Using the Chombo Adaptive Mesh Refinement Model in Shallow Water Mode to Simulate Interactions of Tropical Cyclone-like Vortices

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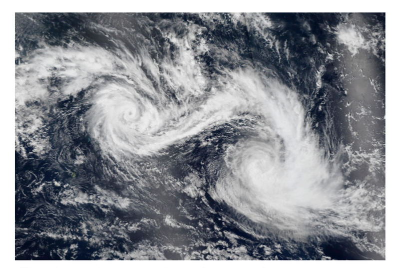
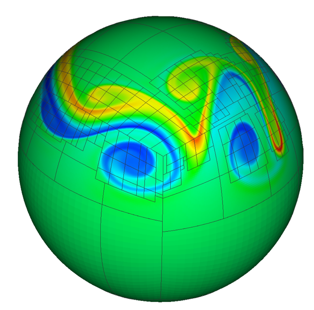
Introduction

- Current global climate and weather models are challenged by multi-scale intense atmospheric phenomena such as tropical cyclones
- Adaptive mesh refinement (AMR):
  - Dynamically increases resolution locally over areas of interest when needed
  - Balances benefits of fine-scale resolution with increased computational burden
- Idealized binary vortices in the spherical shallow water equations exhibit many of dynamics and complexities of of atmospheric modeling
- Work to asses the effectiveness of AMR and various refinement strategies within the Chombo-AMR dynamical core in capturing tropical cyclone-like interactions
- Will use these refinement strategies in 3D-dycore simulations with simplified physics parameterization schemes



Pair of tropical cyclones (NASA's Earth Observatory)

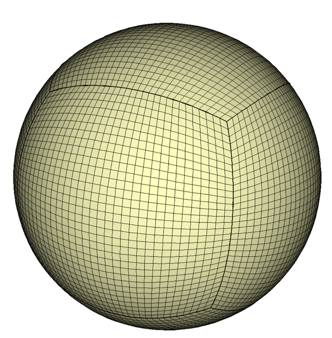


AMR grid example, barotropic instability test case

Model Description

Chombo-AMR Dynamical Core

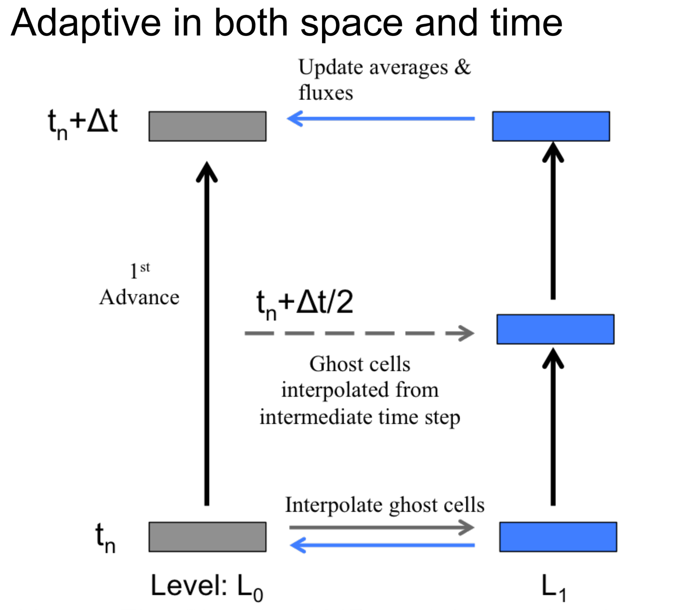
- Development led by Applied Numerical Algorithms Group (ANAG) at LBNL [MU2015] in collaboration with the University of Michigan
- Uses the Chombo framework library, an open-source toolkit for solving PDES on structured grids
- Multi-block grids on a cubed-sphere
- 4th-order finite-volume discretization of the shallow water equations
- Adaptive in both space and time



