torsion tubes and helical springs, SMA wires are often the most practical based on their higher work output density, their ability to be controlled by a simple electric circuit, and faster heating and cooling without the bulk of additional hardware. Furthermore, wires are cheaper to manufacture and more widely available than other forms, and are becoming economically viable for practical, high yield, low cost applications in fields such as appliances and automotive. An alternative approach to packaging SMA is spooling where SMA wires are wrapped around pulleys or mandrels. The technique is useful for packaging long wires into a more compact footprint and for redirecting wires within difficult packaging constraints. Actuators that package SMA wire have been shown to produce useful magnitudes of force and deflection while adhering to packaging constraints in examples including prosthetic and orthotic devices, motion stabilization, panel deployment, automotive latches, and implantable and ingestible biomedical devices. Spooling the wire creates a design tradeoff since the form factor improves as a greater percentage of the actuator wire is wrapped around the mandrel but this improvement is at the expense of losses due to friction between the SMA wire and the surface on which it slips.

Unfortunately, the friction losses are not well understood and have not been rigorously modeled in previous research. As the wire stretches and contracts during phase transitions, slipping occurs between the wire and the spool mandrel, and this slipping motion is opposed by friction. In conventional materials, belt brake models (also known as Capstan models) and belt drive power transmission models are typically used to predict the changing stresses across a belt that wraps a rotating drum. The belt brake and belt drive models are useful for predicting the variation in stress and the torque transmitted between the belt and the drum, but they are specific to a single steady state with a passive material and rotating drum. Existing models for spooled SMA actuators either estimate an efficiency loss based on empirical observations, assume that the wrapped portions of the wire do not contribute to overall motion, or apply the belt-braking model for very specific actuator configurations. The use of an active material creates unique issues that are not addressed by belt models and have not been addressed by current spooling models. Since the stress-strain behavior of SMA is non-linear, the strain does not vary according to the same exponential function as the stress predicted in belt-braking or Capstan models. Additionally, a method for determining the deformation of the wire in each state has not been established and the task is complicated by a need to also account for portions of the wire that gain and lose contact with the mandrel during operation. The current models are not sufficient for analyzing a wide range of spooled actuator designs. To design spooled actuators that properly manage the tradeoffs between friction losses and the actuator footprint, a generic model is needed that can provide a basis for a systematic design methodology.

This paper introduces a predictive model for rotary spooled SMA actuators and describes an experimental validation study and the related findings. Achievable actuation stroke is defined with respect to general geometric parameters (including mandrel diameter, wire length, diameter, and wrap angle), material friction parameters, and the external loading profile. The model assumes that when the wire changes phase, it undergoes unilateral stretching or contraction across the wire, which is valid for wrap angles below a “binding” limitation. The model predictions for rotation as a
function of applied load and for rotation as a function of wrap angle were experimentally validated by measuring the actuator displacement between martensite and austenite through a range of applied loads (1 – 30 N, ~10 – 300 MPa) and with several lengths of wire and corresponding wrap angles between ~30 – 720 degrees. Based on the good correlation between the modeling and experimental studies, this paper provides a useful analytical predictive tool, which is a key element for designing and evaluating spooled SMA actuators that overcome the packaging limitation of current technologies.

2. PACKAGING ARCHITECTURE AND OPERATION STATES

To develop an actuator model that can be used for analysis of a wide variety of rotary spooled actuators, a generic architecture was derived with respect to three key operation states:

- **State 0**: the zero strain reference state (no external load applied),
- **State 1**: the SMA wire is in the martensite phase (martensite phase fraction $\xi^{(M)}=1$) with the external load applied to the actuator,
- **State 2**: the SMA wire in the austenite phase ($\xi^{(M)}=0$) with the external load applied.

