Experimental analysis of vehicle–bridge interaction using a wireless monitoring system and a two-stage system identification technique

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\textbf{A B S T R A C T}

Deterioration of bridges under repeated traffic loading has called attention to the need for improvements in the understanding of vehicle–bridge interaction. While analytical and numerical models have been previously explored to describe the interaction that exists between a sprung mass (i.e., a moving vehicle) and an elastic beam (i.e., bridge), comparatively less research has been focused on the experimental observation of vehicle–bridge interaction. A wireless monitoring system with wireless sensors installed on both the bridge and moving vehicle is proposed to record the dynamic interaction between the bridge and vehicle. Time-synchronized vehicle–bridge response data is used within a two-stage system identification methodology. In the first stage, the free-vibration response of the bridge is used to identify the dynamic characteristics of the bridge. In the second stage, the vehicle–bridge response data is used to identify the time varying load imposed on the bridge from the vehicle. To test the proposed monitoring and system identification strategy, the 180 m long Yeondae Bridge (Icheon, Korea) was selected. A dense network of wireless sensors was installed on the bridge while wireless sensors were installed on a multi-axle truck. The truck was driven across the bridge at constant velocity with bridge and vehicle responses measured. Excellent agreement between the measured Yeondae Bridge response and that predicted by an estimated vehicle–bridge interaction model validates the proposed strategy.

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1. Introduction

Bridges represent a critical structural link within a nation’s transportation system; in the case of the United States, there are 603,307 bridges currently in operation [29]. Bridges support the economic activity of a nation by providing a convenient means of moving people and goods over bodies of water, mountain valleys, and other topological obstructions. The collapse of the I-35W Bridge in Minneapolis, MN (August 1, 2007) underscored the economic importance of bridges; the loss of the I-35W Bridge is estimated to have resulted in a total economic loss of $200 million to the Minneapolis region [20]. In many developed nations, there is growing concern over the general health and well-being of growing inventories of aging bridges. For example, the United States is facing a serious baby-boomer bridge problem; the average age of the nation’s bridges is 43 years old with the vast majority of these bridges designed for 50-year service lives [1]. Over time, daily traffic and extreme environmental factors naturally lead to structural deterioration. To better understand
the behavior of bridges under traffic loads and to comprehend causal relationships in bridge deterioration processes, there has been renewed interest in the study of vehicle–bridge interaction as it pertains to structural health.

The dynamic behavior of bridge systems under the influence of a moving load (i.e., vehicle) is one of the oldest problems in the civil engineering profession. The first reported study of the influence of a moving vehicle on a bridge dates back to 1849 when R. Willis reported on the behavior of a massless beam loaded by an unprung moving mass [28]. In the Willis study, and the many that followed [5,6], the inertia effect of the vehicle was ignored. To begin to account for the vehicle dynamics in the response of the bridge, a variety of solutions were proposed in the 1920s including models that analyze the vehicle as a moving harmonic force [27] and those that accounted for the vertical inertia effect of the vehicle mass [10]. Hillerborg [8] was one of the earliest to offer a solution to the problem of a sprung mass moving across an elastic beam. Other notable contributions to the moving oscillator problem have been reported by Biggs et al. [2], Fryba [4] and more recently by Pesterev and Bergman [21]. With the emergence of computers in the 1960s, finite element method (FEM) models were used to study the dynamic interaction between vehicles and bridges [33]. The use of FEM models in the study of vehicle–bridge interaction was significant because FEM allows for countless variations in: (1) the type of vehicle (e.g., multi-axle trucks, railroad trains), (2) vehicle suspension system, and (3) bridge type (e.g., multi-girder bridge, truss bridge, cable-stayed bridge). The FEM approach entails the creation of two separate models that analyze the vehicle and bridge dynamics. Given the profile of road roughness, the vehicle and bridge displacements are solved in an iterative manner to satisfy both dynamic models at each time-step.

While great advances have been made in the analytical and numerical modeling of vehicle–bridge interaction, comparatively less research has been aimed towards experimental observation. The measurement of vehicle and bridge dynamics in real, operational bridges is critical towards refining existing model frameworks and to understanding how repeated vehicular loading leads to long-term structural deterioration of the bridge. Researchers have previously instrumented bridges to measure bridge responses to controlled vehicular loading [3,15,7,9,22]; however, few researchers have simultaneously measured the position and vibratory behavior of the vehicle. One challenge associated with measuring the vehicle behavior is that the vehicle is moving. As such, the use of a wired monitoring system to record the response of the bridge and vehicle is infeasible. Alternatively, independent monitoring systems can be installed in the bridge and vehicle [19], but accurate time synchronization of the two monitoring systems can prove challenging.

