

# **Robot Navigation with Model Predictive Equilibrium Point Control (MPEPC)**

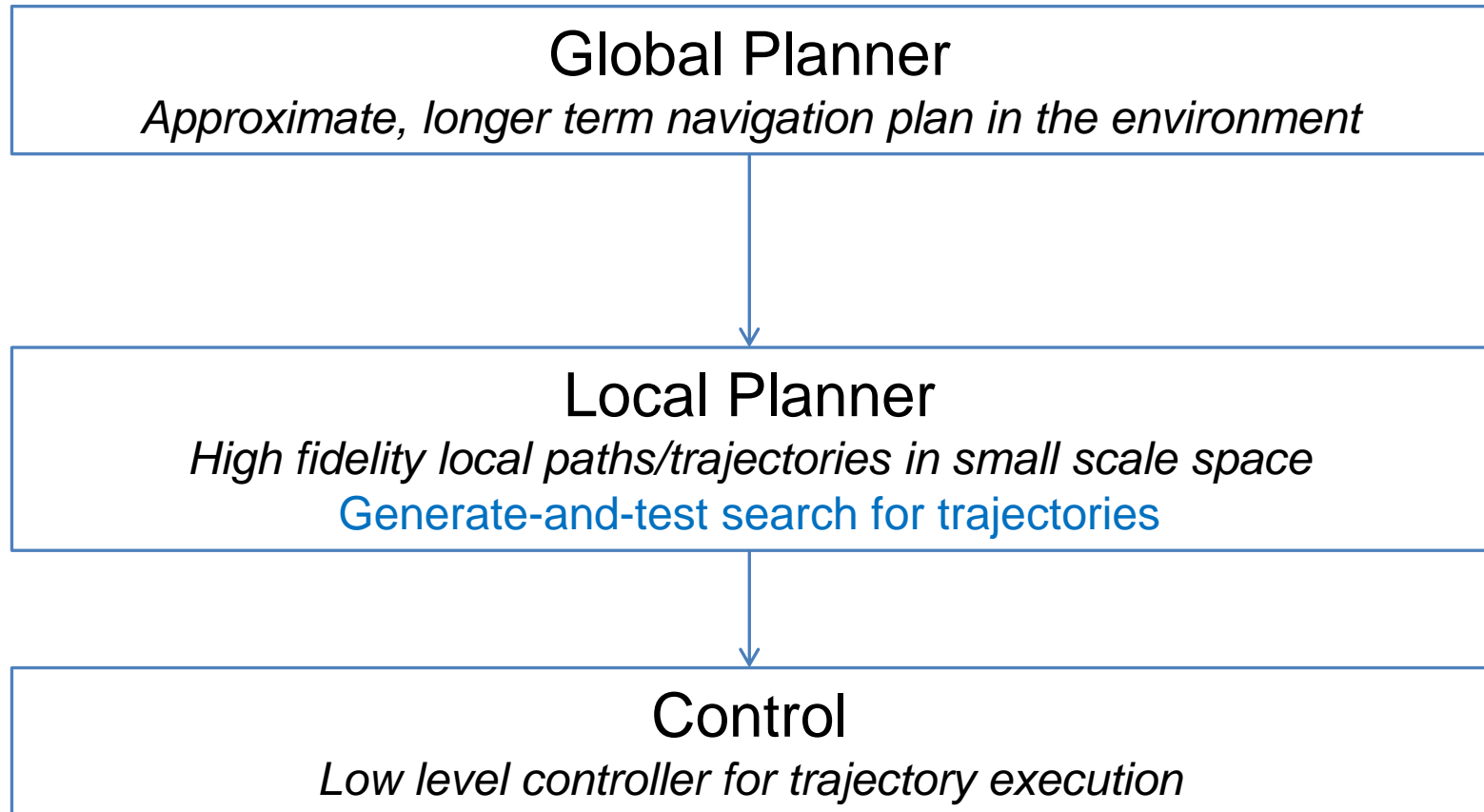
Jong Jin Park, Collin Johnson and Benjamin Kuipers  
University of Michigan, USA

# Robot Navigation Faces Dynamic and Uncertain Environments



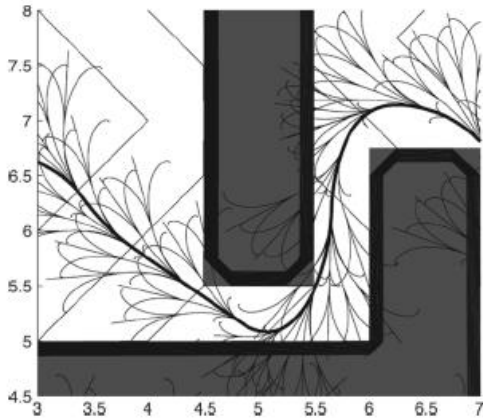
- Tight rectilinear spaces require high precision motion control
- Pedestrians and inaccurate robot model introduce dynamics and uncertainty
- Need to accommodate user preferences, e.g. aggressiveness and comfort

# Hierarchical Motion Planning Is Needed in Dynamic and Uncertain Environments

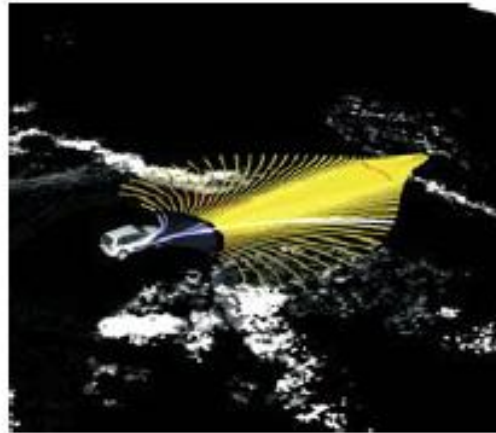


# The Space of Trajectories is Continuous and Infinite

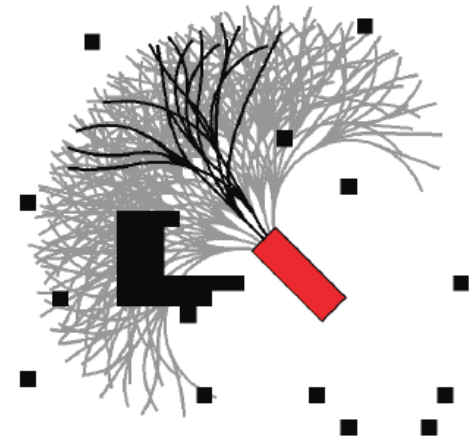
- Many current leading algorithms rely on a finite set of pre-determined candidate trajectories/paths.