For each state, the general architecture is defined based on the configuration shown in Figure 1 where the SMA wire is composed of three continuous segments: 1) the input tail with one end fixed to ground and the other end in tangential contact with the spool, 2) the wrapped portion in frictional contact with the spool, and 3) the output tail with one end in tangential contact with the spool and the other end attached to the rotary arm where output motion occurs. The arm rotates about the spool center and interacts with the external system, which exerts a moment $M_{\text{ext}}$. Based on the basic states and general architecture, the key variables affecting the actuator stroke can be defined.

2.1 State 0 – Zero-strain reference

State 0 is attained by heating the spooled wire to austenite while no load is applied. Assuming the wire does not undergo a two-way effect, the actuator remains in State 0 upon cooling. For the purposes of evaluating strain ($\varepsilon$) and rotation ($\phi$) throughout the actuator operation cycle, the zero-strain length of the wire ($\ell_0$) and the actuator rotation angle ($\phi$) defined equal to zero in State 0 are referenced to State 0. The initial length of the wire is the sum of the input tail length $\ell_{\text{in}}$, the output tail length $\ell_{\text{out}}$, and the wrapped length $\ell_w$. The wrapped length is related to wrap angle $\theta_w$ by the equation

$$\ell_w = \frac{\theta_w}{\rho_{\text{SMA}}},$$

where $\rho_{\text{SMA}}$ is the curvature of the wire, related to the spool radius $R$ and the wire diameter $d_{\text{SMA}}$ by the equation

![Figure 1. Diagram of a general spooled rotary actuator and its operation states.](image-url)
\[ \rho_{SMA} = \frac{1}{(R + \frac{1}{2} d_{SMA})}. \]  

### 2.2 State 1 – Martensite wire, external load applied

State 1 is achieved by stretching a martensite wire such that the wrapped portion of the wire slips along the spool by applying the external load to the output arm with the load either constant or varying according to a load function dependent on position (\( \phi \)). The stretching can occur from State 0 by applying a load to the actuator while the wire is martensite or by cooling an austenite wire under load so that the phase changes to martensite. While the tail lengths do not change between states, the stretching causes an increase in the wrap angle \( \theta_{w}^{(M)} \) and corresponding wrapped length \( \ell_{w}^{(M)} \). The angular position \( \phi^{(M)} \) relates to the wrap angle and wrap length according to the equation

\[ \phi^{(M)} = \theta_{w}^{(M)} - \theta_{w}^{(0)} = \rho_{SMA} \left( \ell_{w}^{(M)} - \ell_{w}^{(0)} \right), \]

where the superscript “M” indicates State 1 martensite.

### 2.3 State 2 – Austenite wire, external load applied

The State 2 configuration is reached only from State 1 when the wire contracts along its entire length due to a phase change from martensite to austenite initiated by heating the wire. The load function \( M_{ext} \) applied to the output arm in State 1 continues to be applied in State 2. The contraction causes a rotation to the angular position \( \phi^{(A)} \), which relates to the wrap angle and wrap length according to the equation

\[ \phi^{(A)} = \theta_{w}^{(A)} - \theta_{w}^{(0)} = \rho_{SMA} \left( \ell_{w}^{(A)} - \ell_{w}^{(0)} \right), \]

where superscript “A” indicates State 2 austenite. Typical operation involves rotation of the output arm between the State 1 and State 2 positions due to heating and cooling of the SMA wire, and the actuator’s range of motion is defined to be

\[ \delta \phi = \phi^{(M)} - \phi^{(A)}. \]

In the derivation of the actuator’s range of motion, the actuator angles \( \phi^{(M)} \) and \( \phi^{(A)} \) are determined independently for each state based on the analytical modeling approach presented herein and the difference taken between them (Equation 5) to define the final actuator motion.

### 3. ANALYTICAL MODELING

The analytical model for spooled SMA actuators was derived to predict the actuator angle in each state based on the actuator geometry, material properties, and the applied load. The model builds on similar mechanics as belt braking models for passive materials, but takes additional steps to model the active material which introduces complexities to the problem that cannot be accounted for with the typical closed-form equations resulting from belt braking models for passive materials. Additional factors that influence the actuator behavior such as bending of the wire and binding due to friction were not modeled in this paper but limits of these effects are defined.