The traditional wired monitoring system architecture can be abandoned in favor of a wireless system capable of simultaneously monitoring a bridge and the moving vehicles that is loading the bridge. Preliminary work has validated the feasibility of using wireless sensors to measure the position and vibratory behavior of a vehicle while driving on a bridge instrumented with a permanent wireless monitoring system [12]. This paper presents a unified wireless monitoring system capable of monitoring the dynamic response of bridge structures loaded by moving vehicles instrumented with wireless sensors. The wireless monitoring system architecture deploys accelerometers and tactile sensors on the bridge deck to measure deck accelerations and the time when the vehicle crosses fixed points, respectively. Simultaneously, the wireless sensors installed in the vehicle are used to measure the vehicle's horizontal acceleration, vertical vibratory acceleration, and gyroscopic pitching motion. To showcase the merits of the proposed vehicle–bridge wireless monitoring system architecture, the 180 m long Yeondae Bridge (Icheon, Korea) is instrumented with a permanent set of wireless sensors. The bridge is loaded during forced-vibration testing using a heavy truck instrumented with a set of wireless sensors that can freely communicate with the wireless monitoring system installed on the bridge.

The proposed wireless monitoring system provides a rich input–output data set from which the dynamic coupling between a vehicle and bridge can be analyzed. The second half of the study focuses on the extraction of the forcing function of the bridge due to the moving, sprung mass (i.e., the vehicle) loading the bridge. The vibratory response of the bridge subjected to a moving random force is effectively a non-stationary random process [5]. In the context of system identification, vehicle–bridge interaction can be considered as a complex time-variant problem that prohibits direct application of popular batch system identification algorithms [31,30] that require the system to be linear, time-invariant. Alternatively, classical parameterized model-based system identification algorithms such as the prediction-error method [17] can be utilized. However, such methods are challenging to apply to time-varying systems with poor convergence expected during recursive optimization in the time-domain. In this study, a novel two-stage approach to system identification is proposed. In the first stage, the free-vibration response of the bridge is used to characterize the dynamic properties of a linear time invariant state-space model corresponding to the unloaded bridge. In the second stage, vehicle position and vibratory response time-histories are used to identify a linear, time-variant model that encapsulates the vehicle–bridge interaction. A key innovation of the second stage of the system identification process is the use of a kernel approximation, which allows the continuous moving load trajectory to be spatially mapped to a finite number of nodes in the bridge model. The proposed two-stage system identification process is applied to the Yeondae Bridge data set so that the accuracy of the proposed system identification process can be quantified.

2. Experimental method

2.1. Wireless sensors for bridge and vehicle monitoring

At the core of the proposed wireless system for monitoring vehicle–bridge interaction is a low-cost wireless sensor node. The use of wireless sensors for structural monitoring has received considerable attention in the research community because
they are inexpensive, have computational resources for sensor-based data processing, and are relatively easy to install in large structural systems [24,23,18]. Although many academic and commercial wireless sensor nodes have been explored for structural monitoring, this study utilizes the Narada wireless sensor node. Narada (Fig. 1) is a wireless sensor node design and fabricated at the University Michigan for structural health monitoring applications [25]. The node is designed to achieve a level of performance suitable for structural monitoring including high resolution digitalization (e.g., 16-bits or higher) and long range communication (e.g., 500 m or longer). To achieve these unique requirements, commercial electrical components are selected and assembled into a single, low-cost package. The sensor node offers a 4-channel sensing interface that can digitize any analog voltage input between 0 to 5 V using a 16-bit analog-to-digital converter (Texas Instruments ADS8341). To coordinate the collection of data from the on-board analog-to-digital converter, a low-power 8-bit microcontroller (Atmel ATmega128) clocked at 8 MHz is included in the node design. While the microcontroller has a large read-only memory bank (128 kB flash memory), random access memory is limited (4 kB). Hence, an additional 128 kB of random access memory is added to the Narada design for the local storage of raw sensor data. The circuit associated with the analog-to-digital converter and microcontroller is fabricated on a single 4-layer printed circuit board as shown in Fig. 1a. For radio communication, an IEEE 802.15.4 wireless transceiver (Texas Instruments CC2420) is designed on a separate printed circuit board. The short nominal communication range of the radio is overcome by including a 10 dB power amplifier on the radio, thereby giving the radio an impressive 700 m communication range (line-of-sight). A standard omni-directional whip antenna (Fig. 1b) or a uni-directional planar antenna can be used with the radio. The accuracy and reliability of the Narada wireless sensor node within realistic bridge settings has been previously established by Kim et al. [13].