[Ogren and Leonard 05]



[Hundelshausen et al. 08]



[Knepper and Mason 12]

- How to construct a good evaluation function is also an important question.
  - Determination of weights in multi-objective function, etc.

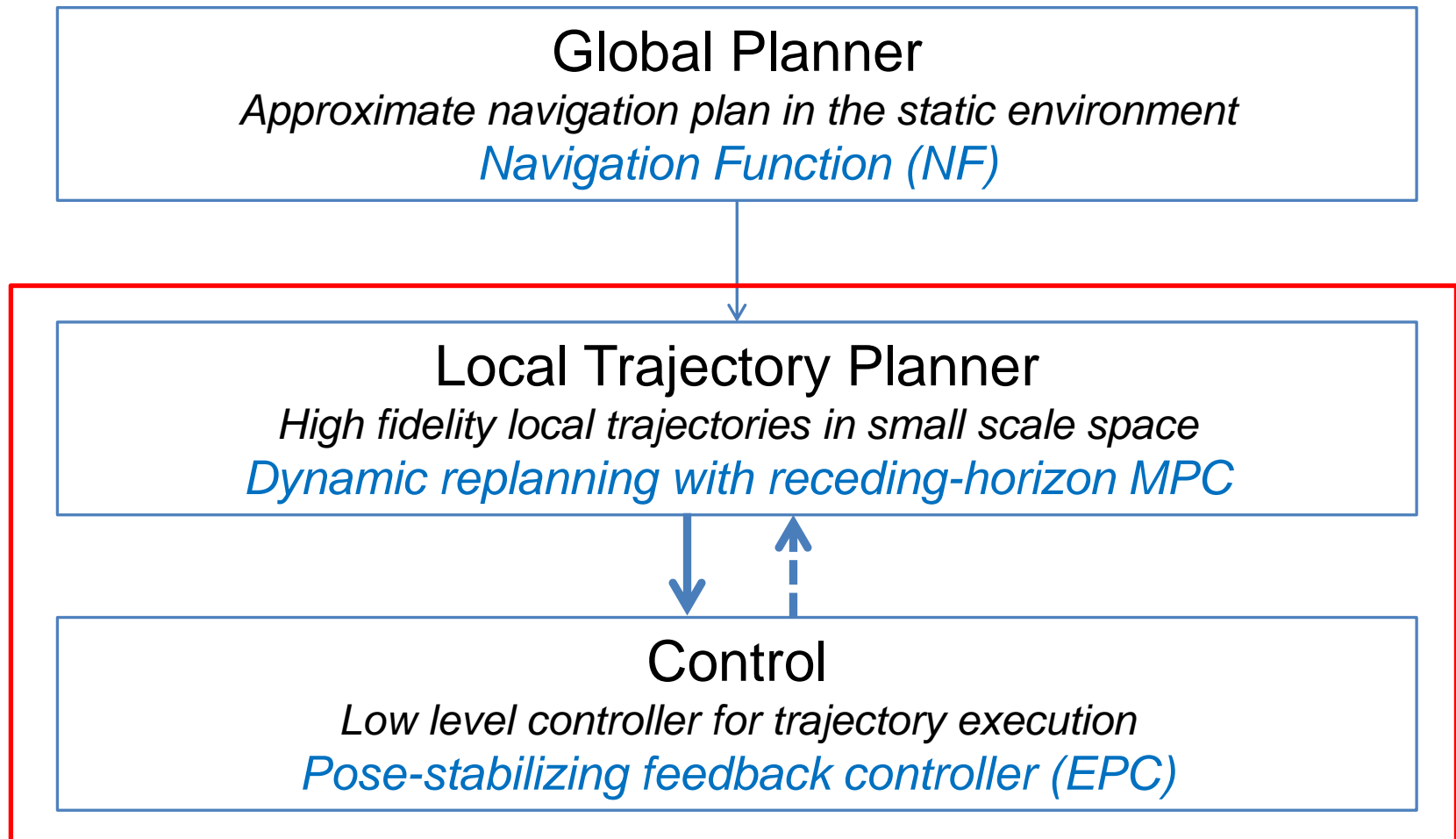
# Our MPEPC Approach: Objectives

- Efficient search for candidate trajectories
- Efficient evaluation of candidate trajectories, considering robot and pedestrian motion uncertainties
- Easy and straightforward implementation
- Accommodation of user preferences

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# Our MPEPC approach to Hierarchical Motion Planning and Control



# Pose-stabilizing Feedback Control

- We have developed a controller that allows the robot to reach an arbitrary target pose in a smooth curve.

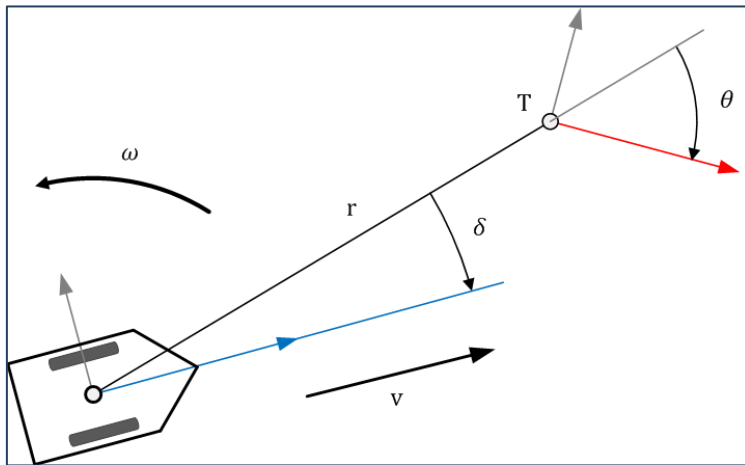
[Park and Kuipers, *ICRA-11*]

- While satisfying linear and angular velocity bounds, slowing down at high curvature points;
  - Without singularity at the target.
  - Target pose is exponentially stable.
- It allows us to compactly parameterize smooth and realizable robot trajectories in terms of the target pose and the gain value (4D).