#### 3.1 Generalized constitutive law

The model is derived using a general form of the constitutive law so that constitutive laws with varying levels of accuracy, simplicity, and ease of use can be applied depending on the application and required accuracy. For this analysis, the strain is assumed to be a function of angle (\( \theta \)), stress (\( \sigma \)) and material phase fraction (\( \zeta \)) according to the general strain function

\[ \varepsilon \{ \theta, \zeta \} = f_{SMA} \{ \sigma \{ \theta \}, \zeta \}, \]

where \( f_{SMA} \{ \} \) is the material constitutive law and the functional dependencies are indicated with curly brackets: \{ \}. Stress varies with position on the wire \( \theta \), and is assumed to be constant in the tails. Tail stress is given by the stress function by evaluating at \( \theta = 0 \) for the input tail and depending on the state at \( \theta = \theta_{w}^{(M)} \) or \( \theta = \theta_{w}^{(A)} \) for the output tail. Assuming that the wire is fully martensite in State 1 (\( \zeta^{(M)} = 1 \)) and fully austenite in State 2 (\( \zeta^{(A)} = 0 \)), the strain can be represented by the simplified functions for strain...
\[
\begin{align*}
\epsilon^{(M)}(\theta) &= f_{\text{SMA}}^{(M)} \left\{ \sigma(\theta), \epsilon^{(M)} = 1 \right\}, & \text{and} & & \epsilon^{(A)}(\theta) &= f_{\text{SMA}}^{(A)} \left\{ \sigma(\theta), \epsilon^{(M)} = 0 \right\} \\
&= f_{\text{SMA}}^{(M)} \left\{ \sigma(\theta) \right\} & & & = f_{\text{SMA}}^{(A)} \left\{ \sigma(\theta) \right\}
\end{align*}
\]  

(7,8)

where \( f_{\text{SMA}}^{(M)} \) and \( f_{\text{SMA}}^{(A)} \) are the constitutive laws for fully martensite and fully austenite wires.

To derive actuator rotation, the change in length of the SMA wire was modeled in each state to relate rotation to the actuator geometry, material properties, and applied load. The generalized constitutive functions are expressed using the simplified functions for strain (Equations 7 and 8) allowing the flexibility to apply different constitutive laws later on. The model considers losses due to friction, the functional dependence of stress and strain on the position along the SMA wire, and a formulation that relates the length of the entire wire to the model’s parameters and the applied load. The model assumes quasi-static motion, Coulomb friction, and motion in the wire in a single direction in each state (i.e., assumption of unilateral contraction and extension).

### 3.2 Derivation of State 1 actuator displacement

Based on the assumption that State 1 wire is stretching along its entire length and that Coulomb friction applies, the wire slides clockwise along the spool with respect to the configuration in Figure 1, so that a differential element of the wire in contact with the spool undergoes the tensile, normal, and friction loads shown in Figure 2a. Based on the free body diagram, the quasi-static force balance in the radial direction (normal to the spool) is

\[
\Sigma F_r = N - (2F_{\text{SMA}} + dF_{\text{SMA}}) \sin (\frac{1}{2} \theta) = 0, \tag{9}
\]

where \( N \) is the normal reaction force and \( F_{\text{SMA}} \) is the tension in the wire. The quasi-static moment balance is

\[
\Sigma M_\theta = \frac{(dF_{\text{SMA}} - \mu N)}{\rho_{\text{SMA}}} = 0 \tag{10}
\]

where \( \mu \) is the coefficient of friction between the spool and the wire. Combining the equations for the force and moment balances (Equations 9 and 10) and assuming that the resulting second order term \( dF_{\text{SMA}} d\theta \) is negligible, the normal reaction forces \( N \) can be cancelled yielding the expression

\[
\mu d\theta = \frac{dF_{\text{SMA}}}{F_{\text{SMA}}}. \tag{11}
\]