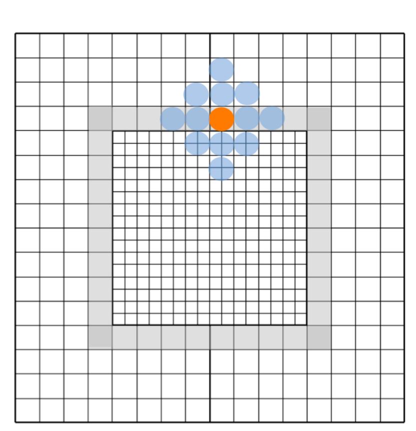
Above: A uniform c32 resolution cubed-sphere grid. The length of each panel edge is 32 grid cells.

Resolution	Δx
c32	313 km
c64	156 km
c128	78.2 km
c256	39.1 km
c512	19.5 km
c1024	9.78 km

Table: Equivalent grid spacing at the equator for cubed-sphere grid resolutions.



Above: The Chombo-AMR model uses the classical 4th order Runge-Kutta temporal discretization. Refined grid levels are sub-cycled in time.



Above: Grid cells are contained in a hierarchy of nested levels for different resolutions. Ghost cells (in grey) are calculated by interpolated stencils of nearby cells (light blue).

Shallow Water Equations

- The 2D shallow-water equations exhibit many of the complexities observed in 3D general circulation model (GCM) dynamical cores
- Can serve as an effective method for testing dynamical cores and the refinement strategies of adaptive atmospheric models

Continuous Form

$$\frac{\partial}{\partial t}(J\mathbf{U}) + \nabla\cdot(J\tilde{\mathbf{F}}) = J\mathbf{\Psi}$$

Discretized Form

$$\frac{d}{dt}\langle J\mathbf{U}\rangle_{i,j} = -\frac{1}{\Delta\alpha}\left(\langle J\mathbf{F}^{\alpha}\rangle_{i+\frac{1}{2},j} - \langle J\mathbf{F}^{\alpha}\rangle_{i-\frac{1}{2},j}\right) - \frac{1}{\Delta\beta}\left(\langle J\mathbf{F}^{\beta}\rangle_{i,j+\frac{1}{2}} - \langle J\mathbf{F}^{\beta}\rangle_{i,j-\frac{1}{2}}\right) + \langle J\mathbf{\Psi}\rangle_{i,j}$$

$$\mathbf{U} = \begin{pmatrix} h \\ hu^{\alpha} \\ hu^{\beta} \end{pmatrix}, \quad \mathbf{F}^k = \begin{pmatrix} hu^k \\ hu^k u^{\alpha} + g^{k\alpha}\frac{1}{2}Gh^2 \\ hu^k u^{\beta} + g^{k\beta}\frac{1}{2}Gh^2 \end{pmatrix}, \quad \mathbf{\Psi} = \begin{pmatrix} 0 \\ \mathbf{\Psi}^{\alpha}_M + \mathbf{\Psi}^{\alpha}_C + \mathbf{\Psi}^{\alpha}_B \\ \mathbf{\Psi}^{\beta}_M + \mathbf{\Psi}^{\beta}_C + \mathbf{\Psi}^{\beta}_B \end{pmatrix}$$

Equation 1: Conservative coordinate-invariant form of the 2D shallow water equations on the sphere.

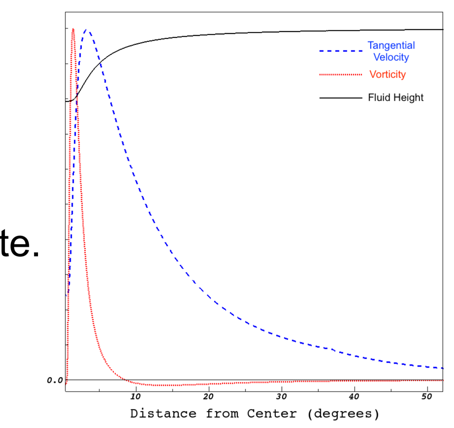
Binary Vortices Interaction with AMR

Vortex Initialization

- Continuous vortex profiles are derived from the gradient wind balance of the cylindrical shallow water equations with an f-plane approximation. [HD1993]
- Creates nearly balanced initial state.
- Initial height perturbation equation (**top**) and initial tangential velocity (**bottom**).

