Behavioral Model and Experimental Validation for Multiple Spool Packaging of Shape Memory Alloy Actuators

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ABSTRACT

Shape memory alloy (SMA) wire actuation can provide lightweight, low cost actuation to a variety of applications due to the material’s particularly high energy density and the simplistic architectures it allows. Yet, practical execution of the technology is often hindered by difficulty in packaging long lengths of wire that are required for moderate deflections within compact footprints or tight, pre-existing form constraints. To overcome this challenge, the SMA wire can be spool-packaged by wrapping it around pulleys and mandrels to redirect the wire within application-specific form constraints or to reduce the overall actuator footprint dimensions. A generalized architecture for multiple mandrel spool-packaged SMA wire actuators is presented, which allows for a wide range of actuators to be designed with varying topologies (number and placement of mandrels), geometries (specific dimensions of the mandrels and the SMA wire), applied load, and material properties. Performance losses due to friction and bending accompany the enhanced packaging, posing a design tradeoff. Foundational models, which account for the SMA material behavior, losses due to friction, and effects resulting from binding, are developed to predict the range of motion of multiple mandrel spool-packaged SMA wire actuators with respect to their geometry, loading, and material parameters. To verify the mechanics of multiple spool actuators, an experimental study was conducted regarding the effect of the number of mandrels and applied load for multiple mandrel configurations. Based on the promising results of this study, the multiple mandrel spooling technique and the accompanying behavioral model provide a foundation for designing compact, form customizable, and high performance SMA wire actuators.

Keywords: shape memory alloy (SMA), spooling, actuator, actuator packaging, GM/UM SMS Collaborative Research

1. INTRODUCTION

The need for compact, lightweight, and cost-effective actuation is common – often critical – to applications in the industrial, military, and medical sectors. While conventional actuators (motors, solenoids, hydraulics, etc.) are typically used to produce motion and forces in devices, SMA actuators

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are becoming an increasingly attractive alternative based on their high energy densities, simplistic architectures, and low cost. While many of the technical challenges regarding SMA (actuation speed, mechanical attachment/connections, performance shakedown over time, etc.) are currently being addressed and resolved [1-8], the need to package SMA compactly without sacrificing its high energy density continues to be an issue. SMA wires can be packaged more compactly by wrapping the wires around pulleys or mandrels, allowing for long lengths of SMA wire to be spooled into a more convenient form factor. While the spooled-packaged architecture does allow for more compact and customizable packaging, spooled actuators undergo some reductions in performance and fatigue lifetime due to friction on the SMA wire and bending strains resulting from the induced curvature in the spooled portions of the wire. Despite these losses, spooled actuators have been demonstrated to produce useful levels of motion in a range of applications including active latches, handgrip stabilization, a hood lifter for pedestrian protection, and biomedical devices [4,6,7,9-12]. Additionally, these losses are predictable and have been modeled for single spool actuators [13-15]. This earlier work focused on developing the fundamental models based on single spool architectures, but further design flexibility and a greater ability to customize the actuator form factor is possible based on multiple spool architectures that redirect SMA wires around the perimeter of a form factor or along a non-linear pathway, which can be specific to the needs/allowances of a particular application. Reduced fatigue lifetime can result from the spooled packaging due to tight curvatures and increased bending strains in the wire. Yet, there are many categories of applications that require less than a few thousand cycles and can therefore accept the reduced lifetimes associated with packaging around very small mandrels [4,7,16-19].

In this paper, a model for a generalized multiple spool SMA wire actuator is derived which relates the motion of the SMA actuator to its packaging configuration and the related geometry of the actuator, the external applied load profile, the interfacial friction and the constitutive material properties. The generalized architecture includes a single SMA wire attached to ground at one end with intermediate portions of the wire wrapped around one or more mandrels. The opposite end of the wire is attached to a rotational or linear single degree-of-freedom motion constraint where the SMA wire interacts with the external system. The model is derived to predict the actuator’s overall displacement in each of three operation states (martensite, austenite, and a zero-strain reference state), and captures complexities introduced by the use of an active material that cannot be accounted for with the typical closed-form equations resulting from belt braking models for passive materials. The model also predicts the effects of binding due to accumulated friction and bending due to the induced curvature, and adjusts and limits the predicted performance accordingly. The model was validated experimentally with respect to varying configuration-related parameters including the number of spools and the wrap angle around each spool. Based on the packaging technique and model presented in this paper, compact and customizable SMA actuators with predictable, high performance can be designed for a broad range of applications bringing about lightweight, low-cost, and simplistic actuation as a promising alternative to conventional actuation strategies.

2. ARCHITECTURE AND OPERATION

Shape memory alloy actuators exploit the material’s unique thermo-mechanical behavior to produce reversible strain deformations and overall gross motion as the material is thermally cycled between the stiff austenite phase at high temperatures and the more compliant martensite phase at low temperatures. Below the material’s characteristic transformation temperature, the martensite SMA can be subjected to residual strains up to 8% without inducing permanent plastic strain. Upon heating, the material reverts to the stiffer austenite phase, and the residual strains are recovered. Spool-packaged SMA wire actuators can vary widely according to their topology (number and placement of spools), type of output motion (rotational or linear), external loading profile, and
frictional and constitutive material properties. Yet, the broad range of potential designs can be generalized according to a basic architecture. To illustrate the basic architecture and operation for spool-packaged SMA wire actuators, the fundamental single mandrel case is described first, and then expanded to the multiple mandrel generalization.

