

Formal Varieties of Logical Notation

De Morgan's Laws, as stated in various formal theories and notations

1. Set Theory

The Complement of the Intersection of any number of sets is Equivalent to the Union of their Complements.

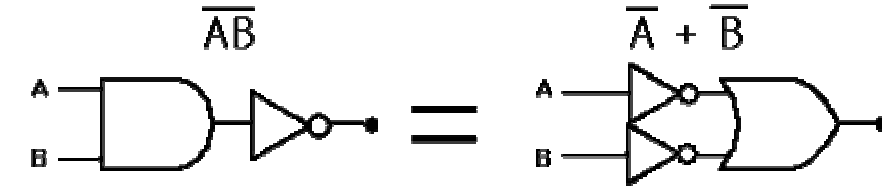
$$\overline{\bigcap_j A_j} \equiv \bigcup_j \overline{A_j}$$

The Complement of the Union of any number of sets is Equivalent to the Intersection of their Complements.

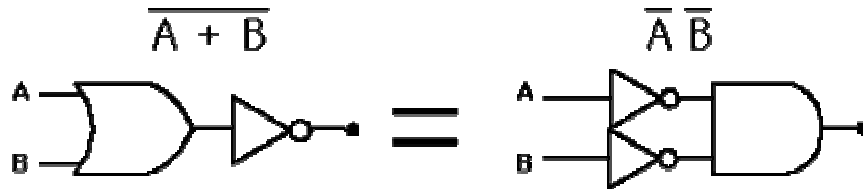
$$\overline{\bigcup_j A_j} \equiv \bigcap_j \overline{A_j}$$

2. Circuit

Diagrams



A NAND gate is equivalent to an inversion followed by an OR



A NOR gate is equivalent to an inversion followed by an AND

3. Propositional Calculus

(Not (p And q)) is Equivalent to *((Not p) Or (Not q))*

$$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$$

'Standard Notation' (*Principia Mathematica*)

$$\text{ENKpqANpNq}$$

'Polish Notation' (Łukasiewicz)

(Not (p Or q)) is Equivalent to *((Not p) And (Not q))*

$$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$$

PM

$$\text{ENApqKNpNq}$$

Łuk

4. Modal Logic

(p is Not Necessary) is Equivalent to *(Not p is Possible)*

$$\neg \Box p \equiv \Diamond \neg p$$

PM

$$\text{ENLpMNp}$$

Łuk

(p is Not Possible) is Equivalent to *(Not p is Necessary)*

$$\neg \Diamond p \equiv \Box \neg p$$

PM

$$\text{ENMpLNp}$$

Łuk

5. First-Order Quantified Predicate Calculus

(Not (For Every x, φ(x))) is Equivalent to *(For Some x, Not φ(x))*

$$\neg(\forall x) \phi(x) \equiv (\exists x) \neg \phi(x)$$

PM

$$\text{EN}\Pi x \phi x \Sigma x N \phi x$$

Łuk

(Not (For Some x, φ(x))) is Equivalent to *(For Every x, Not φ(x))*

$$\neg(\exists x) \phi(x) \equiv \forall(x) \neg \phi(x)$$

PM

$$\text{EN}\Sigma x \phi x \Pi x N \phi x$$

Łuk