Consumption, Retirement and Social Security:
Evaluating the Efficiency of Reform that Encourages Longer Careers

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7–21–2008

Abstract

This paper proposes, and analyzes, a Social Security reform in which individuals no longer face the OASI payroll tax after some specified age or career length, and their subsequent earnings have no bearing on their benefits. We first estimate parameters of a life–cycle model. Our specification includes non-separable preferences and possible disability. It predicts a consumption–expenditure change at retirement. We use the latter’s magnitude, together with households’ retirement–age decisions, to identify key structural parameters. Simulations indicate that reform could increase retirement ages one year or more, equivalent variations could average $4,000 per household, and income tax revenues per household could increase $8,900.

* Osamu Aruga and Lina Walker provided excellent research assistance with the CEX and HRS data sets, respectively. We are grateful to David Blau, John Bound, Miles Kimball, Jeff Liebman, Richard Rogerson, and Matthew Shapiro, and to conference and seminar participants at the Retirement Research Consortium, American Economic Association, Arizona State, Michigan, Minnesota, NBER Spring PE Meetings, NYU, Osaka, SITE, Stanford, Texas, Tokyo, UCLA, Washington University, and Yale. This work was supported by a grant from the Social Security Administration through the Michigan Retirement Research Center (Grant # 10-P-98358-5) and NIH/NIA grant R01-AG030841-01. The opinions and conclusions are solely those of the authors and should not be considered as representing the opinions or policy of the Social Security Administration or any agency of the Federal Government or either the Retirement Research Consortium or MRRC.
1. Introduction

As the U.S. population ages and the moment approaches when Social Security benefit outlays will exceed payroll tax receipts, many discussions of Social Security reform naturally focus on the system’s solvency. This paper argues that issues of efficiency deserve attention as well. The current Social Security rules may generate, or exacerbate, labor-supply distortions; these distortions may contribute substantially to the system’s social cost; and, recent demographic trends may augment their importance. This paper proposes, and evaluates, a simple Social Security reform aimed at alleviating distortions to private retirement decisions that the Social Security system may have created.

The proposed reform would establish a long vesting period (say, 34-40 years of contributions). Subsequent to vesting, a worker would no longer face the old-age and survivors insurance (OASI) payroll tax and his/her benefits would be fixed. In fact, we would maintain the existing benefit formula, but base it only on earnings prior to the vesting age. Individuals who continue to work after “vesting” would thus receive a 10.6 percent payroll tax reduction. To maintain revenue neutrality within the system, there would be a slight increase in the payroll tax during the vesting period.¹

Following the tradition of Auerbach and Kotlikoff [1987] and others, we evaluate this reform in the context of a certainty equivalent life-cycle model. In contrast to the tradition, we estimate the parameters of the model using microeconomic data on earnings, consumption, and retirement. We employ what we think is a novel estimation strategy that provides quite precise estimates of key parameters. The strategy uses both panel data from the Health and Retirement Study (HRS), including linked Social Security annual earning records, and pseudo panel consumption expenditure data from the Consumer Expenditure Survey (CEX). Simulations of the estimated model suggest that the proposed reform could raise retirement ages by more than a year, on average; equivalent variations from the reform could average $4,000 per household (2005 dollars, present value age 50) or more; and, society’s additional income tax revenues could average $8,900 per household.

The logic of the proposed reform echoes a literature on age-dependent taxation that points to efficiency gains from using age to target lower tax rates at households with higher elasticities of labor supply.² Intuitively, the reform aims to eliminate the substitution

¹ Similar reforms have been proposed elsewhere, both in an earlier version of this paper (ASSA annual meeting [2006]), and in work by others (Shah et al. [2006], Burtless and Quinn [2002]). This paper is, as far as we know, the first to evaluate the effects of this reform with a model with estimated parameters — see below.

² See, for example, Kremer [2002], Erosa and Gervais [2002], Lozachmeur [2006], and Weinzierl [2007]. Banks and Diamond [2008, p. 54-57], provides a summary.
effects of Social Security taxes late in life, when labor supply is especially elastic, while leaving other potential distortions of the system unchanged. To see how it can work, note first that our simple model is consistent with long–run balanced growth; it predicts that secular technological progress will not affect the average retirement age. The same characteristic implies that the income and substitution effects of Social Security taxes offset one another on average. However, Social Security benefits also generate an income effect, which leads to earlier retirement, and a substitution effect, which leads to later retirement. In the case of benefits, the income effect tends to dominate; the present value of Social Security benefits is quite insensitive to marginal earnings for households with long work histories — for many households, benefits are effectively lump-sum. On balance, therefore, the existing Social Security system tends to promote an increase in leisure, in the form of earlier retirement. Our proposed reform dismantles the payroll tax late in careers — but before many households’ optimal retirement age. While the reform’s revenue neutrality preserves the income effect of Social Security taxation, reducing the OASI tax rate to zero late in life eliminates the taxation’s substitution effect, which encourages early retirement. Thus, we can hope to take advantage of the potentially high elasticity of labor supply at the age of retirement to offset work disincentives of the current Social Security system.

To quantify the effects of our reform, this paper develops a life–cycle model in which households choose their retirement age as well as their lifetime consumption/saving profile, jobs require full-time work, and retirement is permanent. The benefit to a household of later retirement is greater lifetime earnings; the cost is forgone leisure — and, more generally, lost time at home. A household derives a flow of services from its consumption expenditure and leisure. The service flow, in turn, yields utility through a conventional isoelastic utility function. Although our baseline model ignores health considerations, we present a second formulation with a stochastic, but insurable, chance of disability.

The model predicts a discontinuous change in expenditure at a household’s retirement, due to the abrupt change in leisure and the intratemporal complementarity of expenditure and leisure. A number of empirical studies have described a drop in household consumption expenditure at the time of retirement (Banks et al. [1998], Bernheim et al. [2001], Hurd and Rohwedder [2003, 2005], Haider and Stephens [2005], Aguiar and Hurst [2005], Blau [2006], and others). We use the magnitude of the drop, which this paper measures from CEX data, as well as age of retirement, measured from the HRS, to identify the model’s key parameters.

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3 This non-separability may be interpreted as deriving either from tastes or from the production technology of consumption, including home production. See section 2.1.  
4 We first described our strategy in Laitner and Silverman [2005]. Hall [2006] develops a similar method for estimating the curvature of the utility function. In a similar vein,
Our analysis predicts substantial behavioral and welfare consequences from reform. We find, for example, that stopping the Social Security OASI payroll tax after a vesting period of 34 years of contributions could lead households to postpone their retirement by about one year, on average. We calculate that consumers, on average, would pay approximately $4,000 (2005 dollars, in present value at age 50) to participate in the post-reform system. When we account for the social gain from income taxes on longer careers, the total social benefit could increase to almost $13,000 per household.

Certain assumptions of our model — such as jobs requiring full-time work, the permanence of retirement, and the absence or insurability of many forms of risk — may amplify the behavioral consequences and efficiency gains from reform. Some aspects of a richer model might dampen the effects. However, we believe that estimated magnitudes of the gains in our model indicate that this paper’s reform is worth further consideration.

This paper joins a large literature aimed at evaluating the effects of Social Security on labor supply. See Feldstein and Liebman [2002] for a review. By applying an explicit life-cycle model, we differ from much of this literature, which seeks reduced form estimates. Implementing a structural model allows us, most importantly, to evaluate the life-cycle effects on retirement and consumption of counterfactual reforms. By estimating the parameters of a fully-specified model, our paper also joins a smaller literature that provides structural estimates of life-cycle models of retirement (see, for example, Gustman and Steinmeier [1986], Rust and Phelan [1997], Bound et al. [2005], French [2005], and van der Klaauw and Wolpin [2005]). Our work is distinguished from this literature by its emphasis on a particular reform and by its use of both earnings and consumption data. Our estimation differs from many recent structural models of retirement in its certainty equivalent approach. Policy simulations, however, often employ such a framework, and we believe that it provides a rich yet tractable formulation — permitting analytic as well as numerical insights. In fact, our paper is an effort to bridge structural econometric and policy-oriented literatures.

The organization of this paper is as follows. Section 2 describes our basic model and its formulation with stochastic disability. Section 3 discusses our pseudo-panel data on consumption expenditure, our HRS data on lifetime earnings and retirement ages, and our parameter estimates. Section 4 qualitatively and quantitatively analyzes the Social Security reform outlined above. Section 5 concludes.

Chetty [2006] shows how to estimate risk aversion, in part, from the change in consumption associated with a random change in labor supply.

If, for example, we allowed adjustments to labor supply during the vesting period, reactions to the small increase in the payroll tax early in life might in part counteract efficiency gains from removing the tax late in life.
2. The Model

Section 2.1 presents our basic model; 2.2 elaborates the framework to accommodate uncertain disability.

2.1 Basic Model

In our model, each household maximizes utility subject to a lifetime budget constraint. We focus on married couples and assume unitary decision making for each household and no divorce. At a household’s inception, both spouses learn their earning power — that is, the lifetime profile of their wage rates. At that time, the wife sets her lifetime labor force participation schedule, and the household chooses fertility and family structure paths. We treat these plans as predetermined as households make their remaining household consumption and labor force decisions. The real interest rate is $r_t = r$ all $t$; and, life spans are certain.\footnote{Some papers assume households face earnings uncertainty (Hubbard et al. [1994], Gourinchas and Parker [2002], Scholz et al. [2006], and many others); some assume households learn about their earning abilities (e.g., Guvenen [2007]), perhaps eventually knowing more than outside observers; and, some assume a nonstochastic, representative lifetime profile (e.g., Auerbach and Kotlikoff [1987]). The present paper assumes idiosyncratically different lifetime profiles, but no uncertainty about them on the part of households themselves or observers. As noted in the Introduction, we view this certainty-equivalent approach as a natural one for an initial evaluation of the proposed reform. If the reform does not generate substantial behavioral or welfare gains in the absence of uncertainty, it is unlikely that adding uncertainty will enhance the gains.}

For now, we also abstract from health risk. We assume that men either work full-time in the labor market or not at all.\footnote{See, for example, Rust and Phelan [1997, p.786], Hurd [1996]. An indivisible workday is consistent with the fact that U.S. data show little trend in male work hours or participation rates after 1940, except for a tendency toward earlier retirement 1940-80 — e.g., Pencavel [1986], Blundell and MaCurdy [1999], and Burkhauser et al. [1999].} At the husband’s retirement age $R_i$, both spouses stop working.\footnote{See Gustman and Steinmeier’s [2000] evidence that, in fact, couples often retire together.} We analyze a household’s choice of $R_i$ and consumption expenditure at all ages.

Following convention, we assume isoelastic utility. Letting $x_{is}$ be the consumption expenditure of household $i$ when its adult male is age $s$, letting $n_{is}$ be the household’s size in “equivalent adults,” and assuming that time at home is complementary to consumption of goods, the utility flow of household $i$ at age $s$ is...
\[ u(x_{it}, i, s) = \frac{1}{\gamma} \cdot n_{is} \cdot \left[ \frac{\lambda_{it} \cdot x_{it}}{n_{is}} \right]^{\gamma}, \quad \gamma < 1, \] (1)

where for complementarity parameter \( \lambda > 1 \) (estimated below),

\[ \lambda_{is} \equiv \begin{cases} 1, & \text{if } s < R_i, \\ \lambda, & \text{if } s \geq R_i. \end{cases} \] (2)

We treat the effects of family composition as in Tobin [1967], setting

\[ n_{is} = 1 + \chi^S(i, s) \cdot \xi^S + \chi^K(i, s) \cdot \xi^K, \] (3)

where

\[ \chi^S(i, s) \equiv \begin{cases} 1, & \text{if the household includes a spouse when it is age } s, \\ 0, & \text{otherwise}, \end{cases} \]

and \( \xi^S \) and \( \xi^K \) are adult-equivalent weights (to be estimated).