Since the total wrapped length cannot be known a priori, the derivation departs from typical belt-braking models by integrating from an unknown intermediate position on the wrapped wire \( \theta (0 < \theta < \theta_{\text{w}}) \) where wire tension = \( F_{\text{SMA}}(\theta) \) to the upper limit at the output tail \( \theta^{(w)}_\text{t} \) where wire tension is known with respect to geometry and the applied moment \( M_{\text{ext}} \), such that

\[
\int_\theta^{\theta^{(w)}} \mu d\theta = \int_{F_{\text{SMA}}(\theta)}^{\sigma_{\text{t, out}}} \frac{dF_{\text{SMA}}}{F_{\text{SMA}}}, \tag{12}
\]

where \( A_{\text{SMA}} \) is the cross-sectional area of the SMA wire and \( \sigma_{\text{t, out}} \) is the stress at the output tail. The stress in the output tail is caused by the external moment and is governed by the equation

\[
\sigma_{\text{t, out}} = \frac{M_{\text{ext}}}{A_{\text{SMA}} \rho_{\text{SMA}}}. \tag{13}
\]

Solving the integrated expression (Equation 12) for \( F_{\text{SMA}}(\theta) \) and dividing by the cross-sectional area \( A_{\text{SMA}} \) yields the State 1 stress as a function of position,

\[
\sigma^{(M)}(\theta) = \sigma_{\text{t, out}} e^{\mu (\theta - \theta^{(w)})}. \tag{14}
\]

Based on the general strain function for the State 1 actuator (Equation 7) and the martensite stress function (Equation 14) the strain as a function of position is

\[
\epsilon^{(M)}(\theta) = f_{\text{SMA}}^{(M)} \left\{ \sigma_{\text{t, out}} e^{\mu (\theta - \theta^{(w)})} \right\}. \tag{15}
\]
The strain function predicts the strain along the portion of the wire in contact with the mandrel, but can also predict for the remaining input and output tail portions by evaluating $\varepsilon(M)\{}\theta=0\}$ for the input tail strain and evaluating $\varepsilon(M)\{}\theta=\theta_{w}^{(M)}\}$ for the output tail strain. Thus, the strain function can be used to predict strain across the entire length of the wire.

Since the State 1 wrap angle cannot be known beforehand, a compatibility equation was derived that relates the deformed length of the SMA wire in State 1 ($\ell_{tot}^{(M)}$) to the undeformed reference length in State 0 ($\ell_{tot}^{(0)}$) based on the strain function (Equation 15). For a differential element of wire, the definition of strain is

$$\varepsilon = \frac{ds^{(M)} - ds^{(0)}}{ds^{(0)}} \quad (16)$$

where $ds^{(0)}$ is the length of a differential element of State 0 wire (undeformed) and $ds^{(M)}$ is the deformed length of the differential element in State 1 (martensite). Solving for $ds^{(0)}$ and integrating across the wire’s length, the equation for strain becomes

$$\int_{0}^{\ell_{tot}^{(0)}} ds^{(0)} = \int_{0}^{\ell_{tot}^{(M)}} (1 + \varepsilon)^{-1} ds^{(M)} \quad (17)$$

Expanding the right hand side of the equation into three integrals for the input tail, wrapped length, and output tail portions of the wire, the State 0 wire length (Equation 17) becomes

$$\ell_{tot}^{(0)} = \int_{0}^{\ell_{in}^{(0)}} [1 + \varepsilon^{(M)}\{}\theta=0\}^{-1} ds^{(M)} + \int_{0}^{\ell_{w}^{(M)}\text{inside}} [1 + \varepsilon^{(M)}\{}\theta=\rho_{SMA}^{(M)}\}^{-1} ds^{(M)} + \int_{0}^{\ell_{out}^{(M)}\text{outside}} [1 + \varepsilon^{(M)}\{}\theta=\theta_{w}^{(M)}\}^{-1} ds^{(M)} \quad (18)$$