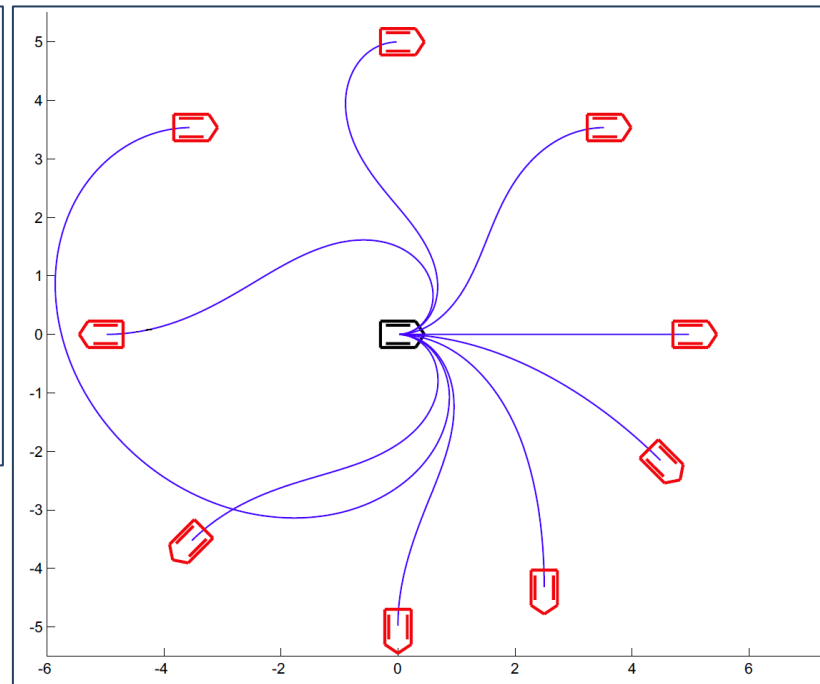


# Pose-stabilizing Feedback Control

$$\omega = -\frac{v}{r} \left[ k_2(\delta - \arctan(-k_1\theta)) + \left(1 + \frac{k_1}{1 + (k_1\theta)^2}\right) \sin \delta \right]$$



- $(r, \theta, \delta)$  describes the target T viewed from the vehicle in terms of the line of sight (LOS).
- At  $r = 0$ , LOS is aligned with T.



[Park and Kuipers, ICRA-11]

# Pose-stabilizing Feedback Control

- Curvature-dependent choice of linear velocity

$$v(\kappa) = v(r, \theta, \delta) = \frac{v_{\max}}{1 + \beta |\kappa(r, \theta, \delta)|^\lambda}$$

- Guarantees bounded linear and angular velocities

- Slowdown rule near target pose

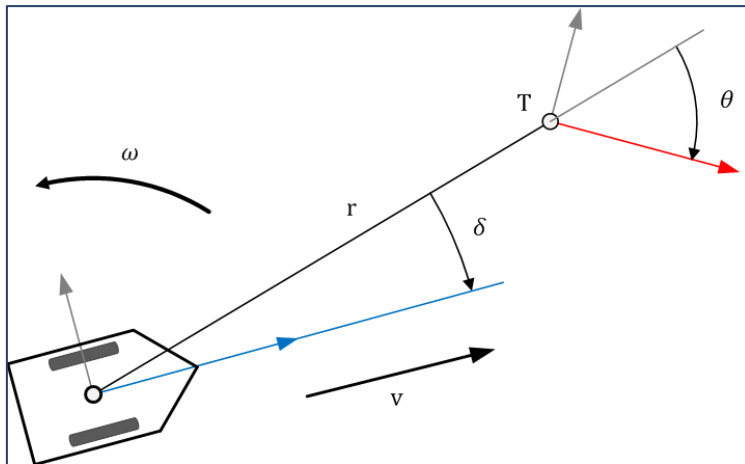
$$v = \min\left(\frac{v_{\max}}{r_{\text{thresh}}} r, v(\kappa)\right)$$

- Removes singularity at  $r \rightarrow 0$
- Target pose is exponentially stable
- $v_{\max}$  can be viewed as a gain value

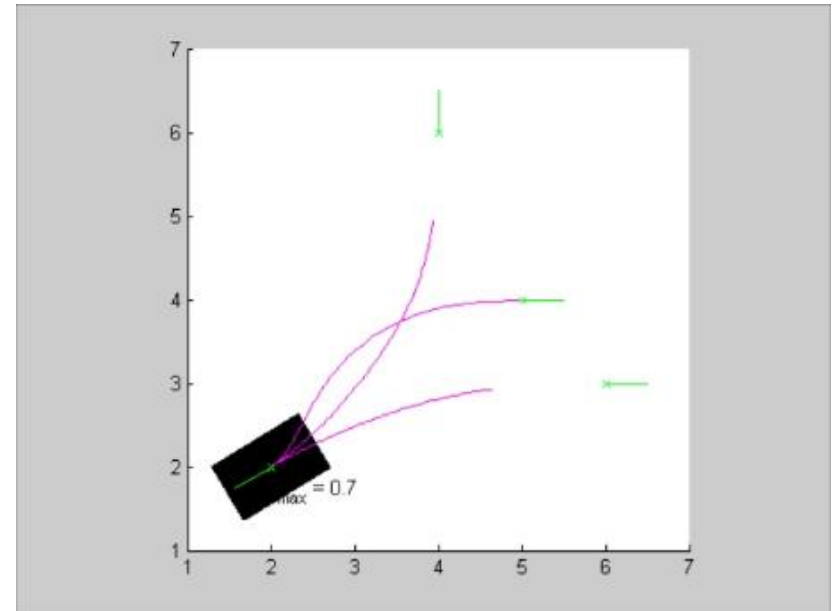
[Park and Kuipers, *ICRA-11*]

# Combined Controller-Robot Model

- Closed-loop robot dynamic simulation with the controller target and gain,  $z_* = (r, \theta, \delta, v_{\max})$ 
  - Non-holonomic, motor saturations, and P-controller for velocities (joystick)
  - $z_*$  parameterize the simulated responses of the robot system under the feedback controller.