To understand the role played by the number of equivalent adults \( (n_{is}) \), temporarily set \( \lambda = 1 \) and \( r = 0 \). Then if we equate the marginal utility of a household’s expenditure at ages \( s \) and \( t \),

\[ [n_{is}]^{1-\gamma} \cdot [c_{is}]^{\gamma-1} = [n_{it}]^{1-\gamma} \cdot [c_{it}]^{\gamma-1} \iff c_{is}/c_{it} = n_{is}/n_{it}. \]

In this way we see that \( n_{is} \) provides the scaling factor that a household uses to adjust its year-to-year expenditure in response to family composition changes.

To understand the complementarity captured by condition (2), temporarily set \( n_{is} = 1 \). Suppose market expenditure \( x_{is} \) yields “service flow” \( c_{is} \), upon which household utility ultimately depends. Then our model assumes

\[ c_{is} = \lambda_{is} \cdot x_{is}. \] (4)

In other words, \( \lambda_{is} \) governs a household’s productivity in transforming consumption expenditure into household service flows. Several interpretations of the change in productivity at retirement are possible. (i) Upon retirement, a household no longer incurs the transportation, wardrobe, and other costs of going to work (Cogan [1981]); it can relocate for advantages in climate, cost of living, and proximity to retirement amenities; it gains more complete control over its schedule (e.g., Hamermesh [2005]); and, it can take maximal advantage of off-peak prices. The manifestation of these benefits in our model is \( \lambda > 1 \).
(ii) A household has more time for leisure and/or home production after retirement (e.g., Aguiar and Hurst [2005] and House et al. [2008]), and non-labor-market time and $x$ are complementary.

Our empirical work assumes, more specifically, that household service flows are produced from consumption expenditure and leisure using a Cobb-Douglas technology:

$$c_{is} = f(x_{is}, \ell_{is}) \equiv [x_{is}]^\alpha \cdot [\ell_{is}]^{1-\alpha}, \quad \alpha \in (0, 1),$$

where $\ell_{is}$ denotes leisure time. In fact, approaches (4) and (5) are equivalent. To see this, normalize $\ell = 1$ for pre-retirement ages and let $\ell = \bar{\ell} > 1$ post retirement. Since $\alpha > 0$, $\alpha \cdot u(\lambda_{it} \cdot x_{it})$ induces the same preference ordering as $u(\lambda_{it} \cdot x_{it})$, which we write as $\alpha \cdot u(\lambda_{it} \cdot x_{it}) \sim u(\lambda_{it} \cdot x_{it})$, we have

$$\frac{1}{\bar{\gamma}} \cdot [f(x_{it}, \ell_{it})]^{\bar{\gamma}} = \frac{\alpha}{\alpha \cdot \bar{\gamma}} \cdot [x_{it}]^\frac{1-\alpha}{\alpha} \cdot [\ell_{it}]^{\frac{1-\alpha}{\alpha}} \sim \frac{1}{\gamma} \cdot [\lambda_{it} \cdot x_{it}]^\gamma$$

$$\iff \gamma = \alpha \cdot \bar{\gamma} \quad \text{and} \quad \lambda_{it} = [\ell_{it}]^\frac{1-\alpha}{\alpha}. \quad (6)$$

In other words, with a properly scaled $\gamma$ and with

$$\lambda = [\frac{\ell_{it}}{\alpha}], \quad (7)$$

formulations (4) and (5) make the same predictions. The first part of (6) also shows that our formulation has intratemporal nonseparability — similar to, for example, Auerbach and Kotlikoff [1987] and Cooley and Prescott [1995].

Household $i$'s life-cycle problem is to solve

$$\max_{R_i, x_{is}} \int_{S_i} R_i e^{-\rho \cdot s} \cdot u(x_{is}, \ell_{is}, s) ds + \varphi(a_{i,R_i} + B_i(R_i) \cdot e^{r \cdot R_i}, i, R_i) \quad (8)$$

subject to: $\dot{a}_{is} = r \cdot a_{is} + y_{is} - x_{is}$,

$$a_{iS_i} = 0,$$

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9 As noted above, retirement ages for U.S. males have shown little trend since 1980. Proposition 4 and its Corollary below show that the Cobb-Douglas specification has the advantage of being consistent with such an outcome in the sense that long-run growth from technological progress will not affect the average $R_i$.

10 See also Heckman [1974], King et al. [1988], Hurd and Rohwedder [2003].
\[ y_{is} = \begin{cases} [e_{is}^M + e_{is}^F] \cdot w \cdot (1 - \tau - \tau^{ss}), & \text{for } S_i \leq s < R_i, \\ 0, & \text{otherwise.} \end{cases} \]  

(9)

where \( \rho \) is the subjective discount rate; the household’s adult male supplies \( e_{is}^M \) “effective hours” in the labor market per hour of work time; the adult female supplies \( e_{is}^F \) “effective hours;” the wage rate per effective hour is \( w \); the income–tax rate is \( \tau \); the combined Social Security OASI and Hospital Insurance tax rate is \( \tau^{ss} \); and, household net worth is \( a_{is} \). “Effective hours” exogenously change with age, reflecting an individual’s maturity and economywide technological progress.

The function \( \varphi(.) \) gives the continuation value associated with retirement at age \( R_i \) and satisfies

\[ \varphi(A + B_i(R_i) \cdot e^{r \cdot R_i}, i, R_i) \equiv \max_{x_{is}} \int_{R_i}^{T} e^{-\rho \cdot s} \cdot u(x_{is}, i, s) \, ds \]  

(10)

subject to: \( \dot{a}_{is} = r \cdot a_{is} - x_{is} \),

\[ a_{iR_i} = A + B_i(R_i) \cdot e^{r \cdot R_i} \quad \text{and} \quad a_{iT} \geq 0, \]

where the age–0 present value of capitalized Social Security and Medicare benefits is \( B_i(R_i) \). A household takes \( r, w, \tau, \tau^{ss}, e_{is}^M, e_{is}^F, \) and \( B(.) \) as given. In our calculations, Social Security benefits begin at age \( \max\{R_i, 62\} \); Medicare benefits begin at age 65; Social Security benefits depend upon retirement age and are taxed at rate \( \tau/2 \); and, Medicare benefits are not taxed.

There may be inducements to retire at particular ages implicit in some defined benefit pension plans (or, indeed, in some employer–provided health insurance packages) — e.g., Ippolito [1997]. We adopt the view that both employers and workers are heterogeneous in their preferences about retirement ages and that workers choose employers whose preferences match their own. It follows that \( B(R_i) \) reflects Social Security alone — and earnings in (8) are gross of all private–employer benefits.

Basic features of the optimal consumption path are summarized in Proposition 1.

**Proposition 1:** Let household \( i \) begin at age \( S \) and choose to retire at age \( R = R_i \). Then a solution of (8)-(10) has

\[ x_{is}/n_{is} = (x_{iS}/n_{iS}) \cdot e^{r \cdot \theta \cdot (S-S)} \quad \text{all} \quad s < R, \]  

(11)

\[ x_{iR_+}/n_{iR_+} = [\lambda]^{\tau / \gamma} \cdot (x_{iR_-}/n_{iR_-}), \]  

(12)

8
\[
x_{is}/n_{is} = (x_{iR+}/n_{iR+}) \cdot e^{\frac{r-R}{1+\tau}}(s-R) \quad \text{all } s > R.
\]

**Proof:** See Appendix I.

The novel result is (12), which shows how changes in expenditure at retirement are a simple function of the curvature of the utility function (\(\gamma\)) and the enhanced productivity of expenditure after retirement (\(\lambda\)). To develop intuition for this result, note that the model assumes that the productivity of consumption expenditure rises after retirement (\(\lambda > 1\)). If \(u(.)\) were linear, a household would desire to increase its expenditure after retirement to take advantage of this complementarity. When \(u(.)\) is concave, a second force arises: a household desires to “smooth” its service flow intertemporally; so, it tends to want to decrease \(x_{it}\) at retirement to offset increases in service flow \(c\) that would otherwise occur. Condition (12) reveals these competing tendencies. With isoelastic utility, the outcome is precise: “productivity” predominates if \(\gamma \in (0, 1)\), but “smoothing” wins out for \(\gamma < 0\).

A number of papers have noted a systematic drop in consumption expenditure at retirement — a point to which Section 4.4 returns. When \(\gamma < 0\), (12) predicts such a decline. As the discussion of the intuition for Proposition 1 shows, the drop is a consequence of our nonseparable utility function — and indivisible workday.\(^{11}\) Conditions (11)-(13) each play a role in our estimation.

A household’s first-order condition for optimal retirement age gives our second proposition.

**Proposition 2:** Let a household’s optimal retirement age be \(R_i \in (S_i, T)\). Then at \(R = R_i\), we have

\[
\frac{\partial u(x_{i,R-}, i, R)}{\partial x} \cdot [y_{iR} - x_{i,R-} + x_{i,R+} + B_i'(R) \cdot e^{r-R}] = u(x_{i,R+}, i, R+) - u(x_{i,R-}, i, R-).
\]

**Proof:** Proposition 2 is a special case of Proposition 4 below.

The first-order condition given by (14) may be interpreted as follows. Suppose a household contemplates delaying its retirement from \(R\) to \(R + dR\). The left–hand side of (14), times \(dR\), gives the household’s utility gain from delay: \(y_{iR}\) measures the gain in earnings; \(x_{iR+} - x_{iR-}\) is an adjustment to the extra earnings from the corresponding change in optimal expenditure (see (12)); and \(B_i'(R) \cdot e^{r-R}\) measures incremental Social Security benefits from building a longer work history. Multiplying the sum of these dollar

\(^{11}\) The Auerbach and Kotlikoff [1987] model, for example, would have made the same prediction had it assumed an indivisible workday.
figures by the marginal utility of consumption converts the left-hand side of (14) to units of utility. The right-hand side, times $dR$, captures the household’s loss in production of utility if it delays its retirement until $R + dR$, and thus forgoes both the direct value of leisure in retirement and that leisure’s beneficial effects on the productivity of expenditure.

2.2 Disability

Disability among older workers is a potentially important impediment to reforms aimed at lengthening careers. Substantial fractions of the population may, because of poor health, find it very difficult to extend their worklives. Our quantitative investigation will accommodate effects of disability on labor supply. In anticipation of that aspect of the empirical work, this section augments our basic model to include a stochastic chance of disability. For simplicity, we consider the case with actuarially fair insurance and assume disability is exogenous, that one’s health status is objectively verifiable, and that disability is a permanent state that prevents labor market work.