With constant strains in the input and output tail and combining with the strain function (Equation 15), the State 0 wire length simplifies to

$$\ell_{tot}^{(0)} = \frac{\ell_{in}^{(0)}}{\int_{0}^{\ell_{w}^{(M)}\text{inside}} [1 + \varepsilon^{(M)}\{}\theta=\rho_{SMA}^{(M)}\}^{-1} ds^{(M)}} + \frac{\ell_{out}^{(0)}}{\int_{0}^{\ell_{w}^{(M)}\text{outside}} [1 + \varepsilon^{(M)}\{}\theta=\theta_{w}^{(M)}\}^{-1} ds^{(M)}} \quad (19)$$

This expression is a compatibility condition that relates the State 0 wire length ($\ell_{tot}^{(0)}$) to the deformed State 1 strain distribution ($\varepsilon^{(M)}\{}\theta\}$, Equation 15) based on the actuator’s geometry ($\ell_{in}^{(0)}$, $\ell_{out}^{(0)}$, $\rho_{SMA}$), material friction ($\mu$), and applied
load \((M_{ext})\) in addition to the equations for the output tail stress (Equation 13) and material stress-strain relationship (Equation 7). Solving for the unknown State 1 wrap angle \(\theta^{(M)}\) is not straightforward since it appears in the limits of integration and within the strain function, which is a transcendental function within the integrand. Since a closed-form solution for \(\theta^{(M)}\) is not possible, iterative solving techniques can be used to satisfy the State 1 compatibility condition (Equation 19).

### 3.3 Derivation of wrap angle compatibility equation for State 2 actuator

The motion and loads of the austenite State 2 actuator are similar to those of the martensite State 1, except that the wire contracts along its entire length rather than stretching, and the motion of the wire is counter-clockwise relative to the drum. Based on the free-body diagram in Figure 2b, the stress and strain functions of angle are derived with the same procedure as used to derive those for State 1 (Equations 14 and 15), except that the opposite contractile motion changes the sign of the exponent, yielding

\[
\sigma^{(A)}\{\theta\} = \sigma_{r,\text{out}}e^{-\mu\theta_{\text{w}}},
\]

and

\[
\varepsilon^{(A)}\{\theta\} = \int_{\theta_{\text{in}}}^{\theta_{\text{out}}} \sigma_{\text{SM}}\{\theta = \rho_{\text{SM}}\varepsilon^{(A)}\} \, ds^{(A)},
\]

The compatibility condition is derived in a similar manner, and results in the equation

\[
l^{(0)}_{\text{tot}} = \ell_{\text{in}} + \int_{\theta_{\text{in}}}^{\theta_{\text{out}}} \left[1 + \varepsilon^{(A)}\{\theta = \rho_{\text{SM}}\varepsilon^{(A)}\} \right]^{-1} ds^{(A)} + \ell_{\text{out}}.
\]

which can be solved to find the State 2 wrap angle \(\theta^{(A)}\) using an iterative algorithm since a closed-form function for the wrap angle cannot be found. The actuator angle equation (Equation 4) is used to find the State 2 actuator angle \(\phi^{(A)}\).

### 3.4 Limitations of the model

The assumption that the wire is only stretching or only contracting in each state ensures that the loads on the wire are in the same direction as those shown in Figure 2, and the derivation of the stress as a function of angle \((\sigma^{(M)}\{\theta\} \text{ and } \sigma^{(A)}\{\theta\})\) is based on this assumption. It is possible for SMA wire wrapped through large angles on the spool to violate the assumption of unilateral motion, and thus an expression was derived to predict whether the unilateral motion assumption is valid. At the input tail \((\theta=0)\), increasing wrap angle causes the strain in martensitic State 1 to decrease (Equation 15) and the strain in austenitic State 2 to increase (Equation 21). Resulting from the increased wrap angle, the strains at the input tail in each state \((\varepsilon^{(M)}\{\theta=0\} \text{ and } \varepsilon^{(A)}\{\theta=0\})\) approach one another as shown in Figure 3a, until they are equal at a critical angle (illustrated in Figure 3b). This critical angle is defined as the binding angle \(\theta_B\), where \(\varepsilon^{(M)}\{\theta=0\} = \varepsilon^{(A)}\{\theta=0\}\), or

\[
\theta_B = \theta^{(A)} \Leftrightarrow \varepsilon^{(M)}\{\theta = 0\} = \varepsilon^{(A)}\{\theta = 0\}.
\]