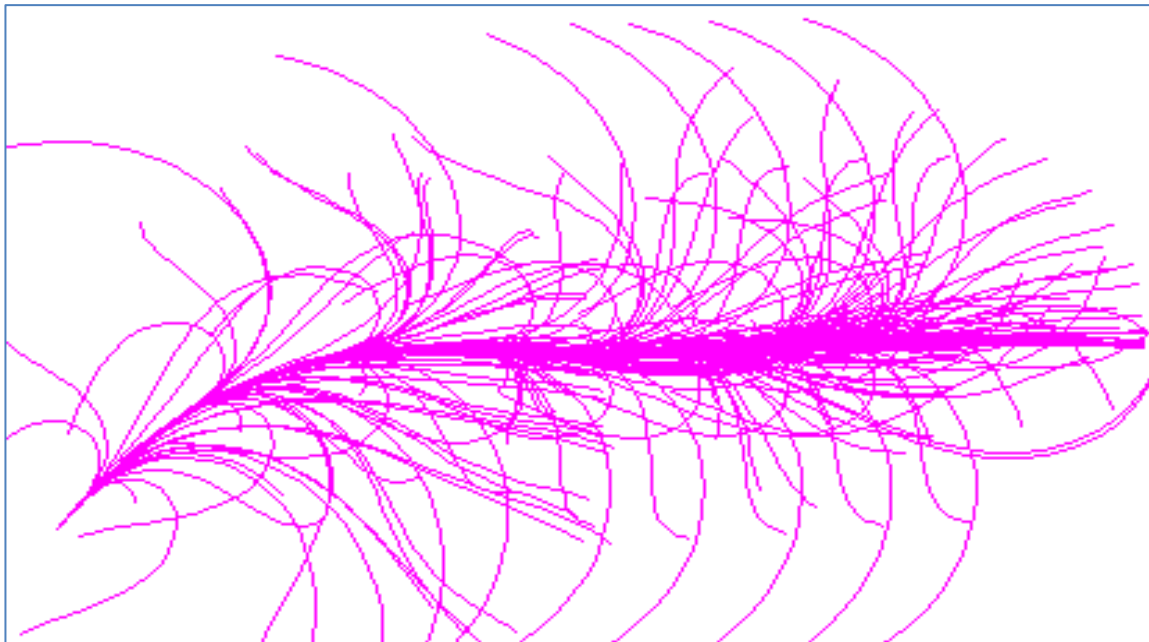


[Park and Kuipers, ICRA-11]



# Defining Our Search Space: Controller-based Trajectory Parameterization

- Our 4D parameterization  $z_* = (r, \theta, \delta, v_{\max})$  defines a continuous space of closed-loop trajectories.
  - It identifies a useful subspace of the infinite and continuous space of possible trajectories that are smooth and realizable by construction.
- Compact parameterization allows efficient search.



# Our MPEPC Approach: Objectives

- Efficient search for candidate trajectories
- **Efficient evaluation of candidate trajectories, considering robot and pedestrian motion uncertainties**
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# Trajectory Evaluation

- Trajectories parameterized by  $z_*$ :

$$q_{z_*} : [0, T] \rightarrow C$$

- Overall *expected* cost of a candidate trajectory, considering probability of collision

$$\begin{aligned} J(x, z_*, T) &= E[\phi_{\text{progress}}] + E[\phi_{\text{collision}}] + E[\phi_{\text{action}}] \\ &= E[\phi(q_{z_*})] \end{aligned}$$

- Negative progress over the static plan (Navigation Function, *NF*)
- Penalty for probability of collision
- Quadratic action cost (on velocities)

# Incorporation of Motion Uncertainties Makes the Optimization Easier

- We construct probability weights as a function of robot and pedestrian motion uncertainties
  - We define simple approximations for:
    - Probability of collision and
    - Survivability of a trajectory segment.
  - Probability weights allow us to formulate the problem as unconstrained optimization over a smooth surface.

# Discrete Approximation to Probability of Collision and Survivability

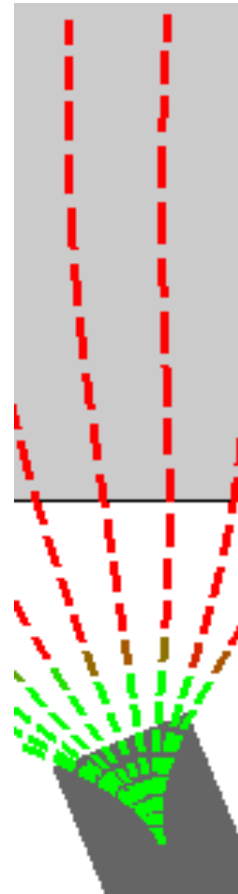
- For  $j$ -th sample along the trajectory, probability of collision to the  $i$ -th object in the map is approximated as:

$$p_c^i(d_i(j), \sigma_i) = \exp(-d_i(j)^2 / \sigma_i^2)$$

- $d_i(j)$  is the minimum distance from any part of the robot body to any part of the  $i$ -th object in the map at time  $j$ .
- $\sigma_i$  are uncertainty parameters.
- Survivability of a trajectory segment is a probability that the trajectory segment will be collision free to any obstacles