2.1. Single mandrel spool-packaged SMA wire actuators

Single mandrel spool-packaged SMA wire actuators are comprised of four key elements: 1) a single SMA wire in tension, 2) a fixed input where the SMA wire attaches to a referenced ground, 3) a cylindrical mandrel fixed to ground (as opposed to a free-wheeling pulley), and 4) a rotational or linear single degree-of-freedom motion output. The SMA wire is fixed to ground at the input tail, has a wrapped portion in frictional contact with the mandrel, and an output tail connected to the motion output. The SMA wire interacts with the external system at the motion output, which is constrained by either a rotational arm assumed to pivot around the center of the spool (Figure 1a) or a linear slider (Figure 1b). The motions for the rotational and linear cases are predicted as two cases of the same model, which are distinguished by the constraint at the motion output and whether the output tail length or wrapped length is constant.

The actuator’s operation cycle results from the thermal cycling of the SMA wire causing it to contract along its length upon heating to austenite and extend along its length upon cooling to martensite. The net contraction and extension of the wire as it is thermally cycled results in intermediate portions of the SMA wire sliding along the mandrels and either linear or rotational motion at the actuator’s output. To predict the actuator’s range of motion through a full operation cycle, the geometry is defined with respect to three main operation states:

- **State 0**: the zero-strain reference state in which no external load is applied to the actuator (state variable $\chi = 0$),
- **State 1**: a martensite phase SMA wire ($\chi = M$, martensite phase fraction $\xi^{(M)}(M) = 1$) with an applied external load ($M_{ext}$ or $F_{ext}$), and
- **State 2**, for an austenite phase SMA wire ($\chi = A$, martensite phase fraction $\xi^{(M)}(M) = 0$) with an applied external load ($M_{ext}$ or $F_{ext}$).

All motions, geometries, and strains of the actuator are referenced to the zero-strain reference state, **State 0** (diagrammed in orange, Figure 1), which is attained by heating the SMA wire to austenite under no load and maintained in **State 0** upon cooling assuming a negligible two-way effect in the SMA material. **State 1** (martensite SMA actuator under an applied load, blue) is achieved by stretching the SMA wire along its length to $\ell_{tot}^{(M)}$, either by applying a load to a martensite **State 0** actuator or by cooling an austenite **State 2** actuator under load. Likewise, the **State 2** (austenite SMA actuator under an applied load, red) is achieved by heating the SMA wire, causing it to contract to $\ell_{tot}^{(A)}$. In the linear and rotational motion cases, different geometric parameters vary throughout operation; whereas the wrapped length is constant and the output tail length varies in the linear motion case, and the wrapped length varies and the output tail is constant for rotational motion actuators.

The overall change in the SMA wire length results in translational output motion $\delta \ell$ for linear motion actuators and in rotational output motion $\delta \phi$ for rotational motion actuators. For the linear actuator, the output motion is equal to the change in the total SMA wire length, according to the equation

$$\delta \ell = \ell_{tot}^{(M)} - \ell_{tot}^{(A)}$$  \hspace{1cm} (1)
where $M_{\text{tot}}$ and $A_{\text{tot}}$ are the State 1 and 2 total wire lengths. Since the input and wrapped lengths of the wire do not vary during operation, the change in length is also equal to the difference between the State 1 and State 2 output tail lengths, $M_{\text{t out}}$ and $A_{\text{t out}}$. For the rotational actuator, the angular output motion is geometrically related to the overall change in the wire length according to the equation

$$\delta \phi = \frac{\delta \ell}{D_w / 2} = \frac{\ell_{M_{\text{tot}}} - \ell_{A_{\text{tot}}}}{D_w / 2}, \quad (2)$$

where $D_w$ is the wrapped diameter of the spooled SMA wire. The wrapped diameter $D_w$ is measured from the centroidal axis of the SMA wire through the mandrel center to the SMA wire centroid on the opposite side such that

$$D_w = D + d_{\text{SMA}} \quad (3)$$

where $d_{\text{SMA}}$ is the SMA wire diameter. For the rotational case, the input and output tail lengths are constant throughout operation. Thus, the change in length is also equal to the difference between the State 1 and State 2 wrapped lengths, $\ell_{M_{\text{w}}}$ and $\ell_{A_{\text{w}}}$. In addition, the change in actuator angle is equal to the difference between the State 1 and State 2 wrap angles, $\theta_{M_{\text{w}}}$ and $\theta_{A_{\text{w}}}$. To predict the actuator’s range of motion for both linear and rotational cases, the SMA wire lengths $\ell_{M_{\text{tot}}}$ and $\ell_{A_{\text{tot}}}$ are determined independently for each state by means of the analytical model presented herein. The arithmetical difference taken between the two lengths, using Eqs. 1 and 2, defines the actuator’s range of motion.

### 2.2. Multiple mandrel spool-packaged SMA wire actuators

To enable the design and analysis of a broader range of spool-packaged SMA wire actuators, the single mandrel actuator architecture can be expanded to multiple mandrels. The multiple mandrel architecture includes the same key elements as for single mandrel actuators, except that more than one mandrel can be used. The components include: 1) a single SMA wire in tension (more complex actuators with multiple wires can be considered to be multiple spool-packaged SMA wire actuators in parallel or series), 2) a fixed input where the SMA wire attaches to a referenced ground, 3) $n$
cylindrical mandrels \((n \geq 1)\) fixed to the referenced ground, and 4) a rotational or linear single degree-of-freedom motion output. The basic architecture, nomenclature, and numbering conventions of a general \(n\) spool SMA wire actuator with a linear output motion are shown in Figure 2. The same architecture and numbering applies to the rotational case, but with a rotating boundary constraint alternatively placed on the output end of the SMA wire.