Let $p(t)$ be the probability that a household becomes disabled (or dies) at age $t$. Assume $p(.)$ is continuous everywhere but at $T$, where it may jump to one. Let $P(s)$ be the probability of becoming disabled after age $s$. Then

$$P(s) \equiv \int_s^T p(t) \, dt = \int_s^T p(t) \, dt - \int_s^T p(t) \, dt = 1 - \int_s^T p(t) \, dt.$$ (15)

We will think of disabled households as having a full time allotment of leisure; thus, if $D_i$ is age of disability, we amend (2) to

$$\lambda_{is} \equiv \left\{ \begin{array}{ll} 1, & \text{if } s < \min\{D_i, R_i\}, \\
\lambda, & \text{if } s \geq \min\{D_i, R_i\}. \end{array} \right.$$ (16)

Disability undoubtedly lowers a household’s utility directly. We could allow for this direct disutility of bad health with an additively separable term in the flow utility function. Such a term does not affect household behavior, however, so we omit it.

If household $i$ becomes disabled at age $D = D_i < R_i = R$ and has disability-insurance payout $X_{ID}$, its cumulative utility for ages $t \in [D, T]$ is

$$\tilde{\varphi}(A + X_{ID}, D, R) \equiv \max_{x_{it}} \int_D^R e^{-\rho \cdot t} \cdot u(x_{it}, i, t) \, dt + \varphi\left(a_{iR} + B_i(R) \cdot e^{r \cdot R}, R\right)$$ (16)

subject to: $\dot{a}_{it} = r \cdot a_{it} - x_{it}$,

$$a_{iD} = A + X_{ID}.$$
In our model, household \( i \) receives capitalized sum \( B_i(R_i) \cdot e^{-R_i} \) at its chosen retirement age whether it is disabled or not; thus, disability insurance need only tide a household over until the planned age of retirement.\(^{12}\) Behavior after retirement is the same as in Section 2.1; hence the continuation value at the planned retirement age, \( \varphi(.) \), is unchanged.

To complete the description of a household’s life–cycle problem for the environment with disability, we now discuss insurance. Risk aversion motivates a household to insure the earnings risk created by uncertain disability. Disability status is verifiable so, at its inception a household can choose its retirement age and sell its expected earnings stream to an insurance company for lump sum (in present value at age 0) \( Y_i \) that satisfies

\[
Y_i = \int_S^R p(D) \cdot \int_S^D e^{-r \cdot s} \cdot y_s \, ds \, dD + P(R) \cdot \int_S^R e^{-r \cdot s} \cdot y_{is} \, ds = \\
\int_S^R \int_t^R p(D) \cdot e^{-r \cdot s} \cdot y_{is} \, ds \, dD + P(R) \cdot \int_S^R e^{-r \cdot s} \cdot y_{is} \, ds = \\
\int_S^R P(s) \cdot e^{-r \cdot s} \cdot y_{is} \, ds . \tag{17}
\]

In addition to earnings risk, uncertain disability also generates risk to household service flows; the complementarities between consumption and leisure imply that optimal expenditure at any age will depend on disability status. As a result, even complete earnings insurance will leave the household subject to service flow risk. Thus, the household has incentive to purchase a sequence of term disability policies at ages \( s < R_i \) to insure its service flow. The payout on each is denoted by \( X_{is} \); the corresponding premium is \( p(s) \cdot X_{is} / P(s) \).

Continue to let \( D = D_i \) and \( R = R_i \).\(^{13}\) To summarize, at inception household \( i \) solves

\[
\max_{R, x_{it}, X_{it}} \int_S^R p(D) \cdot \left[ \int_S^D e^{-\rho \cdot t} \cdot u(x_{it}, i, t) \, dt + \varphi(a_{iD-} + X_{iD}, D, R) \right] \, dD + \\
\left[ 1 - \int_S^R p(t) \, dt \right] \cdot \left[ \int_S^R e^{-\rho \cdot t} \cdot u(x_{it}, i, t) \, dt + \varphi(a_{iR+} + B_i(R) \cdot e^{-R_i}, R) \right] \tag{18}
\]

subject to: \( \dot{a}_{it} = r \cdot a_{it} - x_{it} - \frac{p_t \cdot X_{it}}{P_t} \), \( S \leq t \leq R_i \),

\(^{12}\) In practice, households have SSDI, worker’s compensation, and possibly private disability insurance. Provided a disabled household receives SSDI, the formula for determining Social Security benefits adjusts so as not to penalize work years lost due to disability.

\(^{13}\) In practice, earnings and service flow insurance would likely be sold in combination and appear as incomplete earnings insurance.
The criterion’s first term captures lifetime utility if the household becomes disabled at age $D < R$; its second line captures lifetime utility if the household reaches its optimal retirement age without first becoming disabled.

The new version of Proposition 1 is

**Proposition 3:** Consider the model with disability. Let household $i$ choose, at its inception, to retire at age $R = R_i$. Fix any $X_{it}, S \leq t < R$. Let it become disabled at age $D = D_i$. Then solution of (10) and (16)-(18) has

\[
x_{is}/n_{is} = (x_{iS}/n_{iS}) \cdot e^{\frac{r-\rho}{1-\gamma} (s-S)} \quad \text{all} \quad s < s^* \equiv \min\{D, R\} ,
\]

\[
x_{is+}/n_{is+} = [\lambda]^{\frac{1}{1-\gamma}} \cdot (x_{is-}/n_{is-}) \quad \text{for} \quad s = s^* ,
\]

\[
x_{is}/n_{is} = (x_{is^*}/n_{is^*}) \cdot e^{\frac{r-\rho}{1-\gamma} (s-s^*)} \quad \text{all} \quad s > s^* .
\]

**Proof:** See Appendix I.

The new feature of Proposition 3 is the change in consumption upon pre-retirement disability, condition (20). The intuition is as follows. Households adopt full insurance. The need to pay insurance premiums causes lifetime consumption to be lower. Nevertheless, given insurance, the onset of disability causes a household no further financial hardship. The latter fact implies that a household chooses the same proportionate expenditure change after becoming disabled as at the arrival of its planned retirement age in other circumstances.

The analog to Proposition 2 provides a first-order condition for each household’s utility-maximizing retirement age:

**Proposition 4:** Given a solution to (10) and (16)-(18) and retirement age $R = R_i$, if disability does not occur prior to $R_i$, then at $R = R_i$ we have

\[
\frac{\partial u(x_{iR-}, i, R-)}{\partial x} \cdot [B_i'(R) \cdot e^{r\cdot R} + P(R) \cdot [y_{iR} - x_{i,R-} + x_{i,R+}]] = u(x_{i,R+}, i, R+) - u(x_{i,R-}, i, R-) .
\]
Proof: See Appendix I.

As in Proposition 2, (22) balances lost wages (adjusted for post-career expenditure changes) and sacrificed retirement benefits against an enhanced utility function after retirement. What is new is that only earnings net of the cost of disability insurance now constitute an advantage for postponing retirement.

The assumption of isoelastic preferences allows us to simplify (22) for use in estimation.

Corollary to Proposition 4: Suppose we are given a solution to (10) and (16)-(18) with optimal retirement age $R_i$. Suppose $n_{is}$ is continuous at age $R_i$. Let $\Lambda \equiv [\lambda^{\frac{1}{1-\gamma}}]$. Then if disability does not occur prior to $R_i$, at $R = R_i$ we have

$$\frac{B'_i(R) \cdot e^{rR} + P(R) \cdot yR-}{x_iR-} - \frac{1}{\gamma} \cdot [\Lambda - 1] \cdot [1 - \gamma \cdot P(R)] = 0.$$  

Proof: See Appendix I.

3. Data

As previewed in the introduction, this paper uses both CEX data on household expenditures and HRS data on retirement ages, household composition, and lifetime earnings. Our estimation employs one block of moment conditions based on Proposition 3 and a second based on Proposition 4. The first block uses CEX data; the second the HRS.

For the CEX, we use households of every marital status. This choice is dictated by two considerations. First, since the CEX largely provides only a rotating cross section (the panel aspect of the survey is quite limited, see below) we have no way of knowing which singles will marry in the future or which married couples will eventually divorce. Second, Proposition 3 should apply to singles as well as couples. In contrast, we limit our HRS sample to once-married households. This is feasible because the HRS provides true panel data. Furthermore, in this case, missing data for ex-spouses would hamper our computations.

3.1 CEX Data

The Consumer Expenditure Survey (CEX) provides comprehensive data on U.S. household expenditures. The CEX obtains diary information on small purchases from one set of households; it conducts quarterly interviews cataloging major purchases for a second set. The survey also collects demographic data and self-reports on the value of the respondent’s house. At any given time, the sample consists of approximately 5,000 (7,000 after 1999) households, which each remain in the survey for at most 5 quarters. The survey was conducted at multi-year intervals prior to 1984, and annually thereafter. This paper
employs the data for 1984-2001, using survey weights to combine diary and interview data and to aggregate the quarterly data.¹⁴

Laitner and Silverman [2005] compares CEX annual consumption totals with the National Income and Product Accounts. Total private expenditures predicted from the CEX fall short of NIPA amounts in every year, and the discrepancy has tended to increase over time. Assuming that the NIPA numbers are accurate, that item–nonresponse and other measurement errors of the survey typically make CEX totals too low, and that the relative magnitude of survey errors does not systematically vary with age, for each year we scale CEX consumption categories, uniformly across ages, to match NIPA amounts. Appendix II lists our categories and describes three additional adjustments that we make to housing services, health care, and personal business expenditures. Except in the case of housing, this paper draws no distinction between consumer durable stocks and flows.

Deflating with the NIPA personal consumption deflator, we derive an average consumption expenditure amount, \( \bar{x}_{st} \), for each age \( s \) and year \( t \). We then construct a pseudo panel from differences between \( \bar{x}_{s+1,t+1} \) and \( \bar{x}_{st} \). We organize the CEX data so that a household’s age is the age of the wife for a married couple (and the single household head in other cases). The CEX provides information on the fraction of households that are married and the number of children age 0-17 per household. Figure 1 summarizes these data presenting the average over time (from 1984-2001) of the first differences in \( \ln(\bar{x}) \), and these demographic variables, by age.

[Fig 1 here]

Our estimation also requires information on the fraction of retired households at each age and time. CEX data on retirement is unsatisfactory because the survey only inquires whether the respondent is “retired” if he or she had zero weeks of work in the prior twelve months. We turn, therefore, to the March Current Population Survey (CPS) 1984-2001. We consider a CPS household retired, whether also disabled or not, if the head is over 50 years old and answers that he or she is out of the labor force at the time of the March survey for reasons other than unemployment or, in the case of a male, is not “unemployed” yet reports less than 30 hours per week of work. Figure 1 also plots first differences in the fraction of retired households, by age.

¹⁴ The web site http://stats.bls.gov.gov/csxhome.htm presents aggregative tables, codebooks, etc., for the CEX. This paper uses raw CEX data from the ICPSR archive, and we gratefully acknowledge the assistance of the BLS in providing “stub files” of changing category definitions.
Figure 1: Average Annual Changes in the CEX, by Age

Average annual change in ln of household expenditure, fraction of households married, number of children age 0-17, and fraction retired, by age, CEX 1984-2001
3.2 HRS Data

The HRS is our second main data source. We derive earnings profiles and retirement ages from the original HRS survey cohort, consisting of households in which the respondent was age 51-61 in 1992. A majority of households signed a waiver allowing the HRS to link to their Social Security Administration (SSA) earnings history. Each history runs 1951-1991; our HRS survey data covers 1992, 1994, 1996, 1998, 2000, and 2002. We restrict attention to once-married couples with both spouses alive in 1992, with the husband having linked SSA earnings and remaining in the labor force until at least age 51, and with the wife having linked SSA data or reporting no market work prior to 1992. Men and women must have 8-24 years of education. We consider them adults at the age equaling years of education plus 6, and we drop those reaching this age before 1951. The model assumes that adult men and women live independently between finishing school and marriage. We set age of marriage at the minimum of the reported age and age at first birth. We assume that the children of HRS households leave home at age 18. We assume that men die at the close of age 74 and women at the close of 80. We exclude couples with more than 10 years age difference. Omitting households with incomplete data, our sample is 1083.