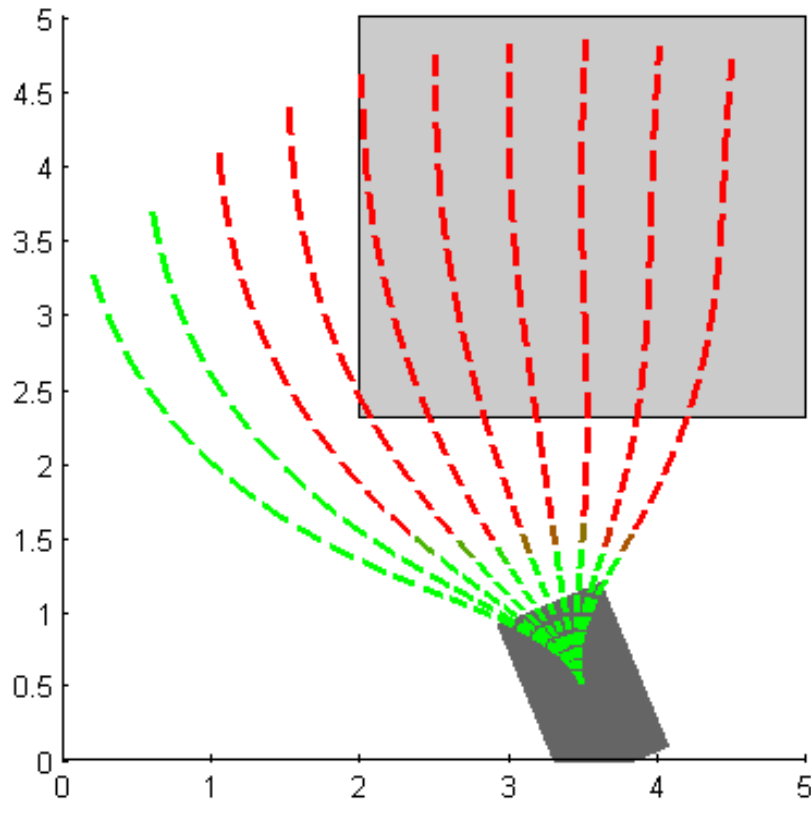
$$p_s(j) \equiv \prod_{i=1}^M (1 - p_c^i(j))$$

- $i \in [1 \dots M]$ ,  $j \in [1 \dots N]$





# Incorporating Probability Weights and Expected Values Creates a Smooth Optimization Surface



- Progress weighted by

survivability

$$p_s(j) \cdot \Delta NF(j)$$

- Collision penalty weighted by  
probability of collision

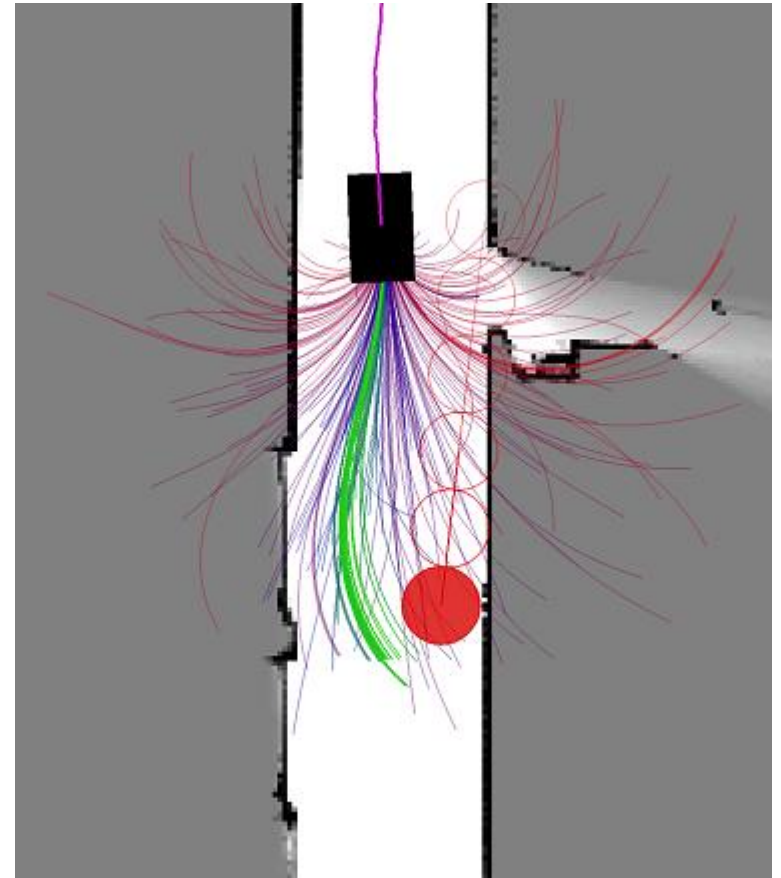
$$\sum_{i=1}^M p_c^i(j) \cdot \phi_{\text{collision}}^i(j)$$

- Additive action cost to modify  
robot behavior

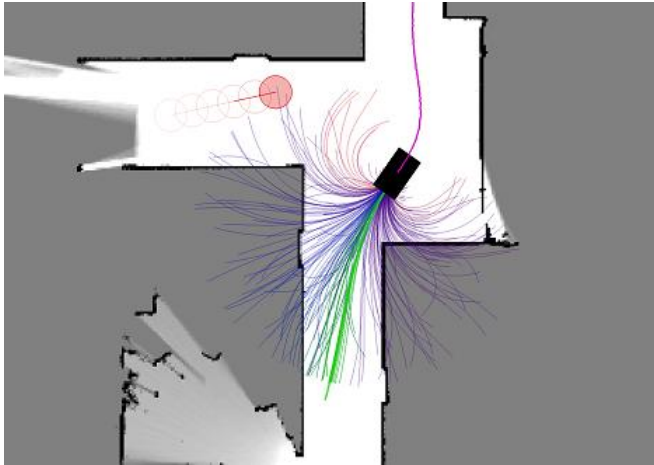
$$c_v v^2(j) + c_\omega \omega^2(j)$$

# Expected Cost of a Trajectory Candidate

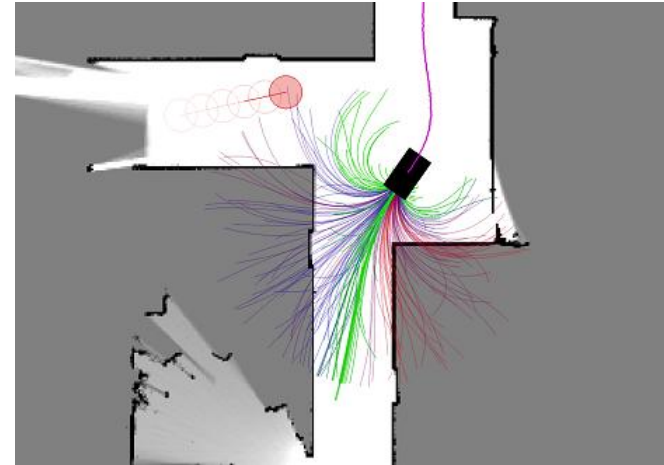
- The expected cost of a trajectory candidate is a probability-weighted time integral over  $[0, T]$
- Probability weights create a smooth cost surface by setting physically meaningful soft boundaries around obstacles
- Weights on action cost can be tuned to match user preferences