2.2. Monitoring strategy of the testbed bridge

The Yeondae Bridge (Fig. 2) is located near Icheon, Korea along an experimental section of the Jungbu Inland Highway. The Korea Expressway Corporation (KEX) constructed this redundant section of the southbound Jungbu Inland Highway in 2002 to monitor the influence of Korean truck loads on the performance of concrete and asphalt pavement systems [14]. The Yeondae Bridge is a continuous steel box girder bridge that is 180 m long with a slight curve in plan at one end (1718 m radius of curvature). The cross-section of the bridge consists of two symmetric trapezoidal box girders that are 2.2 m tall and with widths of 3.1 and 2.1 m at the top and bottom of the trapezoid section, respectively. The box girders are placed in composite action with the 27 cm thick reinforced concrete bridge deck. Along the length of the bridge are three reinforced concrete piers that support the continuous steel box girder using elastomeric pads. The two ends of the bridge are supported on reinforced concrete abutment structures with rubber expansion joints installed between the bridge deck and the abutment structures. The bridge has a 40° skew angle at the abutment supports. KEX was able to close the southbound experimental section of the Jungbu Inland Highway; this provided the research team with uninterrupted access to the bridge for controlled forced vibration testing.

The Yeondae Bridge was instrumented with a dense network of Narada wireless sensor nodes to which accelerometers and piezoelectric tactile sensors have been interfaced (Fig. 3). Twenty Narada nodes, each with an integrated single-axis capacitive accelerometer (Silicon Designs SD2012), were installed along the length of the bridge. The SD2012 is a high-quality accelerometer suitable for bridge monitoring with a high sensitivity (2 V/g), large measurement range (± 1 g), and low noise floor (13 μg/√Hz). Accelerometers were epoxy bonded to the top surface of the bridge deck along the center line of the road approximately 7.5 m apart. The accelerometers were oriented in the vertical direction to record the vertical acceleration response of the bridge. To record the time the truck reached the abutments and piers, five piezoelectric tactile sensors were
bonded to the top surface of the deck at the abutments and piers. The tactile sensors were cut from a thin poly(vinylidene fluoride) (PVDF) sheet to form thin strips 1.5 m long and 0.5 cm wide. The PVDF tactile sensors were designed to generate a voltage when a truck axle applies vertical pressure. The output of each PVDF tactile sensor was interfaced to a Narada wireless sensor node. To enhance the performance of the radio, each Narada node was installed on the top of a 56 cm tall traffic cone with a short shielded wire used to connect the node with each sensor (accelerometer or PVDF tactile sensor). To coordinate the activity of the bridge wireless monitoring system, a wireless server was established in the center of the bridge. The wireless server consisted of a laptop computer with a Texas Instruments CC2420 transceiver attached.

2.3. Monitoring strategy of the truck

A heavy-duty dump truck (Hyundai Trago truck series) was selected for forced-vibration testing of the Yeondae Bridge. The heavy-duty dump truck (Fig. 4) is designed with four axles: the two front axles having two wheels each and the two back axles having four wheels each. The main body of the truck consists of a front passenger cabin and a back payload bin each fixed to a rigid steel frame. The mass of the vehicle body is supported by the 4 axles with a complex suspension system used at each axle-frame connection point. Prior to its arrival on-site, the truck was weighed at a local weigh station. The truck had a total weight of 20.9 metric tons with each axle supporting 4.3, 8.0, 4.6, and 4.0 metric tons from front to back.