We assume that an HRS household retires when its adult male does. In each survey wave, the HRS twice asks if each adult is retired and when retirement took place. Prior to 1992, a male is retired if he reports that status on either question. After 1992, a male who reports being retired and works less than 1500 hours per year, or who works less than 1500 hours and never again more than 1500 hours per year, is “retired.” We exclude households that pass our criterion for retirement in one survey wave but fail to do so in a subsequent wave, or that retire before (male) age 50 or remain unretired at (male) age 70. This reduces our sample to 1023. In sensitivity analysis, Section 4.3 shows that alternative definitions of retirement have little effect on our parameter estimates.

Appendix III provides details on our construction of lifetime earning profiles for men and women. Since HRS earnings are net of employer benefits (including health insurance, pension contributions, and employer Social Security tax), we multiply household earnings for each year by the ratio of NIPA total compensation to NIPA wages and salaries. We derive Social Security benefits after retirement from the statutory benefit formula for 2000. We also incorporate a stream of Medicare benefits after age 65, less participant SMI cost — see Appendix III.

We assume a constant gross-of-income tax real interest rate of 5%/yr. We disregard

government transfer payments other than Social Security. Our income tax rate $\tau$ (recall (9)) comes from government spending on goods and services less indirect taxes (already removed from profits, and implicitly absent from wages and salaries below). Dividing by national income, the average over 1952–2003 is 14.28%/year. In the calculations below, the Social Security benefit formula, including the ceiling on taxable annual earnings, follows the history of the U.S. system.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Coef. Var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age Male Last Works$^a$</td>
<td>60.3007</td>
<td>50</td>
<td>69</td>
<td>.0630</td>
</tr>
<tr>
<td>Couple: Male Age - Female Age</td>
<td>2.7861</td>
<td>-8</td>
<td>10</td>
<td>1.0672</td>
</tr>
<tr>
<td>Male Age Marriage</td>
<td>24.0134</td>
<td>14</td>
<td>56</td>
<td>.1731</td>
</tr>
<tr>
<td>Children per Couple</td>
<td>2.7814</td>
<td>0</td>
<td>10</td>
<td>.5186</td>
</tr>
<tr>
<td>YM50 (thou)$^b$</td>
<td>2,664</td>
<td>808</td>
<td>21,190</td>
<td>.5692</td>
</tr>
<tr>
<td>YF50 (thou)$^b$</td>
<td>549</td>
<td>0</td>
<td>4,314</td>
<td>.9704</td>
</tr>
<tr>
<td>B50 (thou)$^c$</td>
<td>162</td>
<td>91</td>
<td>251</td>
<td>.1447</td>
</tr>
</tbody>
</table>

Source: see text. HRS household weights. Sample size=1023.
(a) Includes cases in which male never retires in sample — see Section 4.
(b) Male (female) present value at male age 50 of male (female) lifetime earnings; gross of benefits; net of OASI, HI, and federal income taxes; 1984 dollars (PCE).
(c) Present value (male) age 50 lifetime OASI benefits, 2005 dollars (PCE).

This paper considers two possible measures of disability-induced retirement. In a sequence of questions about work status, the HRS asks respondents whether they are disabled and, if so, the year of onset. According to our “stringent” definition, a male retires because of disability if he classifies himself as disabled prior to, or within one after, retiring. In a separate sequence of questions on health status, the HRS asks respondents whether they have any health problem that “limits their ability to perform work.” According to our “broad” definition of disability-induced retirement, a male retires because of disability if he classifies himself as disabled and/or as having health problems limiting his ability to work prior to, or within one year after his retirement. (Non-response on the work–limitations question reduces our “broad definition” sample to 944.)

Tables 1-3 provide summary information on our HRS sample. Table 1 presents statistics on earnings and basic demographic information; the second table summarizes information on retirement ages. Table 3 presents cumulative fraction of men who characterize 6%/yr.

16 Auerbach and Kotlikoff [1987], for example, use 15%/year.
themselves as disabled and retired. Recall that the sample is limited to men who retire after age 50 and before 71. Table 3’s cumulative fraction at age \( t \) corresponds to \( 1 - P(t) \) from Section 2.

<table>
<thead>
<tr>
<th>Table 2. Male Retirement Status for HRS Couples 1992-2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
</tr>
<tr>
<td>------------------------------</td>
</tr>
<tr>
<td>Retires without disability</td>
</tr>
<tr>
<td>Never retires in sample</td>
</tr>
<tr>
<td>Retires because of disability^b</td>
</tr>
<tr>
<td>Dies prior to retirement</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

Source: see text. Definitions of “disability” — see text.
(a) Sample omits non-respondents to work–limitations question.
(b) Self-classified as “disabled” one year or less after retirement — see text.

<table>
<thead>
<tr>
<th>Table 3. Cumulative Probability of Male Disability: HRS Couples 1992-2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>51</td>
</tr>
<tr>
<td>55</td>
</tr>
<tr>
<td>60</td>
</tr>
</tbody>
</table>

Source: see text. See text for definitions of “disability.” See Table 2 for sample sizes.

4. Estimation

In order to simulate the consequences of the counterfactual reform, we need to estimate the vector of utility and technology parameters

\[
\beta \equiv (\rho, \gamma, \xi^S, \xi^K, \lambda).
\]  

We adopt a method of moments approach with two blocks of moment conditions, one based on Proposition 3 and CEX data, and the other on Proposition 4 and the HRS. We
estimate block 1 and then block 2. Exact identification and the independence of error terms (described in detail below) imply that sequential estimation does not sacrifice efficiency.

4.1 Block 1

The optimal expenditure path described in Proposition 3 gives our first set of moments which we draw from the CEX data. Approximate the adult equivalent equation (3) with

$$\ln(n_{i,s}) \approx \chi^S(i, s) \cdot \xi^S + \chi^K(i, s) \cdot \xi^K,$$

and define an indicator variable

$$\chi^{RD}(i, s)$$

that equals one if the household is retired or disabled, and zero otherwise. If we denote by $\beta^{CEX}$ a vector of composite parameters that are estimable from the CEX alone,

$$\beta^{CEX} \equiv \left( \frac{r - \rho}{1 - \gamma}, \xi^S, \xi^K, \frac{\gamma}{1 - \gamma} \cdot \ln(\lambda) \right),$$

and set

$$g^{1}_{is}(\beta^{CEX}) \equiv s \cdot \beta^{CEX} + \chi^S(i, s) \cdot \beta^{CEX}_2 + \chi^K(i, s) \cdot \beta^{CEX}_3 + \chi^{RD}(i, s) \cdot \beta^{CEX}_4,$$

then Proposition 3 shows

$$\ln(x_{i,s+t,t+1}) - \ln(x_{ist}) \approx g^{1}_{is}(\beta^{CEX}) - g^{1}_{is}(\beta^{CEX}).$$

In other words, the change in the log of a household’s consumption is a linear function of the composite parameters $\beta^{CEX}$.

The CEX reports average expenditure amounts for each age $s$ and time $t$ ($\bar{x}_{st}$). Given the large CEX sample, we replace the household-level equation (26) with its age-time average analogue

$$\ln(\bar{x}_{s+1,t+1}) - \ln(\bar{x}_{st}) \approx \sum_i g^{1}_{i,s+1}(\beta^{CEX}) - \sum_j g^{1}_{j,s}(\beta^{CEX}),$$

applying sample weights. Finally, defining

$$q^{1}_{st}(\beta^{CEX}) \equiv \ln(\bar{x}_{s+1,t+1}) - \ln(\bar{x}_{st}) - \sum_i g^{1}_{i,s+1}(\beta^{CEX}) - \sum_j g^{1}_{j,s}(\beta^{CEX}),$$

our estimation uses
$$q_{st}^1(\beta^{CEX}) = v_{s+1,t+1} - v_{st},$$  \hspace{1cm} (28)$$

where $v_{st}$ is an iid random variable with mean 0 and variance $\sigma_v^2$.

We think of $v_{st}$ as reflecting measurement error in $\ln(\bar{x}_{st})$. A household survey of consumption expenditure places particularly heavy burdens on respondent memory and understanding of classification systems; systematic errors and omissions can affect even average values such as $\ln(\bar{x}_{st})$. (Recall Section 3 and Appendix II.)

Although we estimate (28) as a GLS regression, for symmetry with our second block we interpret the GLS first–order conditions as method of moments equations. We can write (28), over ages and times, in vector form:

$$q^1(\beta^{CEX}) = \Upsilon,$$

where $\Upsilon$ has covariance matrix $\sigma_v^2 \cdot \Omega$. A Cholesky decomposition yields $L^T \cdot L = \Omega^{-1}$, with $L$ lower triangular. Then the moment conditions that duplicate GLS are

$$[L \cdot q^1(\beta^{CEX})]^T \cdot [L \cdot V_j^1] = 0, \hspace{0.5cm} j = 1, ..., 4,$$  \hspace{1cm} (29)$$

with

$$V_1^1(st) = 1,$$

$$V_2^1(st) = \sum_i \chi^S(i, s + 1) - \sum_j \chi^S(j, s),$$

$$V_3^1(st) = \sum_i \chi^K(i, s + 1) - \sum_j \chi^K(j, s),$$

$$V_4^1(st) = \sum_i \chi^{RD}(i, s + 1) - \sum_j \chi^{RD}(j, s).$$

Solution of (29) yields an estimate of $\beta^{CEX}$, say, $\hat{\beta}^{CEX}$. Standard steps yield an estimated covariance matrix for $\hat{\beta}^{CEX}$, say, $\hat{C}^{CEX}$.

4.2 Block 2

Our second block of moments is given by the first–order condition in Proposition 4 and uses lifetime earnings, demographic information, and retirement ages from the HRS. Given candidate parameter values for $\beta^{CEX}$ and $\gamma$, HRS data for household $k$, and an age of last earnings $R = R^0_k$, Proposition 3 characterizes the household’s optimal expenditure time path $x_{ks}$. Using that expenditure time path and the Corollary to Proposition 4, define
\[
g_k^2(\beta^{CEX}, \gamma, R) = \frac{B_k'(R) \cdot e^{\gamma R} + P(R) \cdot y_{k,R-}}{x_{k,R-}} - \frac{1}{\gamma} \cdot [e^{\beta^{CEX} - 1}] \cdot [1 - \gamma \cdot P(R)].
\]

If \( R_k \) is the optimal retirement age for the household and \( R_k^0 = R_k \), then \( g_k^2 = 0 \).