$$\phi' = \phi_c\left(1 - \exp(-(\frac{r_m}{r})^b)\right)$$

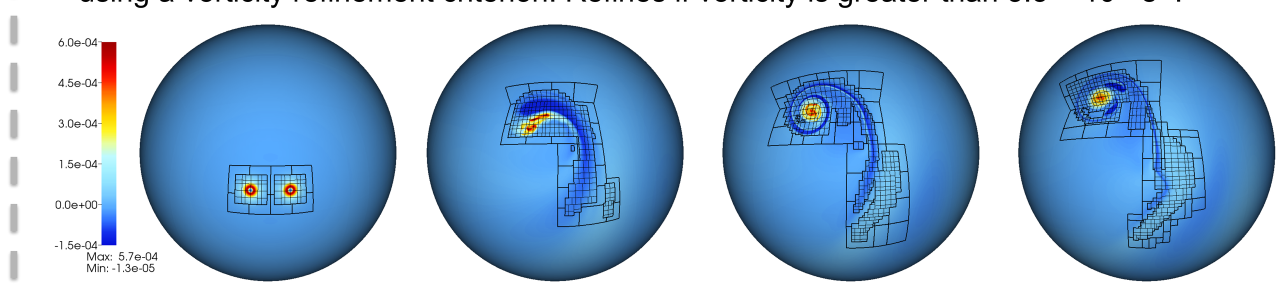
$$v_T = \sqrt{\phi_c b(\frac{r_m}{r})^b \exp(-(\frac{r_m}{r})^b) + \frac{r^2 f^2}{4}} - \frac{rf}{2}$$



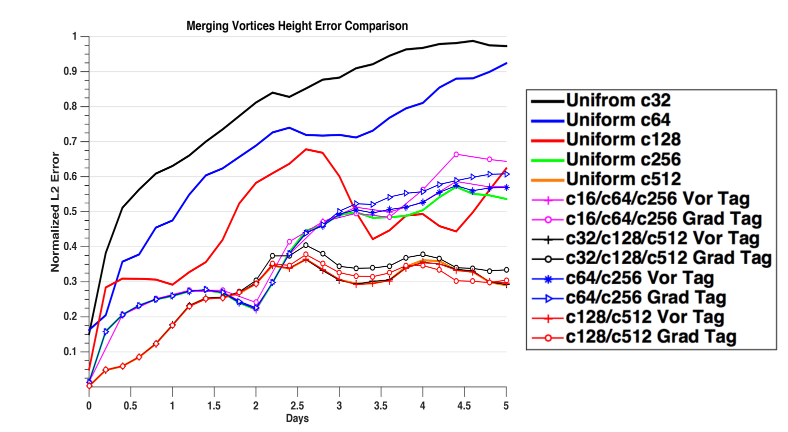
Above: Radial profile of vorticity (red), tangential wind (blue) and height field (black). Note, Max. values scaled to one.

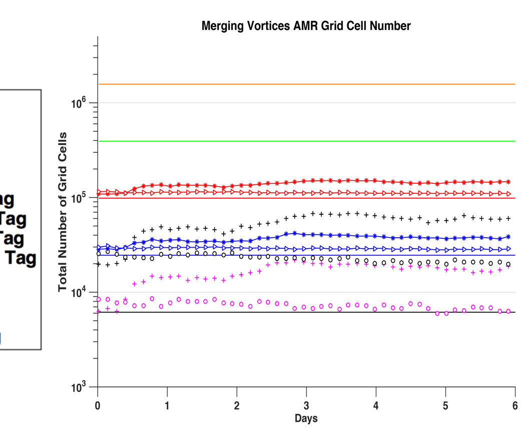
- f is the Coriolis parameter, r is the great circle distance, r_m is radius of max winds and Φ_c is the perturbation height.
- Adjusting size, separation distance, and strength of the vortices results in different interactions (either merging or repelling) that are sensitive to grid resolution. [GL2010]