The SMA wire is composed of \((2n+1)\) regions with \((n+1)\) linear segments and \(n\) wrapped segments. Like the single mandrel case, the linear segment attached to the fixed input \((i = 1)\) is referred to as the “input tail”, and the linear segment attached to the movable output \((i = n+1)\) is named the “output tail”. Lengths of the first \(n\) linear segments are indicated with the variable \(\ell_{lin,i}^{(x)}\), and the output length is indicated with the variable \(\ell_{out}^{(x)}\). As a convention, superscripts indicate operation state but may be omitted if the variable does not change between states. Thus, for a linear actuator only the output tail \(\ell_{out}^{(x)}\) requires the superscript and for a rotational actuator, only variables regarding the \(n^{th}\) wrapped segment \((\theta_{w,n}, \Theta_{w,n}, \phi_{w,n})\) require the superscript. The angular position on the \(i^{th}\) mandrel is indicated using the variable \(\theta_{i}\) and the total wrap angle on the \(i^{th}\) mandrel

**Figure 2. Generalized multiple mandrel architecture for spool-packaged SMA wire actuators with linear output motion.** The multiple mandrel architecture enables the design and analysis of a wide range of spool-packaged architectures by varying the topology (number and position of mandrels), geometry (SMA wire length, mandrel dimensions, and wrap angles), and type of output motion (linear or rotational).
is $\theta_{w,i}$. The cumulative angular position $\Theta$ of the SMA wire on the $j^{th}$ mandrel ($1 \leq j \leq n$) is measured from the running-on point of the first mandrel ($i=1$) such that

$$\Theta = \sum_{j=1}^{j=\text{last}} (\theta_{w,j} + \theta_j) .$$  \hspace{1cm} (4)

For the $j^{th}$ mandrel, the wrap angle is $\theta_{w,j}$ and the cumulative wrap angle is

$$\Theta_{w,j} = \sum_{i=1}^{i=\text{last}} \theta_{w,j} .$$  \hspace{1cm} (5)

The total State 0 length $l^{(0)}_{\text{tot}}$ is defined as a summation of the alternating linear and wrapped segments such that

$$l^{(0)}_{\text{tot}} = \sum_{i=1}^{n} \left( l^{(0)}_{\text{lin},i} + l^{(0)}_{w,i} + l^{(0)}_{i,\text{out}} \right) .$$  \hspace{1cm} (6)

where $l^{(0)}_{\text{lin},i}$ is the length of the $i^{th}$ linear segment, $l^{(0)}_{w,i}$ is the length of the $i^{th}$ wrapped segment, and $l^{(0)}_{i,\text{out}}$ is the length of the output linear segment. Since the wrapped segments are commonly defined according to their angle rather than length, the State 0 total length (Eq. 6) can alternatively be defined as

$$l^{(0)}_{\text{tot}} = \sum_{i=1}^{n} \left( l^{(0)}_{\text{lin},i} + \frac{1}{2} D_{w,i} \cdot \theta^{(0)}_{w,i} \right) + l^{(0)}_{i,\text{out}} .$$  \hspace{1cm} (7)

where $D_{w,i}$ is the mean diameter (sum of mandrel and SMA wire diameters) of the $i^{th}$ wrapped segment and $\theta^{(0)}_{w,i}$ is the wrap angle of contact of the $i^{th}$ wrapped segment.

The overall range of motion results from the change in length of the actuator between States 1 and 2 such that

$$\delta l = l^{(M)}_{\text{tot}} - l^{(A)}_{\text{tot}} = l^{(M)}_{i,\text{out}} - l^{(A)}_{i,\text{out}} .$$  \hspace{1cm} (8)

for linear actuators, and

$$\delta \phi = \frac{l^{(M)}_{\text{tot}} - l^{(A)}_{\text{tot}}}{\frac{1}{2} D_{w,n}} = \theta^{(M)}_{w,n} - \theta^{(A)}_{w,n} .$$  \hspace{1cm} (9)

for rotational actuators, where $\delta l$ is the range of motion for linear actuators and $\delta \phi$ is the range of motion for rotational actuators.

The generalized architecture and basic operation cycle enable a wide range of spool-packaged SMA wire actuators to be defined according to their topology, geometry, and type of output motion. The architecture provides a framework for modeling the mechanics of spool-packaged SMA wire actuators, predicting their motion, and enabling systematic, model-based actuator design with optimized performance.

### 3. ANALYTICAL MODEL

To predict the actuator’s range of motion, an analytical model was derived, which accounts for the actuator’s topology, geometry, material properties, and the applied load. The motion of SMA wire
actuators results from changes in strain as the actuator is thermally cycled between the martensite phase (State 1) and the austenite phase (State 2). This change in strain accumulates to provide gross motion, typically proportional to the overall wire length for a linear (non-packaged) SMA actuator. However, predicting the motion of spooled SMA actuators must also account for variations in strain along the length due to friction and across the cross-section due to bending strains. The model builds on similar mechanics as belt braking models for passive materials, but takes additional steps to model the active material. Additionally, due to portions of the SMA wire that gain and lose contact with the mandrels during operation, the integration of strains along the length is not straightforward and cannot be solved with the same closed-form equations that would be used for solving the deformation of straight SMA wire that is not spooled. In this derivation, the constitutive law is expressed in a simplified form, the effect of bending on strains across the cross-section of the SMA is modeled as a function of applied stress and the degree of bending, the variation in strain along the length that results from friction is modeled, and the range of motion is solved based on a compatibility equation that ensures that all portions of the wire are accounted for including those that gain and lose contact with the different mandrels throughout the operation.