Given the structural configuration of the vehicle, the vehicle dynamics can be modeled by a simplified analytical model with the truck body modeled as a sprung mass supported by four spring-dashpot suspension systems at each of the axles. The dynamics of the truck sprung mass is modeled by the vertical and rotational degrees-of-freedom at the center-of-gravity of the truck body. Each axle is modeled as an unsprung, concentrated mass connected to the truck frame through a spring-dashpot suspension; only a vertical degree-of-freedom is modeled at each axle. In total, the pitch-plane model of the truck consists of six degrees-of-freedom. A sensor was installed in the truck to accurately measure each degree-of-freedom. For example, a single axis ±10 g accelerometer (Analog Devices ADXL105) was installed vertically at each axle and a ±2 g accelerometer (Crossbow CXL02) was installed vertically at the center-of-gravity of the truck body. To capture the rotation of the truck body, a microelectromechanical system (MEMS) gyroscope (Analog Devices ADXRS624) was also installed at the center-of-gravity of the truck body. In addition, a second ±2 g accelerometer (Crossbow CXL02) was installed in the horizontal direction at the truck center-of-gravity to measure the inertial acceleration of the truck. Each sensor installed on the truck was interfaced to a Narada wireless sensor; this allowed the truck-based sensors to freely communicate with the permanent wireless monitoring system installed on the Yeondae Bridge. Specifically, the Narada wireless sensors were accessible to the bridge’s wireless monitoring system through the wireless server coordinating the monitoring system activities.
2.4. Dynamic load testing

Dynamic load testing of the Yeondae Bridge was conducted using the heavy-duty dump truck instrumented with Narada wireless sensors. The stretch of the Jungbu Inland Highway that includes the Yeondae Bridge was closed to regular highway traffic so that only the instrumented truck could load the bridge. The truck was parked approximately 500 m from the northern abutment of the bridge. The wireless server installed at the center of the bridge was used to coordinate the data acquisition activity of the Narada wireless sensor nodes installed on both the bridge and truck. First, the wireless server communicated a beacon packet that was used for time synchronization of the wireless sensor network. Upon receipt of the beacon packet, each wireless sensor node would reset their internal clocks and communicate a confirmation to the wireless server. This form of beacon-based time synchronization has been previously validated with the Narada node to achieve an upper bound error of $30 \mu s$ [26]. Once the wireless sensor confirmed that all of the Narada nodes had reset their clocks, a second command packet was communicated by the wireless server to initiate data collection by all wireless sensor nodes using a sample rate of 100 Hz. After the monitoring system started its data collection activity, the truck was accelerated from rest to achieve a target velocity before entering the bridge. After exiting the bridge, the truck was commanded to slow down and to finally come to rest. Once the last PVDF tactile sensor (PVDF5 in Fig. 3) detected the truck had exited the bridge, the Narada node recording this sensor communicated a packet to the server denoting the truck has exited the bridge. Upon receipt of this packet, the wireless server issued a termination command to all of the Narada wireless sensor nodes and collected the measured response time history from every Narada node in the wireless monitoring system (including those on the heavy-duty dump truck). At the end of the test, the wireless server was in complete possession of the forced-vibration test data including the twenty channels of bridge vertical acceleration, five channels of deck tactile sensing, five channels of truck vertical acceleration, one channel of truck horizontal acceleration, and the one channel of truck rotation.

3. Two-stage system identification of vehicle–bridge interaction

3.1. Mathematical formulation of vehicle–bridge interaction

A simplified physical model of the vehicle–bridge interaction problem is presented in Fig. 5. A discrete-time linear time-variant state space model for this single input, multiple output (SIMO) system can be written as

$$
\begin{align*}
x(k+1) &= A(k)x(k) + B(k)u(k) + w(k) \\
y(k) &= C(k)x(k) + v(k)
\end{align*}
$$

where, $u(k) \in \mathbb{R}$ is the measured bouncing acceleration of the truck body at discrete time step $k$ and is considered a deterministic input to the bridge system. Furthermore, $y(k) \in \mathbb{R}^{x \times 1}$ is the measured bridge vertical acceleration at time step $k$, $x(k) \in \mathbb{R}^{n \times 1}$ is an unknown $n$-dimension state vector of the system, $w(k) \in \mathbb{R}^{n \times 1}$ is the process noise associated with the
system dynamics, and $v(k) \in \mathbb{R}^{1 \times 1}$ is the measurement noise associated with the system observation (i.e., measurement). The noises $v$ and $w$ are assumed to be uncorrelated, zero-mean stationary white noise sequences. Due to the moving nature of the bridge loading, the state space model is written with time varying system, loading, and observation matrices (i.e., $A(k) \in \mathbb{R}^{n \times n}$, $B(k) \in \mathbb{R}^{n \times 1}$, $C(k) \in \mathbb{R}^{1 \times n}$, respectively). However, if it is assumed the bridge response amplitude is small (i.e., moderate vibration) and the mass of the vehicle is trivial compared to the mass of the bridge, then $A$ and $C$ can be treated as time-invariant. However, the position-changing nature of the truck load mandates that the load distribution vector, $B$, remain time-variant:

$$
\mathbf{x}(k+1) = A\mathbf{x}(k) + B(k)u(k) + w(k)
$$

$$
\mathbf{y}(k) = C\mathbf{x}(k) + v(k)
$$

Now, the system identification problem for vehicle–bridge interaction can be stated as the estimation of the state space model system matrices ($A$, $B(k)$, $C$) given the measured system input and output ($u(k)$ and $y(k)$, respectively) over the time trajectory $k=1$ (the discrete time-step when the vehicle enters the bridge) to $N$ (the discrete time-step when the vehicle exits the bridge).

To solve this complex system identification problem, a two-stage system identification approach is proposed as depicted in Fig. 6. First, the time-invariant system, $A$, and observation, $C$, matrices are estimated using the free-vibration response of the bridge (i.e., $y(k)$ for $k > N$). Once the system and observation matrices are identified, the loading matrix, $B(k)$, is estimated at each discrete time-step from $k=1$ to $N$. This second-stage in the system identification process will utilize the system input, $u(k)$ and measured forced-vibration response, $y(k)$ measured from the coupled vehicle–bridge system. Due to the nature of the acceleration measurement, the measured vehicle acceleration, $u(k)$, may be corrupted by the

![Fig. 6. Two-stage system identification strategy that delineates the measured bridge response into free- and forced-vibration components in order to identify time-invariant and time-variant system components, respectively.](image-url)
bridge vertical accelerations, \( y(k) \). Based on the level of acceleration measured on the truck body and that on the bridge, the level of contamination in the measured truck vertical acceleration from the bridge acceleration was considered trivial and was therefore ignored in the analysis.

### 3.2. Stage 1: system identification with free-vibration data

After the vehicle departs the bridge \((k=N+1, N+2, \ldots)\), the bridge undergoes free-vibration without a deterministic input. Hence, the behavior of the bridge can be modeled as a stochastic discrete-time state space model as:

\[
x(k+1) = Ax(k) + w(k)
\]

\[
y(k) = Cx(k) + v(k)
\]

The system identification solution for Eqs. (5) and (6) has been previously derived by Van Overschee and De Moore \cite{30} and named Stochastic Subspace Identification (SSI). Due to complexity of the derivation of SSI, only a brief introduction to the SSI algorithm is presented. The output block Hankel matrix is constructed from the measured bridge free-vibration response and partitioned as past output and future output as follows:

\[
Y_{q:i-1} = \begin{bmatrix}
y_0 & y_1 & \ldots & y_{j-1} \\
\vdots & \ddots & \ddots & \vdots \\
y_{-i} & y_i & \ldots & y_{i-1}\end{bmatrix}
\]

where \( y_j \) is notation for \( y(\cdot) \). Two orthogonal projections of the row space of the future output, \( Y_p \), on the row space of the past output, \( Y_f \), can be determined through LQ decomposition of the output block Hankel matrix:

\[
P_f := Y_f/Y_p; \quad P_{i-1} := Y_f/Y_p
\]

where \( Y_f \) and \( Y_p \) are defined as a one block row down-shift in Eq. (7) as \( Y_{0:i} \) and \( Y_{i+1:2i-1} \), respectively. Since the projection is equal to the product of the extended observability matrix and the non-stationary Kalman filter state sequence \cite{30}, singular value decomposition can be applied to factorize the projection \( P_f \):

\[
P_f = USV^T \cong \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1S_1V_1^T
\]

Now, the extended observability matrix

\[
\mathcal{O}_i = U_1S_i^{1/2}
\]

and the non-stationary Kalman filter state sequence can be calculated respectively as

\[
\hat{x}_i = S_i^{1/2}V_1^T
\]

The one-step shifted state sequence is also calculated as

\[
\hat{x}_{i+1} = (\mathcal{O}_{i-1})^TP_{i-1}
\]

where \( \mathcal{O}_{i-1} \) is equivalent to \( \mathcal{O}_i \) with the last block row omitted. Furthermore, \( *' \) is the pseudo-inverse operator. Finally, estimates of \( A \) and \( C \) can be calculated by a least-squared solution:

\[
\begin{bmatrix} \hat{A} \\ \hat{C} \end{bmatrix} = \begin{bmatrix} \hat{x}_{i+1} \\ \hat{y}_{ij} \end{bmatrix} \hat{x}_i
\]

It should be noted that estimated system \( \{\hat{A}, \hat{C}\} \) is a specific realization of the system with an arbitrary state basis. In other words, the estimated system \( \{\hat{A}, \hat{C}\} \) in Eq. (13) is a black-box mathematical model that best fits the response data used during its extraction.