At the binding angle, the wire neither stretches nor contracts and the assumption of unilateral motion between states is violated. Since no motion occurs, it is possible that there is no change in the stress between states for wire between the output tail at \(\theta=\theta\) and the binding point \(\theta=\theta_B\). Thus, wrapping additional SMA wire beyond the binding angle is hypothesized to contribute no further motion to the actuator.

The model also assumes that the wire is in pure tension and that the SMA wire is flexible such that bending the wire around a curvature has no effect on the behavior. For spool diameters much larger than the wire diameter, the bending strain is negligible. For tighter curvatures, however, the effect of bending is greater and is expected to cause a loss in motion relative to the motion predicted by the model, but analysis and modeling of this effect was beyond the scope of this paper.

### 4. EXPERIMENTAL STUDY

An experimental study was conducted to assess the model’s ability to predict spooled actuator performance and provide insight into the impact of key parameters such as load and wrap angle.
4.1 SMA material properties

The stress-strain behavior of SMA wire can vary widely depending on factors such as composition, manufacturing process, and stress-strain history. In the past 20 years, substantial research has focused on developing constitutive models for SMA including early phenomenological models\textsuperscript{41,43-45}, micromechanical models\textsuperscript{46-50}, coupled thermodynamic and mechanical models\textsuperscript{51,52}, and models predicting the dynamics of phase boundary motion\textsuperscript{53-55}. For this research, a one-dimensional model for the stress-strain behavior is needed for the fully martensite and fully austenite material phases. Many constitutive laws predict the material behavior with respect to temperature, but typical SMA actuators that rely on electrical heating do not have fine temperature control. Rather, the actuator is often heated and cooled to achieve a gross motion without a need to know the transient temperature dependence. In this study, simple polynomial functions for the fully austenite and fully martensite states are used because they can reasonably approximate stress-strain behavior, they greatly reduce the complexity compared to models that are continuous functions of temperature or material phase, and they are quick and easy to evaluate when solving the actuator’s rotation, which is advantageous for iteratively solving the compatibility equations (Equations \ref{eq:compatibility} and \ref{eq:compatibility2}).

A linear stress-strain relation was chosen (slope defined as $E_A$) due to austenite’s linear-elastic behavior below stresses inducing superelastic phase change. A third order polynomial stress-strain relation of the form

$$\sigma^{(M)} = a\varepsilon^3 + b\varepsilon^2 + c\varepsilon + d$$ (24)

was used, where $a$, $b$, $c$, and $d$ are constants such that $d\sigma/d\varepsilon$ is positive for $\varepsilon > 0$. The shape approximates the stress-strain behavior of the material including the martensite plateau, and the function is monotonic and invertible, which is necessary for expressing strain as a unique function of stress (Equation \ref{eq:polynomial}).

To reduce variation in the stress-strain behavior between cycles, the 15 mil, 70°C Flexinol wire (Dynalloy, Inc.) used in this study was shaken-down by cycling it in pure tension through 400 heating/cooling cycles against a 45 N load with maximum strain constrained below a mechanical hard-stop (typically 6% or lower) based on the procedure described by Pathak \textit{et al.}\textsuperscript{7}. The stress-strain curves were measured at room temperature for martensite and heated for austenite by increasing an electrical current until no further motion was observed. The materials constants were determined from the data using least-squares regression. Comparing the experimental stress-strain data to the polynomial fits (Figure \ref{fig:polynomial}), the polynomial provides a reasonable representation of SMA’s material behavior.