3.1. Generalized constitutive law

To relate the stresses in the SMA wire to the strain, which accumulates to provide the actuator’s gross motion, a material constitutive law is needed. A generalized form of the constitutive law is defined to enable the use of a variety of established models (such as phenomenological models [20-23], micromechanical models [24-28], coupled thermodynamic and mechanical models [29,30], or models predicting the dynamics of phase boundary motion [31-33]). Selection of a constitutive model depends on the particular needs of an application with regard to accuracy, simplicity, and rigor. Different material formulations can be applied so long as they relate stress, strain, and material phase with a continuous and monotonic relationship between stress and strain in each state to ensure physical significance and uniqueness to the solution of the actuator’s range of motion. Representing the law in its generalized form, the strain is assumed to be a function of stress ($\sigma$) and martensite phase fraction ($\xi (M)$) according to the function:

$$\varepsilon \{\sigma, \xi (M)\} = f_{SMA} \{\sigma, \xi (M)\}. \quad (10)$$

Since the function is continuous and monotonic, it can be inverted to also describes stress as a function of strain and material phase such that

$$\sigma \{\varepsilon, \xi (M)\} = f_{SMA}^{-1} \{\varepsilon, \xi (M)\}. \quad (11)$$

Stress varies with the position of the wire on the $i^{th}$ mandrel $\theta_i$, and is assumed to be constant in the linear portions. Assuming that the wire is fully martensite in State 1 ($\xi (M) = 1$) and fully austenite in State 2 ($\xi (M) = 0$), the strain is represented by the simplified functions for strain

$$\varepsilon (M) = f_{SMA} \{\sigma \{\theta_1\}, \xi (M) = 1\} = f_{SMA}^{(M)} \{\sigma \{\theta_1\}\}, \quad (12)$$

$$\varepsilon (A) = f_{SMA} \{\sigma \{\theta_1\}, \xi (M) = 0\} = f_{SMA}^{(A)} \{\sigma \{\theta_1\}\}, \quad (13)$$

where $f_{SMA}^{(M)}$ and $f_{SMA}^{(A)}$ are the constitutive laws for fully martensite and fully austenite material.
3.2. Bending strains

To predict the effect of bending, the variation in strain over the wire’s cross-section is modeled by considering the strains normal to the cross-section, which originate from two sources as illustrated Figure 3: 1) the tensile load on the wire, which is assumed to act evenly over the cross-section, and 2) the geometrically induced bending. Assuming plane sections of the wire’s cross-section remain plane, the average strain occurs at the wire’s centroidal axis, and is thus used to determine the overall wire deformation. Based on the principle of superposition, the net strain across the wire’s cross-section is

\[ \varepsilon_{\text{net}}(y) = \varepsilon_{\text{nom}} + \varepsilon_{b}(y) = \varepsilon_{\text{nom}} + \frac{y - e_{\text{N.A.}}}{2D_{w,i}} + e_{\text{N.A.}} \]  

where \( \varepsilon_{\text{nom}} \) is the nominal strain due to the tensile load, \( \varepsilon_{b} \) is the strain due to bending, \( y \) is the distance from the centroidal axis, and \( e_{\text{N.A.}} \) is the distance of the neutral axis from the centroidal axis.

To determine the neutral axis position \( e_{\text{N.A.}} \), equilibrium of tensile forces on the wire must be satisfied such that

\[ F_{\text{nom}} = \sigma_{\text{nom}} \cdot A_{\text{SMA}} = \int_{A_{\text{SMA}}}^{1} f_{\text{SMA}}^{-1} \left\{ \varepsilon_{\text{nom}} + \frac{y - e_{\text{N.A.}}}{2D_{w,i}} \varepsilon \right\} dA_{\text{SMA}} \]  

where \( A_{\text{SMA}} \) is the cross-section area of the wire, \( \sigma_{\text{nom}} \) is the nominal portion of the stress uniformly distributed over the wire’s cross-section, and \( f_{\text{SMA}}^{-1} \) is the constitutive function relating the net strain and material phase fraction to the stress of the SMA wire. If the geometry, nominal load, and material properties are given, the neutral axis position \( e_{\text{N.A.}} \) can be determined from the nominal load equilibrium relationship (Eq. 15). For a particular material constitutive law, the neutral axis position depends on the nominal force \( \sigma_{\text{nom}} \) and the wrap diameter \( D_{w,i} \) represented by the functional dependency

\[ e_{\text{N.A.}} = e_{\text{N.A.}} \left\{ \sigma_{\text{nom}}, D_{w,i} \right\} \]  

The average strain in the wire is determined by evaluating \( \varepsilon \) at the centroid \( (y=0) \), yielding the centroid strain:

\[ \varepsilon_{\text{ctd}} = \varepsilon_{\text{ctd}} \left\{ \sigma_{\text{nom}}, D_{w,i} \right\} = \varepsilon_{\text{nom}} - \frac{e_{\text{N.A.}} \left\{ \sigma_{\text{nom}}, D_{w,i} \right\}}{2D_{w,i} + e_{\text{N.A.}} \left\{ \sigma_{\text{nom}}, D_{w,i} \right\}} \]  

Based on the equation for centroid strain and assuming that the neutral axis shift remains bounded by the SMA wire diameter, the effect of bending becomes negligible for large wrap diameters \( (D_{w,i} \gg d_{\text{SMA}}) \) and has an increasing impact on the centroid strain as the wrap diameter decreases.