In practice, if \( R_k^0 = R_k \), we set \( g_k^2 = \epsilon_k \) where \( \epsilon_k \) is a random variable with mean 0. We interpret this residual as reflecting measurement error — specifically for \( y_{k,R-} \). Although \( B_k'(R) \) and \( x_{k,R-} \) reflect average lifetime earnings, \( y_{k,R-} \) only registers last earnings, which happen to be particularly difficult to assess accurately — recall Appendix III.\(^{17}\)

If disability (or death) causes the household to stop working prior to the optimal retirement age (i.e., if \( R = R_k^0 \leq R_k \)), we assume \( g_k^2 \geq \epsilon_k \).\(^{18}\) Similarly, if at the last wave of the survey household \( k \) has not yet retired, again \( R_k^0 \leq R_k \) — so we assume \( g_k^2 \geq \epsilon_k \).

Assuming that \( \epsilon_k \) is normally distributed, let \( \phi(\cdot, \sigma^2) \) be the normal density and define

\[
q_k^2(\beta^{CEX}, \gamma, R) \equiv \begin{cases} 
g_k^2(\beta^{CEX}, \gamma, R) & \text{if voluntarily retires in sample,} \\ 
\int_{-\infty}^{\infty} g_k^2(\beta^{CEX}, \gamma, R) \cdot e^{\phi(e, \sigma^2) de} & \text{otherwise,}
\end{cases}
\]

\[
q_k^3(\beta^{CEX}, \gamma, R) \equiv \begin{cases} 
[g_k^2(\beta^{CEX}, \gamma, R)]^2 & \text{if voluntarily retires in sample,} \\
\int_{-\infty}^{\infty} g_k^2(\beta^{CEX}, \gamma, R) \cdot e^{2 \phi(e, \sigma^2) de} & \text{otherwise.}
\end{cases}
\]

Then our HRS moment conditions are

\[
\sum_k q_k^2(\beta^{CEX}, \gamma, R_k^0) \cdot 1 = 0 \quad \text{and} \quad \sum_k q_k^3(\beta^{CEX}, \gamma, R_k^0) \cdot 1 = \sigma^2. \tag{30}
\]

\(^{17}\) Note that if, for example, \( y_{k,R-} \) is the actual value but we observe \( y_{k,R-}^* = y_{k,R-}^* \cdot (1 + \eta_k), \eta_k \sim N(0, \sigma^2_\eta) \), then \( \epsilon_k = (y_{k,R-}^*/x_{k,R-}) \cdot \eta_k \) will be homoscedastic because in our formulation, proportionate changes in earnings leave \( y_{r,R-}/x_{k,R-} \) and \( R \) constant.

\(^{18}\) The second—order necessary condition for \( R_k \) implies \( g_k^2 \) is decreasing in \( R \) at \( R = R_k \) (recall the discussion following Proposition 2). Our inequality for \( \epsilon_k \) takes this monotonicity to be global.
Given the composite parameters estimable from the CEX alone, \( \hat{\beta}^{CEX} \), (30) determines \((\gamma, \sigma^2)\). The estimate of \(\gamma\), denoted \(\hat{\gamma}\), and the first and fourth elements of the composite parameter vector \(\hat{\beta}^{CEX}\) together give our estimates of \(\rho\) and \(\lambda\). Thus, we have our estimate of all of the structural parameters, \(\hat{\beta}\). We estimate the covariance matrix of \(\hat{\beta}\), say, \(\hat{C}\), as in Gallant [1987, ch.6], using (29)-(30).\(^{19}\)

4.3 Baseline Results

Table 4 presents our parameter estimates. The first column at the top of Table 4 displays baseline estimates of \(\beta^{CEX}\) from a version of equation (28) modified to include annual time dummy variables for 1984-2000, with coefficients constrained to sum to zero. The bottom of this column presents estimates of \(\beta\) using (30), given \(\beta^{CEX}\) and the stringent definition of disability.

We note first that a sensible interpretation of our model requires \(\gamma < 1\), \(\lambda > 1\), \(\xi^S > 0\), and \(\xi^K > 0\), and that these conditions hold for the baseline estimates. Concerning the preference parameters, our baseline estimate of \(\gamma\) is -0.40 and \(\rho\) is 0.006. The intertemporal elasticity of substitution (IES) corresponding to this estimate of \(\gamma\) is 0.72.

These baseline results may be compared with estimates that have identified the IES from expected changes in interest rates. Using aggregate consumption data Hall [1988], Campbell and Mankiw [1989], and Patterson and Pesaran [1992], for example, estimate the IES for consumption to be very nearly zero. Micro studies tend to estimate larger intertemporal elasticities. Banks et al. [1998], for instance, estimate the average IES for consumption to be approximately 0.5. In another example, Attanasio and Weber [1993] estimate an IES for consumption of approximately 0.75 from micro data.\(^{20}\) Although our calculations rely on a very different source of variation to estimate the IES, our basic results are similar to, if on the larger end of, those obtained in the micro studies.

Our \(\hat{\beta}^{CEX}\) provides an estimate of the average lifetime growth rate for households’

\(^{19}\) Some analyses (e.g., Gustman and Steinmeier [2000]) argue that the random variable \(\epsilon_k\) might reflect a household’s taste for time at home as well as measurement error. Our minimalist orthogonality condition (30) only requires that \(\epsilon_k\) have mean zero. Even if heterogeneity of taste were important, our estimates could remain consistent. This paper’s focus is policy reform, but the nature of the block-2 error term remains an interesting topic for future research.

\(^{20}\) Barsky et al. [1997] use hypothetical questions to estimate an IES distribution for their sample. They find an average IES of 0.2, with less than 20% of respondents having an IES greater than 0.3. Others who have attempted to estimate a distribution of intertemporal elasticities of substitution find evidence that the IES is increasing with wealth (e.g., Blundell et al. [1994]).
Table 4. Estimated Coefficients Equations (28)-(30):
Estimated Parameter (Std. Error/T Stat.)

<table>
<thead>
<tr>
<th>Parameter or Observation Count</th>
<th>Specification of (28):</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Annual Time Dummies(^a)</td>
</tr>
<tr>
<td></td>
<td>Stringent Def. Male Disability</td>
</tr>
</tbody>
</table>

Equation (28): Estimates of \( \beta^{CEX} \) from CEX Data

\[
\begin{align*}
\beta_1^{CEX} &= \gamma - \rho \\
\beta_2^{CEX} &= \beta_3 = \xi^S \\
\beta_3^{CEX} &= \beta_4 = \xi^K \\
\beta_4^{CEX} &= \frac{1}{1-\gamma} \ln(\lambda) \\
\sigma^2_\nu &= 0.008406 \quad 0.000860 \\
\end{align*}
\]

Observations: 935 \quad 55

Equation (30): Estimates of \( \beta \) given \( \beta^{CEX} \); HRS Data; \( \xi^S = \beta_3 \) and \( \xi^K = \beta_4 \) as above

\[
\begin{align*}
\beta_1 &= \rho \\
\beta_2 &= \gamma \\
\beta_3 &= \lambda \\
\sigma^2_\xi &= 0.1499 \quad 0.1637 \quad 0.1546 \quad 0.1646 \\
\end{align*}
\]

Observations: 1023 \quad 944 \quad 1023 \quad 944

Source: see text.

a. Time–dummy coefficients omitted.
per capita consumption (see Proposition 3) of 2.6%/yr. This suggests that between, say, ages 25 and 62, in the absence of retirement, a household’s consumption per equivalent adult rises by a factor of about 2.67. In Auerbach and Kotlikoff [1987] the corresponding factor is about 1.54; in Gokhale et al. [2001], it is 1.74; in Tobin [1967], it is 13.33. (For an infinite–lived representative agent model, e.g., Cooley and Prescott [1995], the growth rate of consumption in a steady–state equilibrium would, of course, match the growth rate of GDP.)

Our baseline estimates of the adult equivalency weights, $\xi^a$ and $\xi^K$, are consistent with substantial returns to scale for larger households. The estimate of $\xi^a$ suggests that the addition of a spouse raises household consumption by 36 percent. This is somewhat smaller than the U.S. Social Security system’s award to retired households of 50 percent extra benefits for a spouse. Our baseline estimate of $\xi^K$ suggests an increase in household consumption of 13 percent for each child age 0-18. Since two parents correspond to about 1.3 “equivalent adults,” a child adds about 20 percent as much as each parent. Mariger [1986] estimates that children consume 30 percent as much as adults; Attanasio and Browning [1995, p. 1122] suggest 58 percent; Gokhale et al. [2001] assume 40 percent; most of the analysis in Auerbach and Kotlikoff [1987] implicitly weights children at zero; and, Tobin [1967] assumes teens consume 80 percent as much as adults, and minor children 60 percent. Our estimates would also be consistent with parents who spend a great deal on their children but reduce expenditures on themselves at the same time — perhaps vicariously enjoying their children’s consumption.

The baseline estimate of $\beta^{C^E}_{4} \gamma$ in Table 4 indicates a 25 percent drop in consumption at retirement. This seems roughly consistent with estimates in Bernheim et al. [2001], Banks et al. [1998], and Hurd and Rohwedder [2003]. Our parameter $\lambda$ is less typical and thus harder to compare with previous estimates. However, a number of papers use formulation (6) — see also (7) — from Section 2. Suppose that a week has $7 \times 12 = 84$ available hours and that a full–time workweek is 40 hours. Then Section 2’s normalization might imply $\bar{\ell} = 84/44 = 1.91$. Auerbach and Kotlikoff’s [1987] favorite calibration has $\bar{\gamma} = -3$ and $\alpha$ (roughly) = 0.4; hence, in our terminology they assume $\gamma = \alpha \cdot \bar{\gamma} = -1.2$ and $\lambda = [1.91]^{6/4} = 2.64$. Altig et al. [2001] use $\bar{\gamma} = -3$ and $\alpha$ (roughly) = 0.5; hence, $\gamma = -1.5$ and $\lambda = [1.91]^{5/5} = 1.91$. Cooley and Prescott [1995] set $\bar{\gamma} = 0$ and $\alpha = .36$; thus, in our terminology, they have $\gamma = 0$ and $\lambda = [1.91]^{64/36} = 3.16$. Table 4 estimates, $\bar{\gamma} = -0.39$ to -0.71 and $\lambda = 2.08$ to 2.58, fall in the general range of the latter three sets of calibrations.

4.4 Sensitivity and Goodness of Fit

This subsection evaluates the consequences for the parameter estimates of changing various modeling assumptions, and assesses the model’s goodness of fit.
We begin with an investigation of the effects on the first block of estimates ($\beta^{CEX}$) of relaxing some of our assumptions regarding uncertainty. The nature of our data precludes an analysis of idiosyncratic shocks to individual households. The coefficients on annual time dummy variables can, however, reflect aggregative shocks to interest and/or wage rates. The F-statistic for constraining all time–dummy coefficients in the first column at the top of Table 4 (see text above) to 0 is $F(16, 915) = 3.2776$, with p-value $0.000014$. Looking at T-statistics on the individual dummy coefficients, the largest in absolute value is $-3.0116$, for 1990, a recession year; the second largest is only $1.5114$. If we drop the dummies, estimates of $\beta^{CEX}$ change only modestly, i.e., $\hat{\beta}^{CEX} = (0.0243, 0.3812, 0.1357, -0.2596)$.

The simple time dummies above implicitly constrain household responses to aggregative shocks to be independent of age. The second column at the top of Table 4 provides estimates after averaging $q^t_{st}$ over $t = 1984-2000$, each age $s$, instead of using time dummy variables at all. There is a drastic sacrifice of degrees of freedom; hence, standard errors for the components of $\hat{\beta}^{CEX}$ rise. Nevertheless, Table 4 shows magnitudes of parameter estimates remaining about as before.

