What *Is* Mathematics?

- Is Mathematics a Science?   A Humanity?
- What's the relationship between Math and Science?
    - Between Math and the Humanities?   Math and Music?
    - How about Math and Computing?   Math and Logic?
- Is Mathematics discovered or invented?
- Where does Mathematics come from?
- What's it really about?   How has it changed over history?
- Why is Mathematics so hard to learn?   Why do so many people hate it?
- We say that $2 + 3 = 5$ is *true*, and that $f = ma$ is *true*.
    - Are these the same sense of *true*?   Can they both be *proven*?
- We say that *there exists* a solution for $x^2 + y^2 = z^2$ in integers *x, y, z,* and that *there exists* an element with 92 protons in its nucleus.
    - Are these the same sense of *exist*?
- Which did you prefer in high school: Geometry or Algebra?   Why?
- What are the odds that in a group of 25 people 2 have the same birthday?
- How about 30 people?   183 people?

We'll be discussing the answers to these and many other questions in this class. For example, do you know...

- ... that there is more than one kind of Infinity?
- ... that geometry and algebra are the same thing?
- ... exactly *why* $(x + y)^2 = x^2 + 2xy + y^2$?
- ... that *calculus* means 'pebble' in Latin?
- ... how many colors you need to make a map?
- ... how many prime numbers there are?
- ... that the sum of the angles in a triangle isn't *always* 180°?
- ... that 'real numbers' are just as imaginary as 'imaginary numbers'?
- ... how to make an unbreakable code?
- ... how Fermat's Last Theorem was proved?
- ... what fractals are and how to make them?

If so, great. You'll love this class. If not, then you've missed out on the really good parts of mathematics, the esthetic parts all mathematicians know about, but rarely discuss, the parts that make mathematics the neglected Humanity.
You may even be a victim of (*shudder*) Math Abuse.

http://www.umich.edu/~jlawler/geb.html

What *Is* Mathematics?

This is a "Math Appreciation" course, which does not attempt to train mathematicians, and does **not** fulfill the Quantitative Reasoning requirement, but rather is a Humanities course that aims to educate those with an interest in math, who may not be proficient in its technique. Mathematics is very badly taught in the United States primary and secondary school system, with the result that many interested people are 'math-abused' – turned off to the entire subject, never realizing that math is a humanity, oriented towards the pursuit of beauty, with very close relations to philosophy, music, and art.

Recently, two very controversial books have appeared that give radical accounts of the origins, ontology, and epistemology of mathematics, and of its role in science: Lakoff and Núñez's *Where Does Mathematics Come From?* proposes a cognitive explanation of how mathematics is derived from human **metaphor**, and Wolfram's *A New Kind of Science* provides nothing less than a complete refiguration of science based on the mathematical concept of **algorithm**.

The books are quite different, with almost no overlap in content, though they are in principle not incompatible with each other. Both books have stirred critical acclaim and critical argument, and they bid fair to combine with other current trends in math, like experimental mathematics, to change both mathematical theory and practice in the 21st Century. This has potentially enormous implications, esthetic and otherwise.

After an introduction to the history and scope of current mathematics and the concepts involved in it, concluding with the "Foundations Catastrophe" that rocked math for three centuries and culminated in Gödel's and Cohen's famous proofs, we will read both books, comparing, contrasting, and critiquing the views they present. We will try to discover what the fuss is all about, and we'll also try to have some fun. Math *is* fun – another well-kept secret in American education.

There will be considerable reading, occasional papers and other assignments, and a term project. Active participation in class is a course requirement, in addition to other assignments.

This class is open to and welcomes students from outside the Residential College, including arts, math, and science students, especially engineers. Despite the fact that this is a RC course, <u>any</u> student from <u>any</u> college may register for it.

Prerequisites: High School Algebra and Geometry.

Class Web page: http://www.umich.edu/~jlawler/geb.html
Email: jlawler@umich.edu
Required Textbooks:

- Lakoff and Núñez, *Where Does Mathematics Come From?*   ISBN 0465037712
- Wolfram, *A New Kind of Science*   ISBN 0824209702
- Plus coursepaks at *Excel* on South U.