Figure 3. Tensile strains acting on a cross-section of spooled SMA wire. The net strain acting on the cross-section is composed of the nominal strain (resulting from tensile loads on the wire) and the bending strain (resulting from the curvature of the wire), combined based on the principle of superposition.
3.3. Friction losses

Whereas the bending losses affect the strain across the SMA wire’s cross-section, the friction losses act along the length of the spooled portion of the wire due to the relative sliding between the wire and mandrel as the wire changes phase between martensite and austenite. Assuming Coulomb friction, unilateral extension of the wire as it transitions to martensite, and unilateral contraction during the transition to austenite, the applied loads and wrapped geometry for spooled SMA wire actuators result in the tensile, normal, and friction loads on a differential element of wire in sliding contact with the mandrel (Figure 4). The relative motion and loads of the State 2 austenite actuator are modeled similarly to those in the State 1 martensite actuator, except that the wire contracts rather than extends such that the friction forces are in the opposite direction (counterclockwise friction acting on the wire for State 1, clockwise friction for State 2 for an actuator oriented as in Figure 1).

From the free body diagram of the differential element of SMA wire in contact with the i-th mandrel (Figure 4), the quasi-static force balance in the radial direction (normal to the spool) is

$$\Sigma F_r = N - (2F_{nom} + dF_{nom}) \sin \left( \frac{1}{2} d\theta \right) = 0,$$

where $N$ is the normal reaction force and $F_{nom}$ is the nominal tension in the wire that results from the applied load. The quasi-static moment balance is

$$\Sigma M_0 = (dF_{nom} \pm \mu N) \frac{D_{w,i}}{2} = 0$$

where $\mu$ is the coefficient of friction between the mandrel and the wire, and the positive or negative sign leading the $\mu N$ term depends on whether the actuator is extending (negative for State 1, $\chi = M$) or contracting (positive for State 2, $\chi = A$). Combining the equations for the force and moment balances, (Eqs. 18 and 19), and assuming that the resulting second order term $dF_{nom} d\theta_i$ is negligible, the normal reaction forces $N$ cancel to yield the expression

$$\pm \mu d\theta_i = \frac{dF_{nom}}{F_{nom}}.$$

This expression relates the variation in nominal wire tension due to friction to the angular position $\theta_i$ of the SMA wire on the mandrel for each of the $n$ wrapped segments such that $0 < \theta_i < \theta_{w,i}$ . Rather than modeling the variation in tension separately for wire in contact with each mandrel, the expression can be simplified by defining the variation in tension with respect to the cumulative angular position $\Theta$. Thus, the variation in tension with respect to cumulative angular position is

$$\pm \mu d\Theta = \frac{dF_{nom}}{F_{nom}}$$

for $0 < \Theta < \Theta_{w,n}$ . Integrating the expression for variation in tension (Eq. 21) from a general angular position $\Theta$ where the wire tension $F_{nom}\{\Theta\}$ is unknown to the upper

Figure 4. Key loads acting on a differential element of spooled wire in sliding contact with the mandrel. In State 1 (martensite), the wire extends, producing counterclockwise friction (based on the actuator configuration in Figure 1). In State 2 (austenite), the wire contracts, producing clockwise friction.
limit $\Theta^{(x)}_{w,n}$ where the applied tension is known, yields the integrated variation in tension:

$$\pm \int_{\Theta}^{\Theta} \mu d\Theta = \frac{\int_{F_{\text{ext}}}^{F_{\text{nom}}} \frac{dF_{\text{nom}}}{F_{\text{nom}}}}{\int_{\Theta}^{\Theta} \mu d\Theta}. \quad (22)$$

Solving the integrals for the nominal tension $F_{\text{nom}}(\Theta)$ and dividing by the SMA wire’s cross-sectional area $A_{\text{SMA}}$ yields the nominal State $\chi$ stress as a function of position:

$$\sigma_{\text{nom}}^{(x)}(\Theta) = \sigma_{\text{t, out}}^{e^{\pm\mu(\Theta - \Theta_{n,n})}}. \quad (23)$$

To determine the average strain of the SMA wire, which occurs at its centroid, the appropriate constitutive law, Eq. 9 or 10 depending on the SMA material phase, is substituted into the equation for centroid strain (Eq. 17), yielding

$$e_{\text{cd}}^{(x)} \left\{ \sigma_{\text{nom}}^{(x)}(\Theta), D_{w,i} \right\} = \frac{\int_{F_{\text{ext}}}^{F_{\text{nom}}} \frac{dF_{\text{nom}}}{F_{\text{nom}}}}{\frac{1}{2} D_{w,i} + e_{N.A.} \left\{ \sigma_{\text{nom}}^{(x)}(\Theta), D_{w,i} \right\}}. \quad (24)$$

Substituting in the nominal stress as a function of cumulative angular position $\Theta$ (Eq. 23), the centroid strain equation (Eq. 24) predicts the motion for all portions of the wire in frictional contact with the mandrels according to the equation

$$e_{\text{cd}}^{(x)} \left\{ \Theta, D_{w,i} \right\} = \frac{\int_{F_{\text{ext}}}^{F_{\text{nom}}} \frac{dF_{\text{nom}}}{F_{\text{nom}}}}{\frac{1}{2} D_{w,i} + e_{N.A.} \left\{ \sigma_{\text{nom}}^{(x)}(\Theta), D_{w,i} \right\}}. \quad (25)$$