Our treatment assumes that adults set their lifetime plans when they finish their education. In analyzing the CEX data, if we set an early starting age, say, $S = 20$, our youngest–age data cells will tend to exclude the highly educated. The dependent variable subsequently will be inflated when the latter join the sample at older ages. On the other hand, a very late starting age has the disadvantages of sacrificing degrees of freedom and observations with family composition changes due to births. To try to steer a middle road, Table 4 uses a sample covering ages 25-79. With a sample limited to 30-79, estimates change only slightly to $\hat{\beta}^{CEX} = (0.0240, 0.3358, 0.1086, -0.2169)$. (With ages 25-74, for instance, $\hat{\beta}^{CEX} = (0.0265, 0.3069, 0.1407, -0.2772)$. Estimates with children leaving home at age 22 instead of 18 are very similar as well.

Turning to goodness of fit of the consumption data, the model appears to capture the life-cycle pattern of expenditure reasonably well. For the specification with time dummies, Figure 2 plots residuals from the left–hand side of (28), averaged over $t = 1984, ..., 1999$, for each age $s$. While their may be some heteroscedasticity at advanced ages (where the CEX sample size tends to be smallest), we see little evidence that the model systematically mispredicts changes in log consumption at particular points in the life-cycle.\footnote{For a recent discussion of problems of estimating linearized consumption first–order conditions, see Attanasio and Low [2004].}

We turn next to the estimates of the structural parameters $\beta$ and equation (30).
Figure 2: CEX Average Annual Expenditure Change Residuals, by Age

Average annual change in ln of expenditure residuals from estimate of equation (28) augmented with year dummies, CEX 1984-1999.children age 0-17, and fraction retired, by age, CEX 1984-2001
Different definitions of disability change the fraction of households whose retirements are classified as censored; however, as can be seen by comparing columns of the second panel of Table 4, the effect on $\tilde{\beta}$ is quite moderate. As with the CEX, we assume children leave home at age 18; changing to age 22 makes little difference.

We can add annual dummies for male birth cohorts, with coefficients summing to zero, analogous to our observation–year dummies in the case of (28). For (30), this could be interpreted as allowing different means for $\epsilon$ across birth–cohorts. To avoid radically uneven cohort sizes, we limit the sample to couples with males born 1928–1941. Wald tests that all dummy coefficients equal 0 yield statistics for columns 1 and 3 of Table 4, respectively, of $\chi^2(13) = 34.4904$ and 32.5341, with corresponding p-values 0.0327 and 0.0578. (The sample size is 976 in both cases.) New point estimates are $(\hat{\rho}, \hat{\gamma}, \hat{\lambda}, \hat{\sigma}^2_e) = (0.0050, -0.4455, 2.2963, 0.1795)$ and $(0.0001, -0.5835, 2.4025, 0.1829)$, which closely resemble Table 4. The pattern of the dummy–variable coefficients is 5 positive, 5 negative, 2 positive, and 1 negative in each case. A possible interpretation is that labor and financial market conditions at the time a cohort reaches retirement age influence the cohort’s retirement outcome — in which case perhaps workers tended to retire slightly later than average in the late 1980s, and earlier than average in the mid 1990s.

We also tried excluding households with careers cut short by disability and death. If health problems occur randomly, this will not induce selection bias. It might, however, reduce the influence of our specialized assumptions about the distribution of $\epsilon$. But, we find only a very slight change in parameter estimates.

Changes to the definition of male retirement change the sample size in Table 2 (by changing the number who do not retire by age 70 and the number who ‘retire’ but subsequently work again) but do not much affect parameter estimates. For example, setting maximal hours per year of work for “retirees” to 1000 modifies column 1, Table 2, to 561, 322, 70, 35, and 988, respectively, but only modifies the estimates of column 1, Table 4, to $(\hat{\rho}, \hat{\gamma}, \hat{\lambda}, \hat{\sigma}^2_e) = (0.0056, -0.4254, 2.3596, 0.1600)$. Similarly, setting maximal hours to 0 modifies column 1, Table 2, to 460, 381, 68, 39, and 948, respectively and modifies the estimates of column 1, Table 4, to $(\hat{\rho}, \hat{\gamma}, \hat{\lambda}, \hat{\sigma}^2_e) = (0.0038, -0.4944, 2.1694, 0.1849)$.

Since we do not base our HRS estimates on a conventional regression equation, we have no exact analog to Figure 2 for evaluating goodness of fit; nevertheless, Figure 3 may serve as a substitute. Figure 3 presents the cumulative distribution of retirement ages for the 628 couples that voluntarily retire in the sample of Table 4, column 1. Since our model permits an arbitrary correlation between $\epsilon_k$ and observables, it will of course just fit the retirement dates of these households. Suppose, however, we replace the residual of each of these households with the mean residual for this group (0.18) and then simulate their retirement ages. Comparing these predicted retirement ages with the actual ages
would provide one way of assessing the predictive power of the model’s basic mechanisms in the absence of a residual. Figure 3 therefore also graphs the cumulative distribution of retirement ages calculated from Proposition 4 for the same couples after replacing each $\epsilon_k$ with 0.18. The mean retirement age for the actual subsample is 59.99; for the simulated subsample it is 59.42. The basic mechanisms of the model seem to simulate retirement behavior fairly well.

[Fig 3 here]

In summary, despite the simplicity of our model, the baseline estimates fit the within-sample data reasonably well. In addition, although F-tests imply that aggregative shocks may be important to both expenditure and retirement behavior, our various attempts to accommodate such shocks do not appreciably affect estimates of the parameters of most interest in this paper.

5. Social Security Reform

This section investigates the consequences of a Social Security reform in which the OASI tax, and all OASI benefit adjustments based on new earnings, cease at a specific age or following a specific span of career years. Although individuals could retire at any age, those who continue working after the Social Security vesting age/span would enjoy a 10.6 percent increase in their aftertax wage. As with the present system, individuals could start collecting Social Security benefits at age 62 or later, with an actuarially fair adjustment for postponed receipt. The reform would be revenue neutral, with the payroll tax raised by the constant amount (at all ages prior to the vesting age) that holds unchanged the average present value of Social Security tax revenues per household less Social Security benefit payments.\(^\text{22}\) Our analysis assumes that wages and interest rates are exogenous — for example, that they depend on international conditions.

In our framework, a transition to reform could be straightforward and uneventful. Government could announce that henceforth all new labor–force participants would be subject to the new system — with existing workers remaining in the old system. The revenue neutrality of the proposed reform means that no special complications would arise from “legacy costs” or other intergenerational transfers.

Table 5 presents simulation outcomes for different vesting ages/spans. We use parameter estimates from Table 4. Different households have different lifetime earning profile shapes, as well as different demographic histories. We need to take this variety into account because an averaged “representative household” would tend to have a smoother,

\(^{22}\) All present value calculations use the net of tax real interest rate 0.0429 from Section 3.
Cumulative fraction of households retired by age of 628 HRS households who voluntarily retired, 1992–2002, and of model simulation for each of these households with residual of equation (30) set to the average for this group (0.18).
<table>
<thead>
<tr>
<th>Vesting Age or Span (years)</th>
<th>Average Change Career Years</th>
<th>Average Equivalent Variation (PV Age 50)</th>
<th>Average Additional Income Tax Revenue Per Household (PV Age 50)</th>
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<tr>
<td><strong>Age</strong></td>
<td></td>
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<tr>
<td>54</td>
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<td>8859</td>
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<td>[8662, 9069]</td>
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<td>59</td>
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<td>1951</td>
<td>4396</td>
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<td>[1885, 2002]</td>
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<td>[85, 96]</td>
<td>[-111, -106]</td>
</tr>
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</table>

a. Based on 1000 random parameter vector draws — see text.

b. Definitions of disability as in Table 3.
and hence possibly quite flat at the peak, profile of lifetime utility as a function of retirement age, which might induce very large responses to changes in tax rates. To preserve realistic heterogeneity, we use our HRS sample of couples. We ask: If each couple had, from birth, faced a reformed Social Security system, how, according to our model, would its retirement age have changed? In other words, we ask what the behavioral responses of the HRS cohorts would have been had government instituted this paper’s reform as they began their careers.

We use parameter values from the first two columns of Table 4. Since the values are only estimates, for each row of Table 5 we draw 1000 random samples for \( \hat{\beta} \), using the asymptotic covariance matrix \( \hat{C} \). Table 5 reports average simulated outcomes and 95 percent confidence intervals. In what follows we both explain these calculations and develop intuition for their magnitudes.

To better understand the simulated behavioral response presented in column 1 of Table 5, focus on row 1, where we have assumed the reform has a vesting age of 54. Absent reform, there are (i) households that retire voluntarily (i.e., without disability) at ages \( \geq 54 \); (ii) households that retire voluntarily at ages \( < 54 \); and, (iii) households forced to retire by disability or death. Rewrite first–order condition (23) as

\[
\frac{B_k'(R) \cdot e^{r-R} + P(R) \cdot y_k,R-}{x_{k,R}} - \kappa(R) = 0. \tag{31}
\]

Fix a parameter estimate \( \hat{\beta} \). Suppose the retirement age of household \( k \) in group (i) is \( R_0^k \). Let the household’s residual — i.e., its actual value of the left–hand side of (31) — be \( \epsilon_k^0 \). Table 5’s simulation reforms the Social Security system at households’ inception — iteratively computing the pre–vesting constant addition to the payroll tax required for revenue neutrality — and determines each household’s new optimal retirement age \( R_1^k \) such that

\[
R = R_1^k \implies \frac{B_k'(R) \cdot e^{r-R} + P(R) \cdot y_{k,R-}^1}{x_{k,R-}^1} - \kappa(R) - \epsilon_k^0 = 0, \tag{32}
\]

where \( B_k^1(\cdot), y_1^1, \) and \( x_1 \) reflect the new Social Security system. In row 1, pre–reform group–(i) averages are roughly \( B'(R) \cdot e^{r-R} = $2200, P(R) \cdot y_{R-} = $62,000 \), and \( x_R = $69,000 \). If we institute reform but maintain \( R = R_0^k \), the \( B'(\cdot) \) term drops to 0 for this group and \( P(R) \cdot y_{R-} \) rises. Gross–of–tax earnings are about $79,000; so, a 10.6% reduction in the payroll tax gives a household almost $8400. Thus, after reform, the numerator of the ratio at the far left end of (31)-(32) rises about 10% at \( R = R_0^k \). If group–(i) households each raise their retirement age one year, on average \( P(R) \cdot y_{R-} \) falls about 3.2%, and Proposition 3 and Table 4 show \( x_{R-} \) rises 2.6%. Hence, a one year increase in \( R \) will not
be enough to achieve equality with 0 in (32). Rather, an average increase in $R$ of 1.6 years is actually needed. Reform does not change $B'(.)$ or $y_{R-}$ for group-(ii) households; thus, their optimal retirement age is negligibly affected. Group (iii) would like to retire later but is constrained by disability. Since about 80% of the row 1 sample falls in group (i), the overall average change in $R$ is about $0.8 \times 1.6 \approx 1.28$, almost exactly the figure that Table 5 reports. Increases are much smaller in rows 2-3 because the relative size of group (ii) rises and, secondarily, because the rate at which $y_{R}$ declines with age increases with $R$.