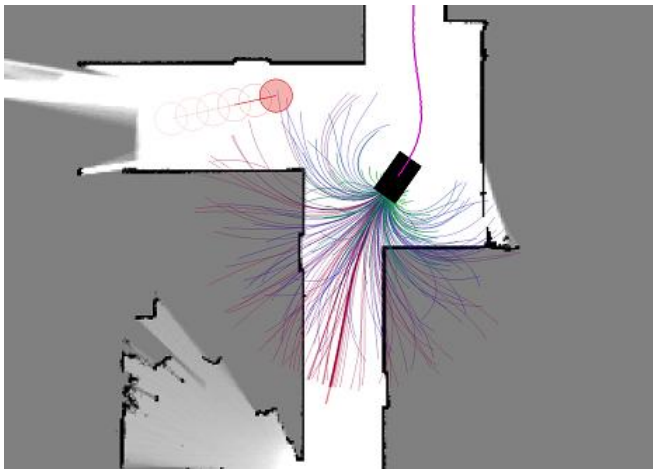
# Expected Cost of a Trajectory Candidate



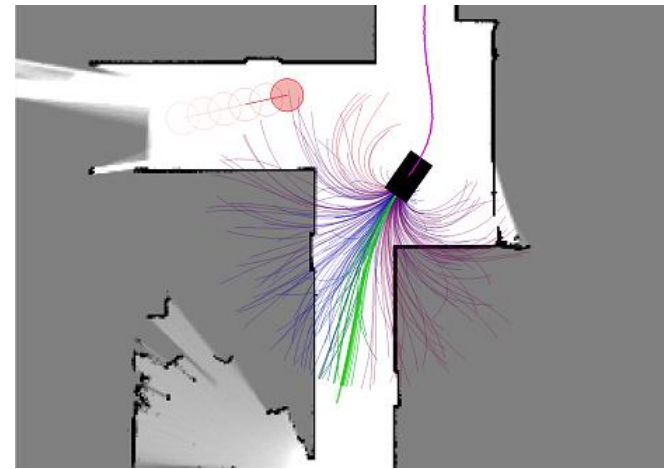
Progress



Collision



Action



Overall

# Our MPEPC Approach: Objectives

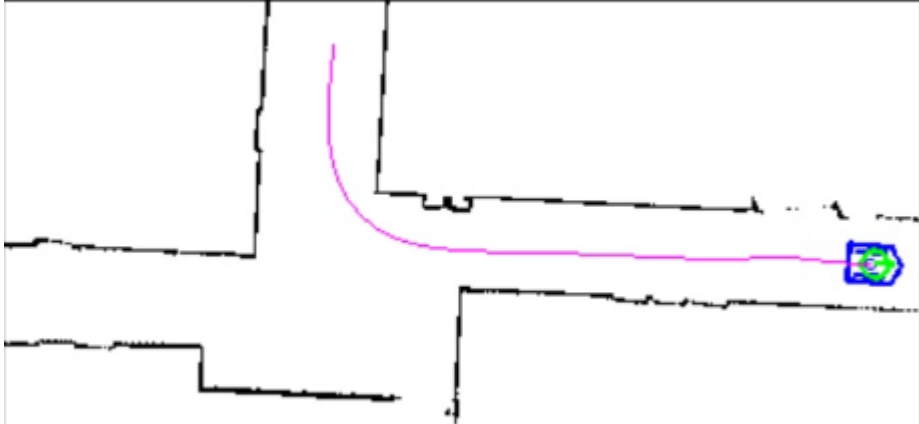
- Efficient generation of motion hypothesis and fine motion control
- Efficient evaluation of candidate trajectories, considering robot and pedestrian motion uncertainties
- **Implementation is easy and straightforward**
- **Action costs express user preferences**

# Implementation is Straightforward

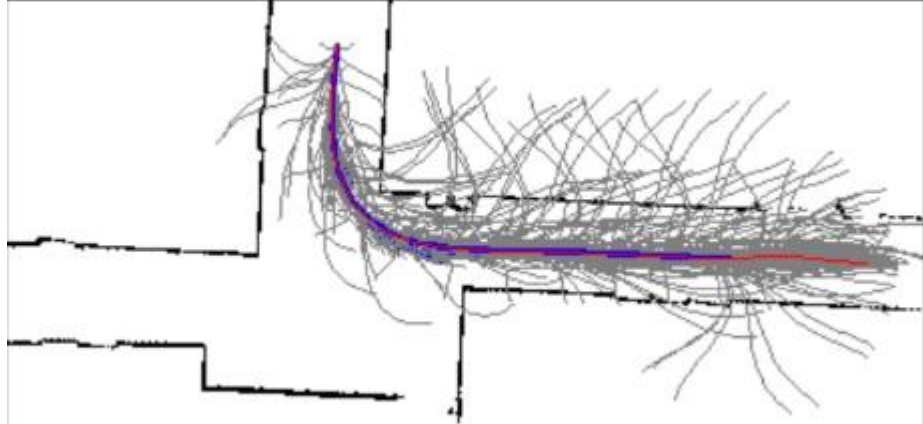
- Off-the-shelf optimization packages
  - Low-dimensional unconstrained optimization on continuous domain
  - No special post processing or optimization techniques
  - Real-time operation (C++)
- Two-phase optimization
  1. Coarse pre-sampling of the search space to find a good initial condition.
  2. Local gradient-based search from the best candidate from the pre-sampling phase.