### 3.3. Stage 2: system identification with forced-vibration data

In the second stage of system identification analysis, the bridge dynamics under the influence of the moving vehicle can be formulated using the system matrices estimated, \( \{\hat{A}, \hat{C}\} \), in Stage 1:

\[
x(k+1) = \hat{A}x(k) + Bu(k) + w(k)
\]

\[
y(k) = \hat{C}x(k) + v(k)
\]
The objective of the second stage of the system identification problem is to estimate the time varying load matrix, \( \mathbf{B}(k) \). This objective is accomplished by formulating the problem as an unconstrained optimization problem over an extended horizon where the difference in measured and predicted model output, \( \mathbf{y}(k) \) and \( \hat{\mathbf{y}}(k) \), respectively, is minimized as follows:

\[
\left\{ \hat{\mathbf{B}}(0), \hat{\mathbf{B}}(1), \ldots, \hat{\mathbf{B}}(N-1) \right\} = \arg\min_{\{\mathbf{B}(0), \mathbf{B}(1), \ldots, \mathbf{B}(N-1)\}} \frac{1}{N} \sum_{k=1}^{N} \| y(k) - \hat{y}(k|k-1) \|^2 
\]

where \( \hat{y}(k|k-1) \) is the one-step ahead prediction of the bridge acceleration calculated at discrete time-step \( k - 1 \). This output prediction is formulated by the underlying system physics encapsulated in Eqs. (14) and (15)

\[
\hat{y}(k|k-1) = \mathbf{\hat{C}} \mathbf{A}^k \mathbf{x}(0) + \sum_{q=1}^{k} \mathbf{\hat{C}} \mathbf{A}^{k-q} \mathbf{B}(k-1) \mathbf{u}(q) \tag{17}
\]

where \( \mathbf{x}(0) \) is the initial state of the system. Since the bridge is initially at rest before the truck arrives, a zero initial state is assumed herein. Hence, Eq. (17) can be further simplified as:

\[
\hat{y}(k|k-1) = \sum_{q=1}^{k} \mathbf{u}(q) \mathbf{\hat{C}} \mathbf{A}^{k-q} \mathbf{B}(k-1) \tag{18}
\]

The predicted system output formulated in Eq. (18) can be combined with the unconstrained optimization problem of Eq. (16):

\[
\left\{ \hat{\mathbf{B}}(0), \hat{\mathbf{B}}(1), \ldots, \hat{\mathbf{B}}(N-1) \right\} = \arg\min_{\{\mathbf{B}(0), \mathbf{B}(1), \ldots, \mathbf{B}(N-1)\}} \frac{1}{N} \sum_{k=1}^{N} \| y(k) - \sum_{q=1}^{k} \mathbf{u}(q) \mathbf{\hat{C}} \mathbf{A}^{k-q} \mathbf{B}(k-1) \|^2 
\]

In order to solve this unconstrained optimization problem by the least-squared method, a kernel approximation based on the identified vehicle position will be taken.

### 3.4. Vehicle position-load effect kernel

Prior work in forced vibration testing of bridge structures focused on estimating an accurate position vector of the truck using a Kalman estimator [11]. In this framework, the accelerometer installed in the truck to measure horizontal acceleration was fused with the output of the PVDF tactile sensors to estimate the truck trajectory. This specific methodology was also used in this study to extract a precise truck position time history function for inclusion in the two-stage system identification analysis. Knowledge on the truck position during forced vibration testing can be leveraged to reduce the complexity of the unconstrained optimization problem posed in Eq. (19). Specifically, the position of the vehicle will be used to estimate a vehicle position-load effect kernel function, \( \Phi(k) \in \mathbb{R}^{n \times 1} \), that will allow the load effect of the vehicle in the system (Eqs. (14) and (15)) to be estimated through a scalar time-varying function, \( \alpha(k) \):

\[
\mathbf{B}(k) := \alpha(k) \Phi(k) \tag{20}
\]