The second column of Table 5 computes equivalent variations. Continue to use row 1 as an illustration. Reform raises the OASI payroll tax for ages through 54 and eliminates it thereafter. Suppose, for the moment, that we fix pre–reform behavior. Then payroll tax changes induce what we might call “transfers.” Because households that retire earlier than average will not benefit from lower taxes late in life sufficiently to offset their higher payroll tax in youth, their transfers are negative. Conversely, households that retire later than average receive positive transfers. Because reform is revenue neutral, the average transfer is 0. Reform also generates efficiency gains — virtually exclusively for group–(i). We compute these as follows. The left–hand side of (23) is proportional to marginal lifetime utility for a non–disabled household with respect to a change in its retirement age. The proofs of Proposition 4 and its Corollary derive the constant of proportionality. We sum the value of a household’s “transfers” from reform and the addition to its lifetime resources yielding the same increment to lifetime utility as a change from $R = R^0_k$ to $R = R^1_k$ in the post–reform environment. We derive the latter increase in utility by integrating the left–hand side of (32) from $R = R^0_k$ to $R^1_k$, and then multiplying by the constant of proportionality. (By integrating the left–hand side of (32), we maintain the original error $\epsilon^0_k$ throughout the analysis.)

The third column of Table 5 computes the gain in federal income tax revenues from Social Security reform. These are social gains, which private household calculations neglect. Looking at row 1, consider group (i). Average gross–of–tax earnings are about $79000. Section 3’s federal tax rate is 14.28%. If reform extends earnings by 1.25 years, the average tax gain is roughly $14000, or, in present value at age 50, about $9000.

Turning to implications of the results, Table 5 shows a labor supply gain of over 1 year from Social Security reform with vesting after age 54. Row 4 shows almost the same outcome for vesting after 34 years of employment. These increases seem substantial: in terms of lifetime labor supply — measuring each year’s quantity with the constant–dollar value of earnings — the increase would be almost 4 percent. The equivalent variations are

---

23 Note that about 11 percent of households fall in group (iii). In row 1 of Table 5, about 8 percent of households fall in group (ii); in row 2, the latter group rises to about 28 percent; in row 3, about 77 percent.
small relative to lifetime earnings; nevertheless, they average $3400-4000 in rows 1 and 4. The corresponding additional income tax revenues are $8000-8900 per household.

We close this section with three more detailed observations.

First, Table 5 shows that average labor–supply increases depend a great deal on reform’s vesting age, and that we need early vesting to obtain strong results. The “full retirement age” for the Social Security system is 66; the early retirement age is 62. Labor supply gains in Table 5 exceeding 1 year require, however, a post–reform vesting age in the mid 50s. Almost half the gain in the labor supply is gone with vesting at age 59. Gains are negligible with vesting at 64.

Second, the fraction of the population that is disabled is evidently quite important. The top two panels of Table 5 adopt this paper’s “stringent” definition of disability — which is the conventional one. The bottom two panels repeat the analysis for our “broad definition.” The latter re-classifies many retirements as involuntary. In our framework, households that retire involuntarily cannot respond to new incentives for longer careers. If the broad definition were judged the most accurate, Table 5 shows that gains in labor supply from reform would tend to be about half as great at each vesting age.

Third, as indicated above, the reform that this paper studies generates “transfers” as well as efficiency gains. Transfers are 0 for households that retire at the average age, but they are negative (positive) for those that retire earlier (later). Despite the fact that empirical retirement ages are quite bunched (e.g., Figure 3), transfers that are large relative to the average equivalent variation occur rather frequently. See Figure 4. That is to say, even our very simple reform has noticeable, unintended redistributional consequences.

[Figure 4 here]

6. Conclusion

In the end, we think the simulated responses of Table 5 are large enough to justify more attention to Social Security reform of the type that this paper studies. In an era of greater longevity, it is paradoxical that older workers do not appear to have extended their careers. If distortions from tax policy contribute to this outcome, we think Social Security reform of the type analyzed here might provide an attractive remedy. Indeed, our increases in labor supply would be almost 50% larger if the entire payroll tax (rather than just its OASI component) were discontinued at our vesting age.

Although this paper’s life–cycle framework is quite simple, its stripped–down nature enables us to estimate parameters rather than relying on calibration. Similarly, our simulations analyze the behavioral responses from individual households in the HRS sample — which can preserve a realistic degree of heterogeneity of earning profile shapes and family
Distribution of simulated equivalent variations (in 2005 dollars) from the reform with vesting age set to 54 years, for the 1032 HRS households in the estimation sample. Model parameters set to those in Table 4, Column 1.
composition. The simulations take into account the standard errors on our parameter estimates.

One of the greatest changes in life styles in recent decades has been the growing labor force participation of married women. While our formulation incorporates both male and female labor supply, one of the elaborations that we are most anxious to pursue is to model female labor force participation decisions in much greater detail.
Bibliography


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Appendix I: Proofs

Proof of Proposition 1: Fix $R_i = R$. Suppress $i$ for convenience. Consider (8). Define a present-value Hamiltonian

$$
\mathcal{H}(a_t, x_t, \mu_t, t) \equiv e^{\rho t} \cdot u(x_t, t) + \mu_t \cdot [r \cdot a_t + y_t - x_t] \quad \text{all} \quad t \in [S, R].
$$

Necessary conditions all $t \in [S, R]$ include

$$
x_t = \arg\max_x \{\mathcal{H}(a_t, x, \mu_t, t)\}, \quad (A1)
$$

$$
\dot{\mu}_t = -\frac{\partial \mathcal{H}}{\partial a_t}, \quad (A2)
$$

$$
\mu_R = \frac{\partial \varphi}{\partial a_R}. \quad (A3)
$$

Define a marginal utility function

$$
m_t \equiv \begin{cases} 
e^{-\rho t} \cdot [n_t]^{1-\gamma} \cdot [x_t]^{\gamma-1}, & s < R, \\
\lambda^\gamma \cdot e^{-\rho t} \cdot [n_t]^{1-\gamma} \cdot [x_t]^{\gamma-1}, & s \geq R. \end{cases} \quad (A4)
$$

At any $t$, (A1) requires

$$
m_{t-} - \mu_{t-} \leq 0 \leq m_{t+} - \mu_{t+}. \quad (A5)
$$

For a solution, $\mu_t$ and $m_t$ are continuous in $t$ (the last by the Weierstrass-Erdmann corner condition) — e.g., Intriligator [1971, p.354]. Thus,

$$
\mu_{t-} = \mu_{t+} \equiv \mu_t, \quad m_{t-} = m_{t+} \equiv m_t, \quad m_t = \mu_t. \quad (A6)
$$

From (A2), $\dot{\mu}_t = -r \cdot \mu_t$; so, $\mu_t = \mu_0 \cdot e^{-r \cdot t}$. This and the last part of (A6) show

$$
e^{-\rho t} \cdot \frac{\partial u(x_t, t)}{\partial x} = \mu_0 \cdot e^{-r \cdot t}, \quad t \neq R. \quad (A7)
$$

(A7) establishes (11). The same logic establishes (13).

Consider age $t = R$. Define $g_t \equiv m_t - \mu_t$. The preceding paragraph shows $g_t$ is continuous and $g_{R-} = 0$. So, $g_R = 0$. Define an analogous Hamiltonian for (10), with costate $M_t$. Define $G_t \equiv m_t - M_t$. The same logic shows $G_t$ is continuous and $G_{R+} = 0$. So, $G_R = 0$. (A3) shows $\mu_{R-} = \partial \varphi/\partial a_R$, and the envelope theorem implies $M_{R+} = \partial \varphi/\partial a_R$. Hence,
\[ m_{R-} = \mu_t = M_t = m_{R+}, \]
which establishes (12).

**Proof of Proposition 3:** Surpress the index \( i \) for simplicity. Fix \( R \). Looking at the first component of the first integral in (18), one has

\[
\int_S^R p(D) \cdot \int_S^D e^{-\rho \cdot t} \cdot u(x_t, t) dt dD
\]

\[
= \int_S^R \int_t^R p(D) \cdot e^{-\rho \cdot t} \cdot u(x_t, t) dD dt \quad \text{from Fubini’s theorem}
\]

\[
= \int_S^R \left[ P(t) - P(R) \right] \cdot e^{-\rho \cdot t} \cdot u(x_t, t) dt.
\]

In the same way, we can rewrite the whole criterion of (18) as

\[
\int_S^R \left[ P(t) \cdot e^{-\rho \cdot t} \cdot u(x_t, t) + p(t) \cdot \bar{\varphi}(a_{t-} + X_t, t, R) \right] dt +
\]

\[
P(R) \cdot \varphi(a_{R-} + B(R) \cdot e^{r \cdot R}, R). \tag{A8}
\]

Set up a Hamiltonian

\[ H(a_t, x_t, X_t, \mu_t, t) \equiv P(t) \cdot e^{-\rho \cdot t} \cdot u(x_t, t) + p(t) \cdot \bar{\varphi}(a_{t-} + X_t, t, R) + \mu_t \cdot \left[ r \cdot a_t - x_t - \frac{p_t \cdot X_t}{P_t} \right], \quad t < R. \tag{A9} \]

The state variable is \( a_t \); the costate is \( \mu_t \); \( x_t \) and \( X_t \) are control variables.

**Step 1.** For \( t < \min\{D, R\} \) or \( t > \max\{D, R\} \), the proof is exactly analogous to Proposition 1.

**Step 2.** Suppose \( t = D < R \). Following the proof of Proposition 1,

\[
P(D) \cdot e^{-\rho \cdot D} \cdot \frac{\partial u(x_{D-}, D-)}{\partial x} = \mu_{D-} = \mu_D.
\]

Similarly, the FOC for \( X_t \) yields

\[
p(D) \cdot \frac{\partial \bar{\varphi}}{\partial X} - \frac{p(D)}{P(D)} \cdot \mu_D = 0.
\]
Set up a Hamiltonian for problem (16), with costate $\bar{M}_t$:

$$D \equiv e^{-\rho_s} \cdot u(x_s, s) + \bar{M}_s \cdot [r \cdot a_s - x_s], \quad R > s \geq D.$$  

The analogue to (A7) and the envelope theorem yield

$$e^{-\rho_D} \cdot \frac{\partial u(x_{D+}, D+)}{\partial x} = \bar{M}_D = \frac{\partial \bar{\phi}}{\partial X}.$$  

Combining these equations,

$$P(D) \cdot e^{-\rho_D} \cdot \frac{\partial u(x_{D-}, D-)}{\partial x} = \mu_D = P(D) \cdot \frac{\partial \bar{\phi}}{\partial X} = P(D) \cdot e^{-\rho_D} \cdot \frac{\partial u(x_{D+}, D+)}{\partial x}$$

$$\iff \frac{\partial u(x_{D-}, D-)}{\partial x} = \frac{\partial u(x_{D+}, D+)}{\partial x}. \quad (A10)$$  

**Step 3.** Suppose $t = R < D$. Following the proof of Proposition 1, we have

$$P(R) \cdot e^{-\rho_R} \cdot \frac{\partial u(x_{R-}, R-)}{\partial x} = \mu_{R-} = \mu_R = P(R) \cdot \frac{\partial \phi}{\partial a_{R-}}.$$  

The analogue of (A7) and the envelope theorem yield

$$e^{-\rho_R} \cdot \frac{\partial u(x_{R+}, R+)}{\partial x} = M_{R+} = M_R = \frac{\partial \phi}{\partial a_{R-}}.$$  

Combining these,

$$e^{-\rho_R} \cdot \frac{\partial u(x_{R-}, R-)}{\partial x} = \frac{\partial \phi}{\partial a_{R-}} = e^{-\rho_R} \cdot \frac{\partial u(x_{R+}, R+)}{\partial x}$$

$$\iff \frac{\partial u(x_{R-}, R-)}{\partial x} = \frac{\partial u(x_{R+}, R+)}{\partial x}. \quad (A11)$$  

(A10)-(A11) establish (20).  