4.2 Laboratory setup

To exercise different parameters in the spooling model, a single spool rotary actuator was built and instrumented to allow for the different model parameters to be varied (including spool diameter, wire diameter, wire length, input tail
length, output tail length, spool-wire coefficient of friction, and external load) while measuring the actuator’s rotation angle and the wire tension at each end. The spooling test apparatus shown in Figure 5 is composed of the input structure, the spool, and the rotary arm where output motion occurs. The input structure is a PVC block that electrically insulates the input end of the wire from ground. The PVC block can be positioned to set the length of the input tail ($l_{in}$) and attaches to the SMA wire with an aluminum connector that crimps the wire and a load cell that measures tension in the input tail. The spool is attached to the aluminum structure that is fixed to ground. While the spool is interchangeable to allow for various diameters and materials to be tested, all the results in this paper were performed on a 1.5” diameter Garolite cylinder. The output arm attaches to the wire with an aluminum connector that crimps the wire and a load cell. The arm rotates concentrically about the spool using rotary ball bearings to reduce friction and is instrumented with a rotary encoder to measure the angular position of the arm. External loads were applied to the output arm using pulleys and weights. Data was recorded using a laptop with a USB data acquisition card. The average coefficient of friction was estimated based on load cell measurements and the Capstan equation to be approximately 0.1 – 0.15.

Preparing for a typical spooling test, the SMA wire and mandrel are selected, one end of the wire is clamped to the input structure (which can be positioned to vary the input tail length and wrap angle), the wire is wrapped around the spool, and the other end of the wire is clamped to the output arm (positioned to give a 20 mm output tail length). A test is conducted by applying a load and cycling the wire 3-5 times through 20 seconds of heating (1.8 A current using a DC power supply) and at least two minutes of cooling to provide sufficient time for the wire to completely transition between austenite and martensite in both directions.

4.3 Experimental results

Two sets of experiments were performed to examine the actuator’s range of motion and how it relates to wrap angle and applied load. The prediction for the actuator’s range of motion $\delta \phi$ was determined based on the compatibility equations for States 1 and 2 (Equations 19 and 22). Since closed form solutions of these equations are not possible, a MATLAB code was written to numerically solve the range of motion given the actuator geometry, coefficient of friction, and applied load. For the longer lengths of wire, the predictions are based on the hypothesis that wrapping the wire...
beyond binding makes no further contribution to the motion of the actuator. Thus, the range of motion prediction was calculated at the binding length rather than the full length of the wire.

### 4.3.1 Effect of applied load

To study the effect of applied load for different lengths of wire, experiments were conducted by varying the applied load between 20-570 Nmm while measuring the range of motion for four lengths of wire (80, 200, 300, and 400 mm) with input and output tail lengths of 20 mm. The wire lengths were chosen to demonstrate the actuator’s performance for scenarios in which the entire wire can slip on the mandrel (unilateral motion) and for scenarios in which portions of the wire bind. Unilateral motion is predicted for the 80 and 200 mm lengths, and binding is predicted for the 300 and 400 mm lengths based on the binding condition (Equation 23).

For the shorter lengths of wire (80 and 200 mm, Figure 6a and b) where no binding occurs, there is good agreement between the prediction and the model with the shape of the data well-matched, 14% median error for the 80 mm wire, and 9% median error for the 200 mm wire based on a friction coefficient of 0.125. Since the coefficient of friction is difficult to measure, the data is compared to a range of friction from 0.1 – 0.15 in the figure, with most of the data bound by the predictions in this friction range, especially in the 200 mm experiments. For the experiments where binding occurs (300 and 400 mm lengths, Figure 6c and d), the range of motion prediction was estimated at the binding length based on the assumption that wrapping beyond the binding length would make no further contribution to actuator motion. The shape of the data agrees approximately with the prediction, but the error is increased from the non-binding cases. The median error was 32% for the 300 mm length and 22% for the 400 mm length. Additionally, there was a

![Graphs](a) 80 mm wire length, 0.35 wraps, no binding predicted (b) 200 mm wire length, 1.4 wraps, no binding predicted

![Graphs](c) 300 mm wire length, 2.2 wraps, binding predicted (d) 400 mm wire length, 3.0 wraps, binding predicted