Below: Evolution of the merging vortices in a 3 level (c64/c256/c1024) AMR run using a vorticity refinement criterion. Refines if vorticity is greater than $3.5 \times 10^{-5}\ s^{-1}$.



Note: The adaptive blocks are outlined in black.

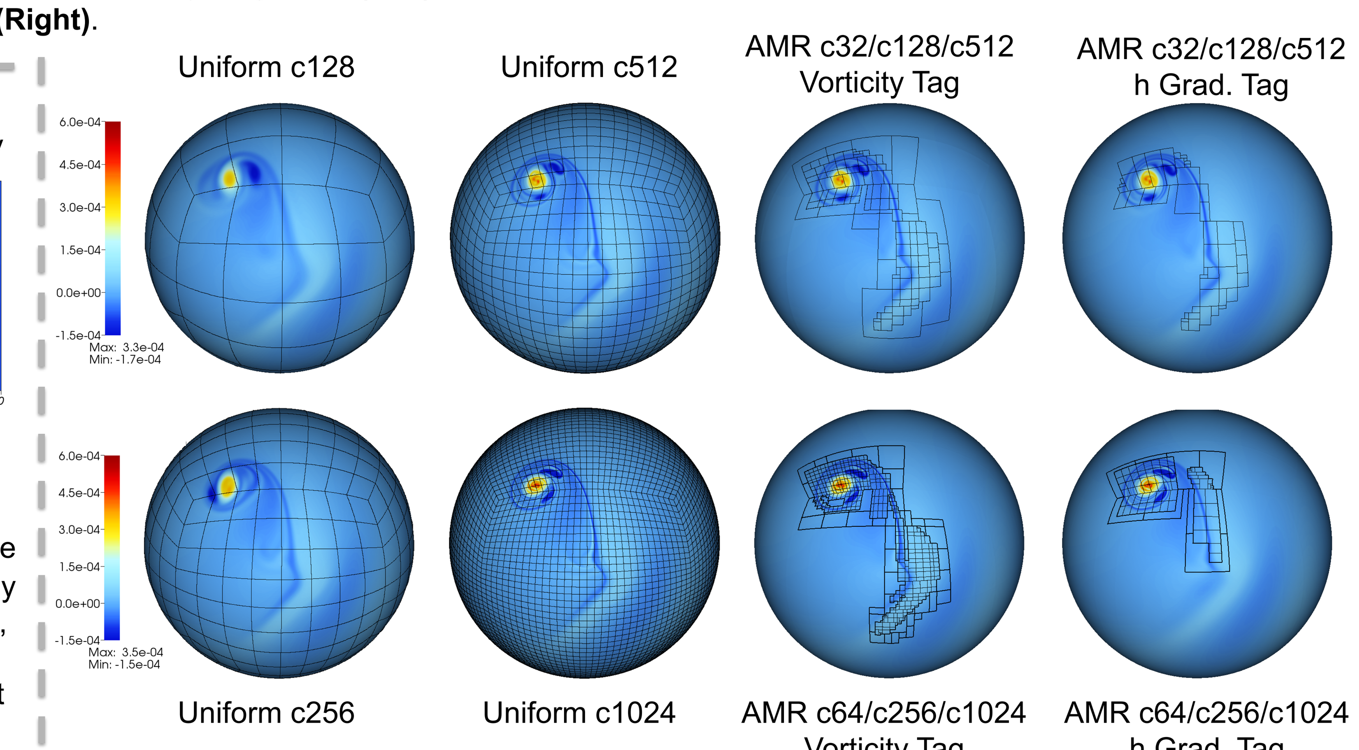




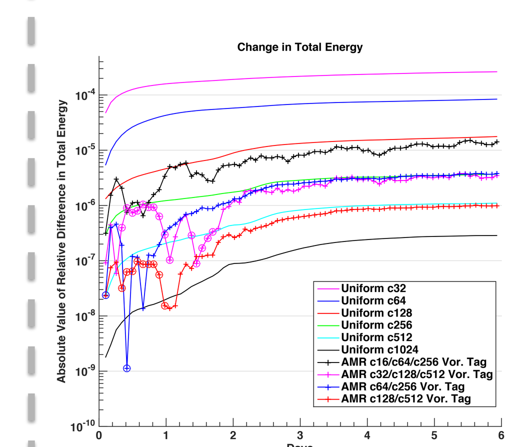
Above: Global normalized L2 height error over time comparison for uniform runs and 1- and 2- level AMR runs tagging on the height gradient and relative vorticity magnitude (**Left**). Total number of grid cells (log-scale) over time in each run (**Right**).

Merging Vortices

- Vortices initialized at 10° N with a separation distance of 15.65° (~1700 km), a radius of max winds of 400km and an initial height perturbation of 800m.
- The two vortices start to rotate about each other, but then merge.
- Creates a fine filament structure of high vorticity around the merged vortex and a leeside wave train.