**Proof of Proposition 4:** Surpress the index $i$ for simplicity.
Step 1. Define

\[ z_s \equiv n_s \cdot e^{r - \rho} \cdot s \quad \text{and} \quad \Lambda \equiv \lambda^{\gamma}. \]

Then for the proper scaling factor \( z \), Proposition 3 implies that optimal expenditures are \( x_s = z \cdot z_s \) for \( s \in [S, s^*] \) and \( x_s = \Lambda \cdot z \cdot z_s \) for \( s \in \left[ s^*, T \right] \). The insured cost of the lifetime expenditure profile (in present value at age 0) is

\[
E(z, R) \equiv \int_S^R p(D) \cdot \int_D^D e^{-r \cdot t} \cdot z \cdot z_t \cdot dt \cdot dD + \int_S^R P(R) \cdot \int_0^T e^{-r \cdot t} \cdot z \cdot z_t \cdot dt + \\
\int_S^R p(D) \cdot \int_0^T e^{-r \cdot t} \cdot z \cdot z_t \cdot \Lambda \cdot dt + \int_R^T e^{-r \cdot t} \cdot z \cdot z_t \cdot \Lambda \cdot dt. \tag{A12}
\]

Define

\[
J(z, R) \equiv \\
[z]^\gamma \cdot \int_S^R p(D) \cdot \int_D^D e^{-p \cdot t} \cdot u(z_t, t) \cdot dt \cdot dD + [z]^\gamma \cdot \int_S^R p(D) \cdot \int_0^T e^{-p \cdot t} \cdot [\Lambda]^\gamma \cdot u(z_t, t) \cdot dt + \\
[z]^\gamma \cdot P(R) \cdot \int_S^R e^{-p \cdot t} \cdot u(z_t, t) \cdot dt + [z]^\gamma \cdot P(R) \cdot \int_0^T e^{-p \cdot t} \cdot [\Lambda]^\gamma \cdot u(z_t, t) \cdot dt. \tag{A13}
\]

Let \( Y = Y(R) \) be as in (17). A household solves

\[
\max_{z, R} J(z, R) \quad \text{subject to:} \quad E(z, R) = Y(R) + B(R). \tag{A14}
\]

Step 2. Hold \( R \) constant. Differentiating (A13) and \( E = Y + B \) with respect to \( Y \),

\[
\frac{\partial J}{\partial Y} = \gamma \cdot [z]^{\gamma - 1} \cdot J \cdot \frac{\partial z}{\partial Y},
\]

\[
\frac{\partial z}{\partial Y} \cdot \frac{E}{z} = 1.
\]

Combining,

\[
\frac{\partial J}{\partial Y} = \gamma \cdot [z]^{\gamma - 1} \cdot J \cdot \frac{z}{E} = \gamma \cdot \frac{J}{Y}. \tag{A15}
\]

By the envelope theorem, \( \partial J/\partial Y \) here should equal \( \mu_0 \), the costate variable from the Hamiltonian \( H \) in the proof of Proposition 3.
Step 3. Differentiate $J$ with respect to $R$:

$$\frac{\partial J}{\partial R} = \gamma \cdot \frac{[z]^{\gamma - 1}}{[z]^{\gamma}} \cdot J \cdot \frac{\partial z}{\partial R} + e^{-\rho \cdot R} \cdot P(R) \cdot [u(x_{R^-}, R^-) - u(x_{R^+}, R^+)]. \quad (A16)$$

Using (17), differentiate $Y(R) + B(R) = E(z, R)$ with respect to $R$:

$$B'(R) + P(R) \cdot e^{-r \cdot R} \cdot y_R =$$

$$\frac{\partial z}{\partial R} \cdot \frac{E}{z} + P(R) \cdot e^{-r \cdot R} \cdot x_R - P(R) \cdot e^{-r \cdot R} \cdot x_R + =$$

$$\frac{\partial z}{\partial R} \cdot \frac{Y}{z} + P(R) \cdot e^{-r \cdot R} \cdot [x_{R^-} - x_{R^+}]. \quad (A17)$$

Combining (A16)-(A17),

$$\frac{\partial J}{\partial R} = e^{-\rho \cdot R} \cdot P(R) \cdot [u(x_{R^-}, R^-) - u(x_{R^+}, R^+)] +$$

$$[B'(R) + P(R) \cdot e^{-r \cdot R} \cdot [y_R - x_{R^-} + x_{R^+}]] \cdot \frac{\gamma \cdot J}{Y}. \quad (A18)$$

At the optimal $R$, (A18) should equal 0. Step 2 shows $\gamma \cdot J / Y = \mu_0$; Proposition 3 shows

$$e^{-r \cdot R} \cdot \mu_0 = \mu_R = e^{-\rho \cdot R} \cdot P(R) \cdot \frac{\partial u(x_{R^-}, R^-)}{\partial x}.$$ 

So, (22) is a necessary condition for an optimal $R$.  

Proof of Corollary 1: Surpress $i$ for simplicity. Notice that

$$[\Lambda]^{\gamma} \cdot [\Lambda]^{\gamma} = [\Lambda]^{\frac{\gamma^2}{1-\gamma}} \cdot [\Lambda]^{\frac{\gamma-\gamma^2}{1-\gamma}} = [\Lambda]^{\frac{\gamma}{1-\gamma}} = \Lambda. \quad (A19)$$

Then provided $n_s$ is continuous at $s = R$, we have

$$\frac{\partial u(x_{R^-}, R^-)}{\partial x} \cdot \left[ B'(R) \cdot e^{r \cdot R} + P(R) \cdot [y_{R^-} - x_{R^-} + x_{R^+}] \right] -$$

$$[u(x_{R^+}, R^+) - u(x_{R^-}, R^-)] = 0 \quad (\text{from 22})$$

$$\iff [n_R]^{1-\gamma} \cdot [x_{R^-}]^{\gamma - 1} \cdot \left[ B'(R) \cdot e^{r \cdot R} + P(R) \cdot [y_{R^-} - x_{R^-} + \Lambda \cdot x_{R^-}] \right] -$$

$$[n_R]^{1-\gamma} \cdot \frac{1}{\gamma} \cdot [[\Lambda]^{\gamma} \cdot [\Lambda]^{\gamma - 1}] \cdot [x_{R^-}]^{\gamma} = 0 \quad (\text{see Prop 3})$$

$$\iff \frac{B'(R) \cdot e^{r \cdot R} + P(R) \cdot y_{R^-}}{x_{R^-}} + P(R) \cdot [\Lambda - 1] - \frac{1}{\gamma} \cdot [\Lambda - 1] = 0 \quad (\text{see A19})$$

$$\iff \frac{B'(R) \cdot e^{r \cdot R} + P(R) \cdot y_{R^-}}{x_{R^-}} - \frac{1}{\gamma} \cdot [\Lambda - 1] \cdot [1 - \gamma \cdot P(R)] = 0,$$
which establishes the Corollary.

Appendix II: CEX Data

We divide the NIPA and CEX data into 11 categories: food, apparel, personal care, shelter, household operation, transportation, medical care, recreation, education, personal business, and miscellaneous. See Laitner and Silverman [2005] for details. Assuming NIPA data is the more accurate, we scale the CEX data by year and category to the corresponding NIPA amount, applying each scaling factor across every age in the CEX.

This paper uses 3 additional adjustments, as follows.

1. We subdivide “shelter” into “services from own house” and “other.” We scale the latter as we do other categories, but we drop the CEX “services from own house” and impute a substitute that allocates the annual NIPA total service flow from residential houses to the CEX in proportion to CEX reported house values.

2. CEX medical expenditures omit employer contributions to health insurance and services that Medicare covers. We annually, proportionately, and for every age, adjust CEX expenditures on private health insurance to match the Department of Health and Human Services total for all premiums for private health insurance; and, we adjust out-of-pocket health spending from the CEX to match annual DHHS totals. Turning to Medicare, funding for the benefits comes from a hospital insurance (HI) tax on wages and salaries, monthly premiums for supplementary medical insurance (SMI) from people currently eligible for benefits, and contributions from general tax revenues to SMI. The CEX registers only SMI premiums from participants; so, we allocate the yearly total of Medicare benefits (both HI and all SMI expenditure) to the CEX sample in proportion to SMI premium payments (principally for people over 65).

3. The NIPA “personal business” category includes bank and brokerage fees, many of which are hidden in the form of low interest on saving accounts, etc., and hence absent from expenditures that CEX households perceive. We assume that bank and brokerage fees make their way into the life-cycle model in the form of lower-than-otherwise interest rates on saving; therefore, we normalize annual personal business expenditures measured in the CEX to match the corresponding NIPA amount less bank and brokerage fees.

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Appendix III: HRS Data

We derive male lifetime earnings as follows. Some male earnings figures are missing (e.g., non–FICA employment); the data are right censored at the Social Security tax cap prior to 1980; and, they are right censored at $125,000 for earnings 125,000-250,000, at $250,000 for earnings 250,000-500,000, and at $500,000 for earnings 500,000+ for 1981-1991. Thus, for men we estimate a so–called earnings dynamics model of earnings, dividing the total HRS sample into 4 education groups, and regressing log constant–dollar earnings on a quartic in age and dummy variables for time. The regression error has an individual effect as well as a random term. The likelihood function takes censoring into account. Laitner and Silverman [2005] present details. Prior to 1991, we impute censored observations from the regression. To protect against non–FICA earnings, we also impute from the regression when our data differ from it by over two–thirds of a standard error (for the regression).

After 1991, survey data is available biennially. The survey data includes work hours. We drop observations with less than 1500 hours per year and all observations past age 60. As a protection against coding errors, we exclude survey earnings greater than twice, or less than half, the earnings dynamics equation prediction for the same age. Then we fit a least squares line to a man’s log earning figures from 1986 onward, constraining the line to match 1986 earnings and have a non–positive slope. Using the line, we interpolate missing data and extrapolate prospective earnings through age 69.

Although we use similar steps for female earnings, there are several differences. A woman who never works remains in our sample. As stated above, we assume a woman retires when her spouse does. We extrapolate non–zero late–in–life earnings only for women who supply market hours in the survey in the last year that their husband works. We are much more concerned than for men that women might have part–time earnings. Prior to 1992, therefore, a woman’s earnings are her SSA earnings unless the latter are censored, in which case we impute from female earnings dynamics equations resembling the men’s (see Laitner and Silverman [2005]). The HRS provides information in 1996 on whether a woman had non–FICA earnings prior to 1992. If a woman had non–FICA jobs and provided beginning and end dates, we impute her earnings from our earnings–dynamics regressions; if she provided only the span of non–FICA employment, we subtract non–FICA employment years 1980-91, which are evident from the data, and probabilistically impute remaining years using our earnings–dynamics regressions; if a woman said she had non–FICA employment but provided no information on when or how long, we drop the couple from the sample on the basis of incomplete data.