# MPEPC in Action

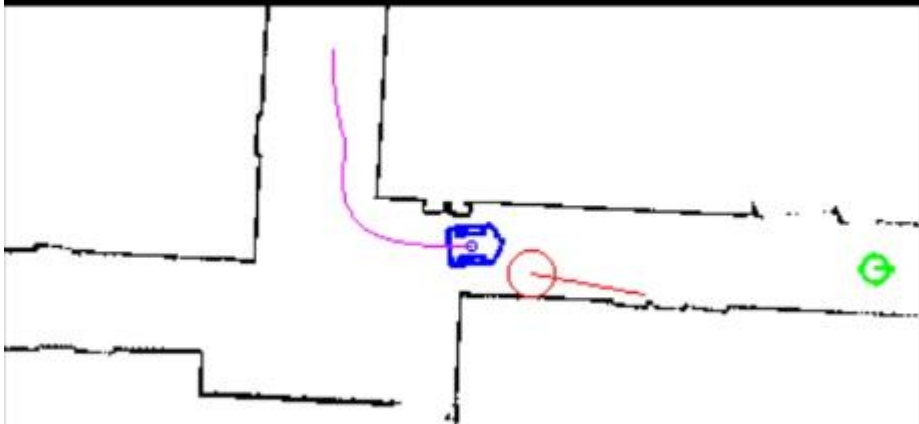
Robot Motion



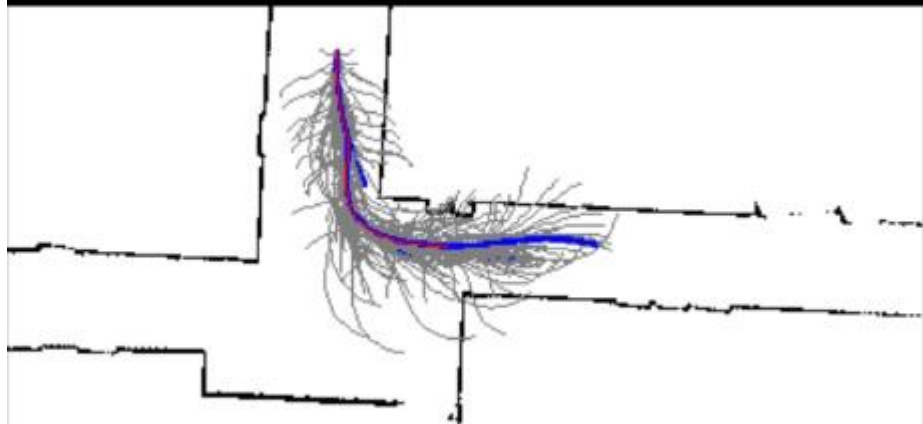
MPEPC Planner



Robot Motion



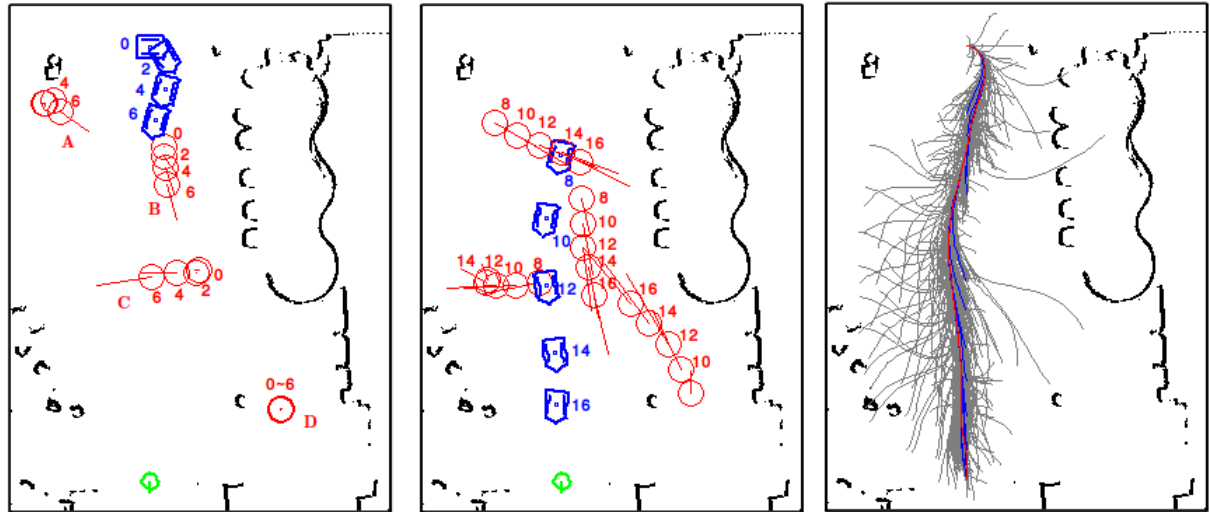
MPEPC Planner



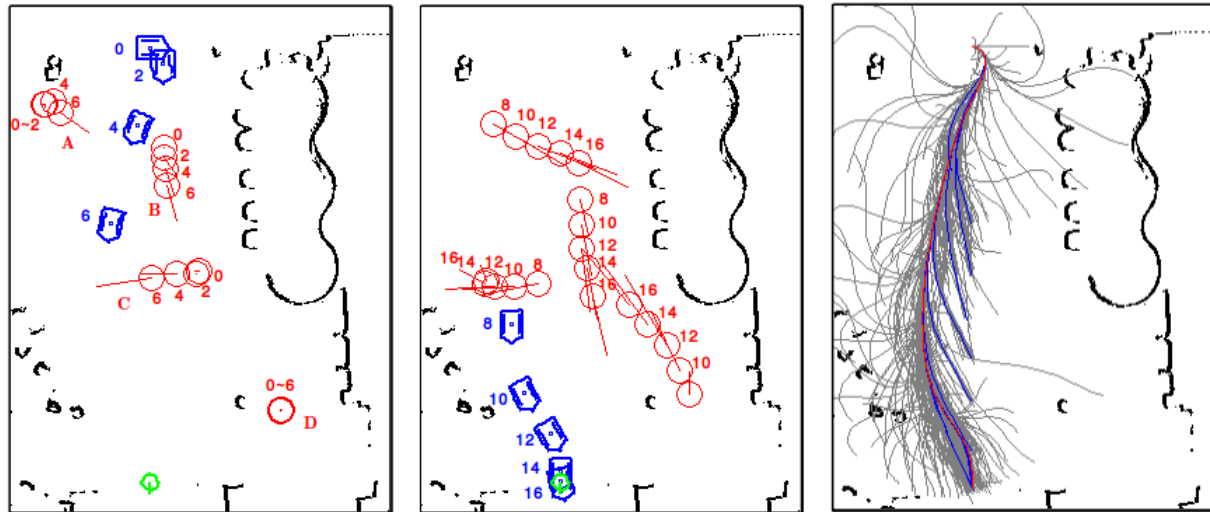
# Different People Have Different Preferences