- Even at c128 merged vortex shape and filaments are unresolved.
- AMR runs converge well to the runs that have uniform resolution the same as the finest AMR level.
- AMR is not creating spurious filaments or degrading the solution at grid boundaries.



Above: Vorticity field at Day 4 for uniform runs and AMR runs with refinement criteria based on vorticity or height gradient. Merged vortex and filament structure are resolved at resolutions above c256. Even with strict refinement criteria as in the runs on the right, AMR is able to resolve the core features.

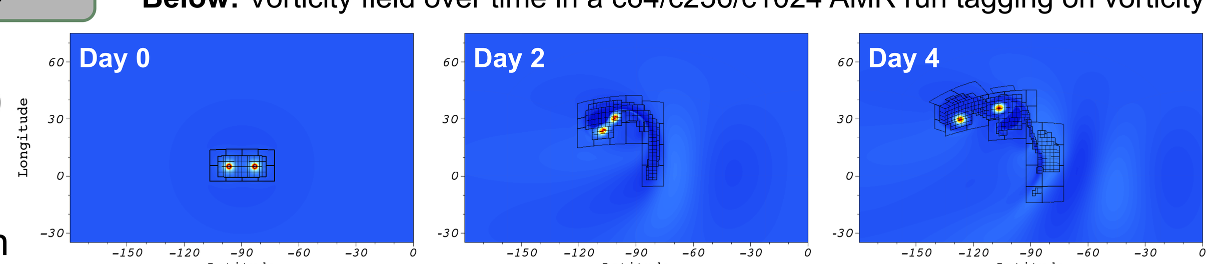


Left: Absolute value of the relative difference in total energy from initial value for merging vortices runs. The relative difference is *negative* at all steps with the exception of a few early time steps in some AMR simulations, as in the runs on the right, the relative difference is positive. Those time steps marked with circles on the graph.