To predict strain in the linear portions of the SMA wire, the equation for centroid strain is evaluated at each boundary between the wrapped and linear regions, which occur at $\Theta_{w,i}$ for all integer values of $i$ such that $0 \leq i \leq n$. Since the bending portion of the strain is absent in the linear segments, the centroid strain (Eq. 25) simplifies to the nominal term yielding

$$e_{\text{l, in, i}}^{(x)} = \int_{F_{\text{ext}}}^{F_{\text{nom}}} \left\{ \sigma_{\text{t, out}}^{e^{\pm\mu(\Theta_{n,i-1} - \Theta_{n,i})}} \right\} \text{ for } 1 \leq i \leq n, \quad (26)$$

in the first $n$ linear segments, and

$$e_{\text{l, out}}^{(x)} = \int_{F_{\text{ext}}}^{F_{\text{nom}}} \left\{ \sigma_{\text{t, out}}^{e^{\pm\mu(\Theta_{n,n})}} \right\} \quad (27)$$

in the output tail ($i = n + 1$). Using the expressions for the strain across the entire length of the wire, Eqs. 25-27, which predict for the effects of friction and bending, the State 1 and 2 lengths of wire can be determined to solve for the overall range of motion.

### 3.4. Actuator range of motion

While the set of equations for strain (Eqs. 25-27) defines the strain along the entire length of the wire, the deformation that results from strain cannot be expressed simply by closed-form equations. In a linear wire actuator, the State $\chi$ change in wire length from State 0 would be determined by multiplying the State 0 length and the State $\chi$ strain $e^{(x)}$, which is constant along the length of a non-packaged wire. But for a spooled SMA wire actuator, the State $\chi$ strain equations are specific to
particular regions of the wire, portions of the wire transition between the wrapped and linear regions throughout operation, and the strain is neither constant along the length nor the cross-section of the wire. These computational difficulties are overcome by deriving a compatibility condition that ensures all portions of the wire are accounted for when integrating strain, including those that gain and lose contact with the mandrel during operation. The derivation begins by relating the State 0 and State \( \chi \) lengths of a differential element of wire, \( ds^{(0)} \) and \( ds^{(\chi)} \), using the definition of strain:

\[
e^{(x)} = \frac{ds^{(x)} - ds^{(0)}}{ds^{(0)}}.
\]

Solving for \( ds^{(0)} \) and integrating across the wire’s length yields the State 0 total wire length, related to the State \( \chi \) strain distribution according to the equation:

\[
\ell^{(0)}_{\text{tot}} = \int_0^{\ell^{(0)}_{\text{tot}}} ds^{(0)} = \int_0^{\ell^{(\chi)}_{\text{tot}}} \left(1 + e^{(x)}\right)^{-1} ds^{(x)}.
\]

Expanding the right-hand-side of the equation into separate integrals for the alternating linear and wrapped portions, the State 0 wire length (Eq. 29) becomes

\[
\ell^{(0)}_{\text{tot}} = \sum_{i=1}^{n+1} \int_0^{\ell^{(x)}_{\text{lin},i}} \left[1 + f^{(x)}_{\text{SM}} \left\{ \sigma_{\text{t, out}} e^{\left[\frac{\varepsilon_{\text{eff}}(\Theta_{\text{in},i-1})}{\Theta_{\text{out,n}} - \Theta_{\text{in},i}} \right]} \right\} \right]^{-1} ds^{(x)} + \sum_{i=1}^{n} \int_{\ell^{(x)}_{\text{lin},i}}^{\ell^{(x)}_{\text{lin},i+1}} \left[1 + e^{(x)}_{\text{el}} \left\{ \frac{s^{(x)}}{2D_{w,i}} \right\} \right]^{-1} ds^{(x)}.
\]

With constant strains in each of the \((n+1)\) linear portion and continuous strains between the \( n \) wrapped portions, the State 0 wire length equation (Eq. 30) simplifies to

\[
\ell^{(0)}_{\text{tot}} = \sum_{i=1}^{n+1} \int_0^{\ell^{(x)}_{\text{lin},i}} \left[1 + f^{(x)}_{\text{SM}} \left\{ \sigma_{\text{t, out}} e^{\left[\frac{\varepsilon_{\text{eff}}(\Theta_{\text{in},i-1})}{\Theta_{\text{out,n}} - \Theta_{\text{in},i}} \right]} \right\} \right]^{-1} ds^{(x)} + \sum_{i=1}^{n} \int_{\ell^{(x)}_{\text{lin},i}}^{\ell^{(x)}_{\text{lin},i+1}} \left[1 + e^{(x)}_{\text{el}} \left\{ \frac{s^{(x)}}{2D_{w,i}} \right\} \right]^{-1} ds^{(x)}.
\]

Upon applying the boundary conditions for either rotational or linear output motion, and given the geometry, material constitutive law, coefficient of friction, and external applied load, only one geometric variable is unknown: the output tail length \( \ell^{(x)}_{\text{lin},i+1} \) for linear actuators, or the \( n \)th cumulative wrap angle \( \Theta^{(x)}_{\text{lin},n+1} \) for rotational actuators. Solving the resulting compatibility equation for the unknown variable, the actuator deformation can be solved based on the change in the unknown variable between States 1 and 2, such that

\[
\delta \phi = \Theta^{(M)}_{\text{lin},n} - \Theta^{(A)}_{\text{lin},n}
\]

for rotational actuators, and

\[
\delta \ell = \ell^{(M)}_{\text{lin},i+1} - \ell^{(A)}_{\text{lin},i+1} = \ell^{(M)}_{\text{lin},i} - \ell^{(A)}_{\text{lin},i}
\]