The proposed navigation algorithm handles multiple dynamic objects. We can shape robot behavior by changing weights in action cost.

Moving **slowly** in a cluttered hall with multiple pedestrians (high weights on action cost)



Moving **aggressively** in a cluttered hall with multiple pedestrians (low weights on action cost)



# Initial Tests on a Physical Platform





# Navigation is a Constant Decision-Making Process

- The navigation problem can be factored by decomposing the task in the hierarchical architecture.
- The search for the optimal trajectory can be made easier by integrating planning and control.
- Motion uncertainties need to be considered explicitly.
- What do they teach in driving school?

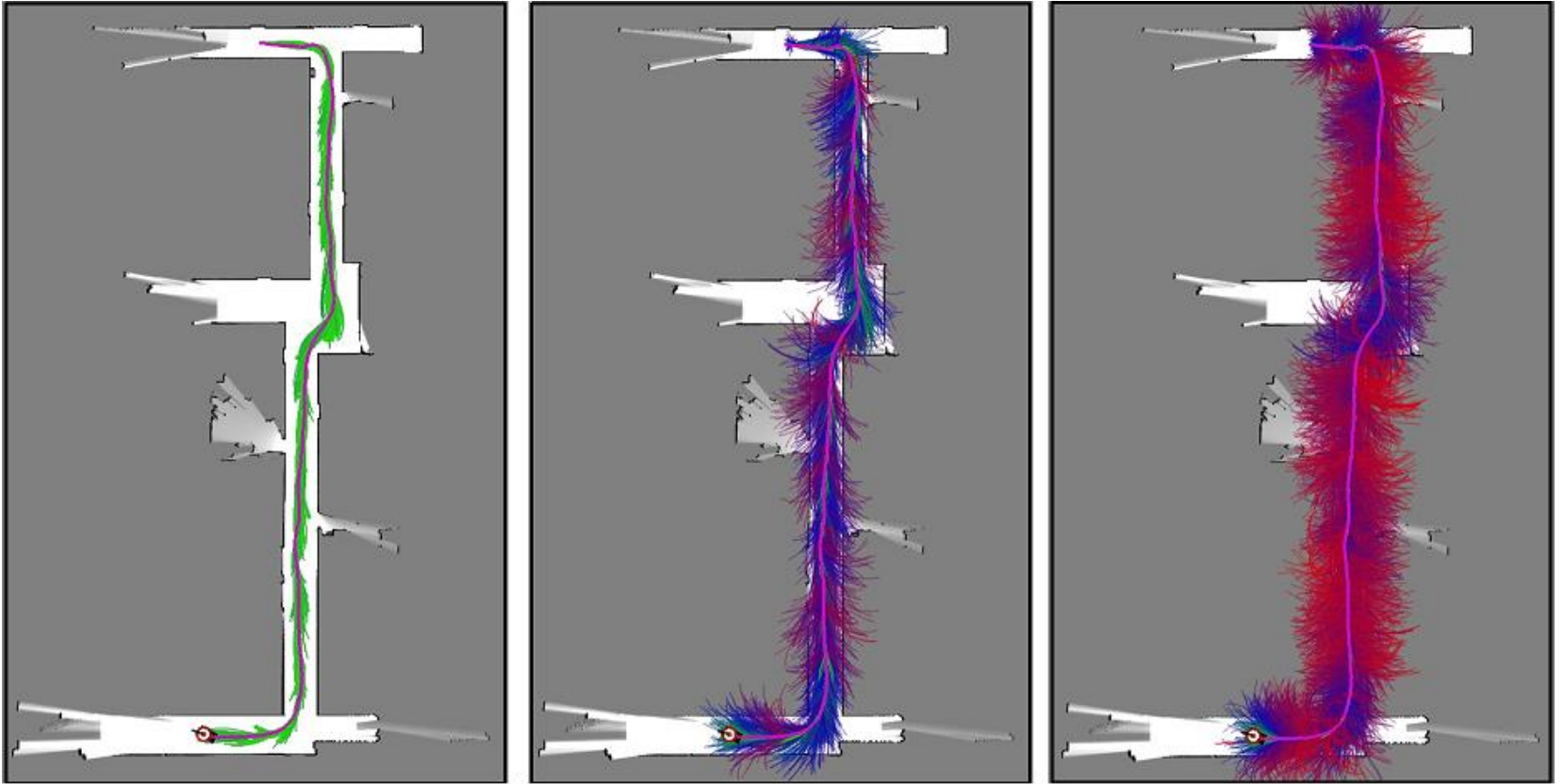
# Navigation is a Constant Decision-Making Process

- The navigation problem can be factored by decomposing the task in the hierarchical architecture.
- The search for the optimal trajectory can be made easier by integrating planning and control.
- Motion uncertainties need to be considered explicitly.
- Identify, predict, decide and execute.
  - Minimize the probability that you might get in trouble, while progressing along the road.

# Conclusion

- We provide a compact representation of a space of smooth and realizable trajectories.
- We formulate local motion planning as an unconstrained optimization problem by computing expected values, using probability weights.
- The formulation allows straightforward low-dimensional optimization on a continuous domain.
- We have simple, easy to understand tunable parameters for qualitative robot behavior.

# Thank You



# References

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