First, consider the conversion between the continuous and discrete time representation of the vehicle–bridge interaction model. A power series approximation for the conversion of the discrete-time system matrix, \( \mathbf{A} \), and load matrix, \( \mathbf{B} \), can be specified as

\[
\mathbf{A} = e^{\mathbf{A} \Delta t} = \mathbf{I} + \mathbf{A} \Delta t + \frac{1}{2!} \mathbf{A}^2 \Delta t^2 + \frac{1}{3!} \mathbf{A}^3 \Delta t^3 + \cdots \tag{21}
\]

\[
\mathbf{B}(k) = \left( \int_{0}^{\Delta t} e^{\mathbf{A} \tau} d\tau \right) \mathbf{B}_c(t) = \left( \mathbf{I} \Delta t + \frac{1}{2!} \mathbf{A} \Delta t^2 + \frac{1}{3!} \mathbf{A}^2 \Delta t^3 + \cdots \right) \mathbf{B}_c(t) \tag{22}
\]

where \( \Delta t \) is the time-step in discrete time, \( \mathbf{A}_c \) is the continuous time system matrix, and \( \mathbf{B}_c(t) \) is the continuous-time load matrix. Using Eq. (21) and the estimated \( \hat{\mathbf{A}} \) in Stage 1, Eq. (22) can be written as

\[
\mathbf{B}(k) = \hat{\mathbf{A}}^{-1} (\mathbf{A} - \mathbf{I}) \mathbf{B}_c(t) = \left( \frac{1}{\Delta t} \ln \hat{\mathbf{A}} \right)^{-1} (\mathbf{A} - \mathbf{I}) \mathbf{B}_c(t) = \mathbf{L} \mathbf{B}_c(t) \tag{23}
\]

where \( \mathbf{L} \in \mathbb{R}^{n \times n} \). Hence, the vehicle position-load effect kernel function in the discrete time-domain, \( \Phi(k) \), is related to the vehicle position-load effect kernel function in the continuous time-domain, \( \Phi_c(t) \), through the same linear operator, \( \mathbf{L} \):

\[
\Phi(k) = \mathbf{L} \Phi_c(t) \tag{24}
\]

Using the known truck trajectory, the vehicle position-load effect kernel function in the continuous time domain, \( \Phi_c(t) \), takes on the value of 1 at the output node closest to the truck position at that time, \( t \), and zero everywhere else (i.e., binary kernel).
The representation of \( B(k) \) in Eq. (20) can be included in Eq. (18) to yield:
\[
\hat{y}(k|k-1) = \sum_{q=1}^{k} u(q) \hat{C}A^{-q} \Phi(k-1) z(k-1)
\]  
(25)

The output predictor of Eq. (25) can be written at each time-step increment in the following manner:
\[
\begin{bmatrix}
\hat{y}(1|0) \\
\hat{y}(2|1) \\
\vdots \\
\hat{y}(k|k-1) \\
\vdots \\
\hat{y}(N|N-1)
\end{bmatrix} =
\begin{bmatrix}
u(1) \hat{C}A^{-0} \Phi(0) & 0 & \cdots & 0 \\
u(1) \hat{C}A^{-1} \Phi(1) & u(2) \hat{C}A^{-0} \Phi(1) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
u(1) \hat{C}A^{-N-1} \Phi(0) & u(2) \hat{C}A^{-N-2} \Phi(1) & \cdots & u(N) \hat{C}A^{-0} \Phi(N-1)
\end{bmatrix}
\begin{bmatrix}
z(0) \\
z(1) \\
\vdots \\
z(k-1) \\
\vdots \\
z(N-1)
\end{bmatrix}
\]  
(26)

Eq. (26) can be symbolized as
\[
\hat{Y} = \Psi z
\]  
(27)

where \( \hat{Y} \in \mathbb{R}^{N \times 1} \), \( \Psi \in \mathbb{R}^{N \times N} \), and \( z \in \mathbb{R}^{N \times 1} \). The optimization problem of Eq. (19) can be written with argument \( z \) and simplified as
\[
\{ \hat{z}(0), \hat{z}(1), \ldots, \hat{z}(N-1) \} = \arg\min_{\{z(0), z(1), \ldots, z(N-1)\}} \frac{1}{N} \sum_{k=1}^{N} \left\| y(k) - \sum_{q=1}^{k} u(q) \hat{C}A^{-q} \Phi(q) z(k-1) \right\|^2
\]
\[
= \arg\min_{\{z(0), z(1), \ldots, z(N-1)\}} \frac{1}{N} \sum_{k=1}^{N} \sum_{q=1}^{k} u(q) \hat{C}A^{-q} \Phi(q) z(k-1)
\]
(28)

where \( Y = [y(1)^T, y(2)^T, \ldots, y(N)^T]^T \) is the stack of the measured output vectors. Then, the least square solution of Eq. (28) is calculated as
\[
\hat{z} = \Psi^+ Y
\]  
(29)