for linear actuators.
3.5. Binding limitation

The assumption that the wire is only stretching or only contracting in each state (unilateral motion) requires that the loads on the wire are in the same direction as those indicated for each state in the differential element in Figure 4. It is possible for SMA wire wrapped through large angles on the spool, often occurring after multiple wraps, to violate the assumption of unilateral motion, and thus an expression was derived to predict whether the unilateral motion assumption is valid. At the input tail \((\Theta = 0)\), increasing the cumulative wrap angle \(\Theta_{w,n}^{(0)}\) causes the input tail strain in the martensite State 1 actuator to decrease accordingly and the input tail strain in the austenite State 2 actuator to increase. Resulting from the increased cumulative wrap angle, the strains at the input tail in each state \(\varepsilon^{(M)}(\Theta = 0)\) and \(\varepsilon^{(A)}(\Theta = 0)\) approach one another as shown in Figure 5a, until they are equal at a critical angle (illustrated in Figure 5b). This critical angle is defined as the binding angle \(\Theta_B\), where \(\varepsilon^{(M)}(\Theta = 0) = \varepsilon^{(A)}(\Theta = 0)\), or

\[
\Theta_B = \Theta_{w,n}^{(0)} \iff \varepsilon^{(M)}(\Theta = 0) = \varepsilon^{(A)}(\Theta = 0).
\] (34)

At the binding angle, the wire neither stretches nor contracts and the assumption of unilateral motion between states is violated. Since no motion occurs, no change in the stress or strain can occur between states for wire between the output tail and the binding point \((0 < \Theta < (\Theta_{w,n}^{(0)} - \Theta_B))\). Thus, wrapping additional SMA wire beyond the binding angle is hypothesized to contribute no further motion to the actuator. For actuators in which binding occurs, the range of motion can be found by analyzing an equivalent actuator in which the wire is rigidly attached to the mandrel at the binding point and all portions between the input and the binding point are discarded.

4. EXPERIMENTAL VALIDATION

To validate the model and provide further insight into the behavior of multiple mandrel spool-packaged SMA wire actuators, an experimental study was conducted. The basic mechanics regarding spool-packaged SMA wire actuators were experimentally confirmed for single spool actuators with respect to a number of actuator parameters including the type of output motion (linear

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**Figure 5. Effect of increase in wrap angle as it leads to frictional binding.** a) As the wrap angle is increased, the strain in the input tail \(\varepsilon_{\text{in}}^{(M)}\) increases in austenite and decreases in martensite in response to the friction opposing the motion between the wire and mandrel as predicted by Eq. 25. b) The binding condition occurs at the angle \(\Theta_B\) where the strains in the input tail are equal in each state, violating the assumption of unilateral motion.
or rotational), applied external load, wrap angle, and mandrel to SMA wire diameter ratio in previous work by the authors [13-15]. Due to the increased complexity of the multiple spool model and the multiple regions that gain and lose contact with the mandrel during operation, additional experiments were conducted on multiple spool actuators. Utilizing an experimental apparatus for multiple mandrel spool-packaged SMA wire actuators, physical actuators were tested with respect to different packaging configurations and applied load.

4.1. Experimental Setup and Procedure

The experimental apparatus (Figure 6) is made up of the same four key elements of a basic multiple mandrel spool-packaged SMA wire actuator: 1) a single 0.38 mm diameter SMA wire in tension, 2) a fixed input where the SMA wire attaches to ground, 3) one or more fixed cylindrical mandrels, and 4) the motion output where the SMA wire interacts with applied loads from the external system. An aluminum fixture rigidly positions four Garolite cylinders (25.4 mm diameter) on a 50.8 mm square grid. Four PVC input fixtures with aluminum crimp blocks attach the input end of the SMA wire to ground and are positioned around the mandrel fixture to allow the number of mandrels in contact with the wire \( n \) to be varied experimentally by selecting the appropriate length of SMA wire and input fixture position (Figure 7). While a mandrel material with a very low friction coefficient can provide the greatest range of motion in practical applications, Garolite was selected due to its good wear and electrical insulation properties and its known, consistent friction properties. The consistent frictional properties help to reduce uncertainty with respect to friction and facilitate validation of the model. The actuator’s linear range of motion was measured by a laser displacement probe. Loads were applied via a Kevlar string attached to the output slider at one end and wrapped over a pulley that allows known weights to be hung off the apparatus to apply the external load.

![Diagram of experimental apparatus (top view).](a)

![Photograph of experimental apparatus (side view).](b)

Figure 6. Diagram and photograph of experimental apparatus.
To reduce variation in the constitutive properties of the SMA wire throughout the experiments, the linear wire (not spooled) was thermally cycled between martensite and austenite while under 45 N of applied tension and under a 6.5% maximum strain constraint. Thermal cycling continued until the motion in each phase stabilized according to the shakedown procedures described by Sun et al [8]. The stress-strain behavior was measured across a range of applied stresses on the linear wire, and linear and third-order polynomial functions were fit to the data for the austenite and martensite phases, respectively as shown in Figure 8. To further ensure consistent performance, the constitutive law was measured periodically throughout the set of experiments.

In a typical experiment, the SMA wire is installed in the experimental test apparatus (Figure 6) with the State 0 wire length $l_{tot}^{(0)}$, spool configuration (input fixture position and number of mandrels $n$), wrap angles $\theta_{w,i}$, and applied load $F_{app}$ set according to the particular experiment. Once the spool configuration and loads are set, electrical current is applied to the wire (regulated and monitored by LabView software and a laptop computer equipped with data acquisition hardware) to resistively heat it to austenite (3A for about 2 seconds), and then cool it to martensite (0A for at least 3 minutes), in each case heating and cooling until a steady state deflection is reached.