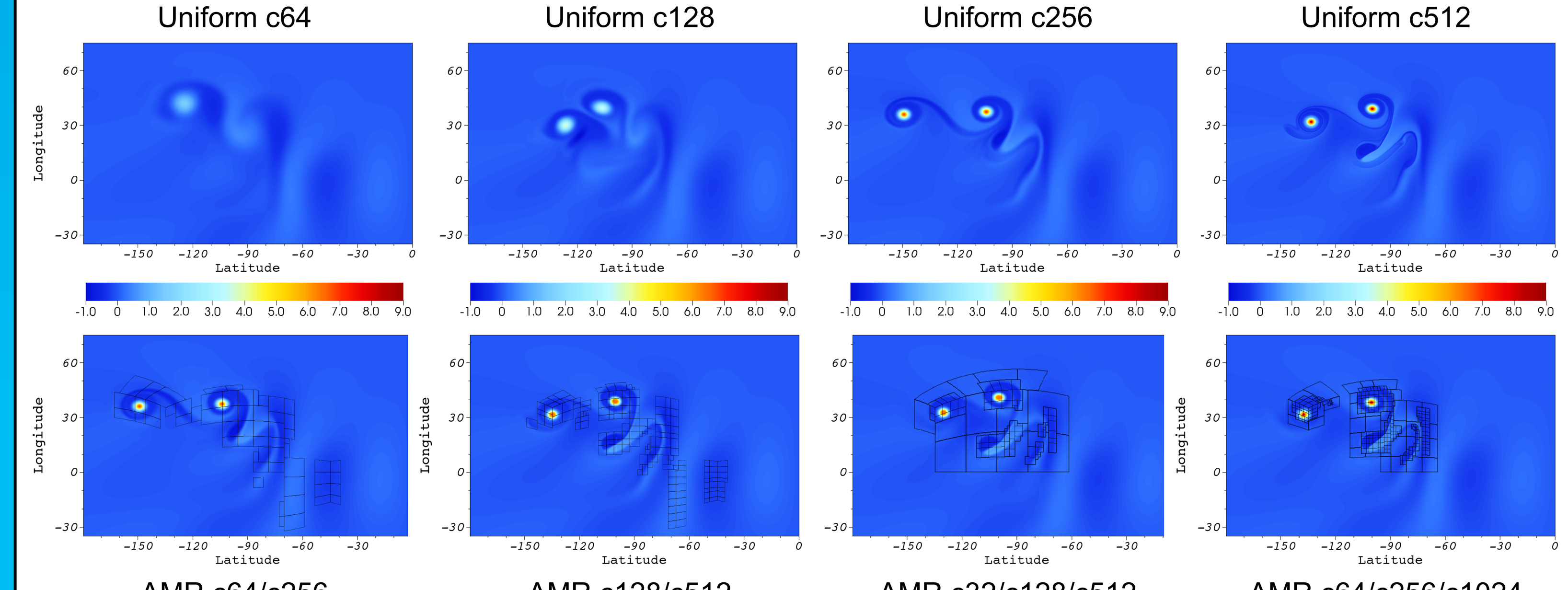
Repelling Vortices

- Vortices initialized at 5° N with a separation distance of 13.5° (~1500 km), a max. winds radius of 250km and an initial perturbation of 800m.
- The two vortices rotate around each other twice and then drift apart.
- Solution very resolution dependent, at coarse resolutions vortices merge.

Below: Vorticity field over time in a c64/c256/c1024 AMR run tagging on vorticity



Below: Vorticity field at day 6 for uniform (**top row**) and AMR runs tagging on vorticity (**bottom row**). For coarse uniform resolutions, vortices are weak, in drastically different positions, or have even merged. Note that the uniform c512 and c1024 runs are virtually the same so only one is shown. In AMR, the vorticity field converges to that of the uniform run with the same resolution as the finest level



Acknowledgements and References

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Conclusions

AMR techniques show promising results in the 2D shallow water tests:

- AMR can capture and track the complex dynamics of the interacting vortices
- Able to reproduce the results of uniform runs with high fidelity (no spurious waves or features)
- It achieves similar errors to uniform runs with significantly fewer grid cells
- Multiple refinement criteria are possible and thresholds are robust.

Future Work: Begin similar analysis using the full 3D Chombo-AMR with the simplified physics parameterization schemes. Focus will be on assessing the dependence of these sub-grid parameterizations on the resolution and grid structure in AMR runs using idealized tropical cyclones and other test cases.