**Figure 6. Effect of applied load on rotation.** The rotation is plotted for four different wire lengths with varying number of wraps. For the cases with no binding (a) or only limited binding (b), the model predicts well, especially at high and low loads. For the cases with binding (c and d), the model was extrapolated based on the maximum rotation of a non-binding actuator at each load. The extrapolated model predicts performance behavior best at high loads, but does not account for the difference in motion between the different lengths of wire. Experiments used 15 mil, 70°C wire conditioned with 45 N load and 4.5% maximum strain.
slight increase in rotation for the 400 mm case despite the prediction that wrapping beyond the binding angle would contribute no further motion. For an actuator with binding, the prediction agrees best at the higher loads, but further model refinement is necessary to provide better predictions for spooled actuators in which binding occurs.

### 4.3.2 Effect of wrap angle

The model’s prediction of the wrap angle was tested by measuring the rotation of a spooled actuator based on a constant length SMA wire with varied wrap angles. Experiments were conducted on a 300 mm wire with 15 N tension applied at the output tail (285 N-mm applied moment). In the experiments, the overall wire length and the output tail length were held constant. The wrap angle was varied by adjusting the length of the input tail such that any length that is not in the input and output tails was wrapped around the mandrel.

The experimental results correlated very well with the theory in both form and magnitude (Figure 7). Results illustrate the actuator’s range of motion initially decreasing and then leveling off as the wrap angle was increased. Additionally, the theory predicts an inflection in the curvature of the rotation angle with respect to the wrap angle, which is supported by the data. Nearly all the data was bounded by the theoretical prediction for the expected range of friction coefficients (0.1 to 0.15). The best-fit theoretical curve was determined with finding the coefficient of friction between the wire and mandrel that minimizes the average error based on a least-squares regression analysis, and has an average error of 5.5%. Experiments for 15 mil, 70°C wire conditioned with 45N load and 6% maximum strain.

![Effect of wrap angle experimental results](image)

**Figure 7.** Effect of wrap angle experimental results. Data represents actuator rotation for a 300 mm SMA wire with varied wrap angles under 15 N load. The best-fit theoretical curve was determined with a least-squares regression analysis, and has an average error of 5.5%. Experiments for 15 mil, 70°C wire conditioned with 45N load and 6% maximum strain.

As predicted by the model, the actuator configurations with the least wrapping had the best performance, and with more compact packaging there was a decline in performance, up to 60% (although even the lowest level of performance was still within a useful range). This trend represents a design tradeoff between packaging and performance. The model presented in this paper can be a useful tool in optimizing the spooled actuator configuration to minimize losses while conforming to a given packaging footprint and represents an important step in addressing the SMA packaging problem.

### 5. CONCLUSIONS

This paper presents the derivation of an analytical model for rotary spooled SMA actuators, which provides a relationship between the active material and frictional losses in addition to the typical architectural geometric parameters and external loading profile. This model did not include bending and binding, but a binding limitation is defined. An experimental study was conducted to assess the model’s ability to predict spooled actuator performance and provide insight into the impact of key parameters such as load (1 – 30 N, ~ 10 – 300 MPa) and wrap angles (between ~30 – 720 degrees). Experiments below the binding limit correlated very well with theory, with errors ranging from 9-14% with an average error of 5.5%, indicating that the key governing mechanics behind spooling are captured in the model for the range of configurations tests. Even in the regime of binding where the assumptions of the model begin to degrade the shape of the behavior was captured though the direct correlation with results decreased as the range of error increased to 22-32%. As predicted by the model, the actuator configurations with the least wrapping had the best performance, and with more compact packaging there was a decline in performance, up to 60% (although even the lowest level of performance was still within a useful range). This trend represents a design tradeoff between packaging and performance. The model presented in this paper can be a useful tool in optimizing the spooled actuator configuration to minimize losses while conforming to a given packaging footprint and represents an important step in addressing the SMA packaging problem.
6. REFERENCES