4.2. Experimental results

The experimental study tested two aspects of the spooling model for multiple mandrel configurations: the effect of varying the number of spooled segments of SMA wire and the effect of applied load for two multiple mandrel configurations. Experiments investigated the accuracy and range of the model providing insights into the behavior of multiple mandrel actuators.
4.2.1. Effect of multiple mandrels

The experiments tested different actuator configurations with a variable number of “quarter-wrap” segments (wrap angles $\theta_{w,i} = \pi/2$ for $1 \leq i \leq n$). Throughout testing, the input tail and State 0 output tail lengths were maintained at $\ell_{\text{lin},1} = 7.1$ mm and $\ell_{\text{out}}^{(0)} = 7.8$ mm; the intermediate linear segments were a constant $\ell_{\text{lin},i} = 50.8$ mm ($2 \leq i \leq n$) due to the set positioning of the mandrels on the fixture. Tension was applied with a 1.5 kg load, subjecting the output tail to 129 MPa of tensile stress. The experiments were performed in random order testing the range of motion of actuators with up to $n = 10$ wrapped segments, thermally cycling the actuator at least 5 times in each configuration.

The experimentally determined range of motion is plotted with respect to the number of wrapped segments (Figure 9), and is well-bounded by theory for the expected range of friction ($0.1 < \mu < 0.15$). The data matches the model well in shape, initially increasing with $n$ and then leveling off due to the onset of binding beyond about $n = 4$ quarter-wraps. Additionally, the data fits the model well in magnitude with 1.6% average error between the data and best-fit theory line (determined using the method of least squares with respect to friction, occurring at $\mu = 0.14$). The experiments demonstrate that the theory predicts the behavior of physical actuators for multiple mandrels, and accounts for the accumulation of friction and the related onset of binding. Additionally, the results demonstrate that there is an initial advantage with respect to range of motion for adding greater lengths of wire to the actuator without increasing the packaging footprint. Yet, as the wrap angle approaches the binding angle, the further increases in the amount of wrapping have a decreasing, and then absent, effect on the actuator’s motion.

4.2.2. Effect of applied load

To validate the model’s ability to predict the motion of multiple mandrel actuators with respect to applied load, experiments were performed for two additional configurations using multiple “half-wrap” segments (wrap angles of $\theta_{w,j} = \pi$ radians) for which non-binding and binding configurations were both tested ($n = 2$ and $n = 4$, respectively). As in the first set of experiments, the input, output, and intermediate linear portions of the SMA wire remained constant ($\ell_{\text{lin},1} = 7.1$ mm, $\ell_{\text{out}}^{(0)} = 7.8$ mm, and $\ell_{\text{lin},i} = 50.8$ mm for $2 \leq i \leq n$).

For both the binding and non-binding cases, the data agree with the theoretical predictions in shape with the characteristic inflection in the theory reflected in the data (Figure 10). Due to larger deviations from the model at low applied stresses, the average error between the data and model was 23% for the $n = 2$ case, and 22% for the $n = 4$ case. For ranges of motion measured at applied stresses above the martensite plateau, error was significantly reduced with average errors of 3.6% for $n = 2$ and 6.1% for $n = 4$. The distribution of error across the range of applied loads is consistent with the error from the experiments testing the effect of $n$ since they were performed at an applied stress above the martensite plateau. The increase in error for the lower range of applied stresses results in part from the shallower slope of the martensite plateau (indicating greater compliance), which causes the strain (and the resulting motion) to be significantly more sensitive to applied stress. Thus, any
inaccuracy in the constitutive law would be amplified for the low applied stresses and the portions of the wire where the martensite stress is low (closer to the input tail). Fortunately, SMA actuators are typically designed to operate at stresses above the plateau where the error is lower to take advantage of the greater strains, and thus, the range of motion.

5. CONCLUSIONS

While spool-packaged SMA wire actuators are attractive alternatives to conventional actuators for a number of reasons including their low cost, high energy density, and simplistic forms, they have been limited in practicality by their difficulty in packaging the long lengths of wire that are often required for reasonable deflections. In this paper, the use of topologies involving multiple mandrels was introduced, which adds a higher degree of tailorability to the design of spool-packaged SMA wire actuators. The generalized architecture introduced in this paper allows for a broad range of actuators to be designed and modeled with varying topologies, geometries, material properties, and load profiles. To verify that the mechanics of single spool actuators extend to the use of multiple mandrels, an experimental study was conducted, successfully demonstrating the expansion of the model. In experiments varying the number of discrete wrapped and linear segments and a second set varying the applied load, the model predicted the stroke well in both form and magnitude. The results of this study enable the use of multiple mandrels in both the design and analysis of spool-packaged actuators, where only single spool architectures were previously modeled and understood. The expanded ability to apply spooled-packaging further broadens the ability to utilize SMA actuation in applications for lighter weight, more compact, simpler, and reduced cost actuation.

Figure 10. Effect of applied stress on multiple mandrel spool-packaged SMA wire actuators. Two different mandrel configurations were tested for range of applied motion with respect to applied load. The best-fit curves represent the model prediction with the least-squares error with respect to the coefficient of friction. a) For the \( n=2 \) case, no binding was predicted. b) For the \( n=4 \) case, the actuator is expected to undergo frictional binding.
6. REFERENCES