Once the discrete-time variation of \( z(k) \) is estimated, \( B(k) \) is easily calculated:
\[
\{ \hat{B}(0), \hat{B}(1), \ldots, \hat{B}(N-1) \} = \{ \hat{z}(0) \Phi(0), \hat{z}(1) \Phi(1), \ldots, \hat{z}(N-1) \Phi(N-1) \}.
\]  
(30)

4. Results and discussion

4.1. Time- and frequency-domain analysis

In total, eleven separate tests were conducted on the bridge during forced-vibration testing: the truck was driven over the bridge at four different speeds (i.e., 30, 50, 65, and 70 km/h) three times each except for 70 km/h which was conducted only twice. During all tests, the wireless monitoring system architecture proved to be an extremely reliable system that yielded a rich set of time-synchronized truck-bridge time-histories. Fig. 7 depicts a representative set of acceleration only twice. During all tests, the wireless monitoring system architecture proved to be an extremely reliable system that yielded a rich set of time-synchronized truck-bridge time-histories. Fig. 7 depicts a representative set of acceleration.

In Fig. 8, the measured dynamic responses of the truck are plotted. The truck time-history responses in Fig. 8 correspond to the same test plotted in Fig. 7. The truck axles underwent high frequency vibration when traveled past the bridge expansion joint at both ends of the bridge (i.e., at PVDF1 and PVDF5). The high frequency response observed in the dynamic interaction between the truck wheels and the bridge due to the road roughness in the local vicinity of the sensor (e.g., localized dynamic response of the concrete slab). In contrast, the high amplitude, low frequency response that is persistent in the bridge accelerations is associated with the global vibration response of the continuous box-girder.
Fig. 7. Bridge vertical acceleration measured at the center of each span of the Yeondae Bridge. Bridge excited with a heavy-duty truck travel crossing the bridge at 65 km/h. Vertical lines denote the time each truck axle crossed the PVDF tactile sensor.

Fig. 8. Measured truck response as the truck crossed the Yeondae Bridge at 65 km/h; response is time synchronized with the bridge response data of Fig. 5. Vertical lines denote the time each truck axle crossed the PVDF tactile sensor.
axles was effectively filtered out by the truck suspension system when analyzing the truck vertical acceleration; however, the high frequency axle responses translated directly to the pitching motion of the truck body as seen in Fig. 8.

To confirm the observations made in the time-domain, the forced-vibration response of the bridge and truck (i.e., the response shown from 34 to 44 s in Figs. 7 and 8, respectively) were converted to the frequency-domain in the form of power spectral density (PSD) functions as shown in Fig. 9. The PSD functions of the mid-span accelerations plotted in Fig. 9a revealed distinct peak frequencies at 2.25, 2.64, 3.08, 3.42, 4.05, and 4.44 Hz. These peak frequencies were in strong agreement with modes (at 2.25, 2.64, 3.35, and 4.00 Hz) extracted during past modal analysis of the Yeondae Bridge as shown in Fig. 10 [13]. The dynamic response of the truck was more complex. The dynamic behavior of the truck axles were defined by high frequency content. For example, the first axle exhibited a dynamic response in the 10 to 16 Hz frequency band while the third axle response had response energy in the 8 to 12 Hz frequency band. While some of the high frequency axle response was observed in the truck body vertical acceleration PSD, axle dynamics were greatly attenuated by the truck suspension system with the vertical bouncing of the truck body defined by lower frequency content centered at 4 Hz. The lower amplitude peaks found at frequencies less than 4 Hz in the truck vertical acceleration PSD were attributed to nonlinearities present in the suspension system [6]. The pitching action of the truck body was defined by narrow band behavior centered at 12.5 Hz.

To illuminate the time dependency of the frequency-domain behavior of the bridge, time-frequency plots (spectrograms) of the bridge vertical accelerations were calculated as presented in Fig. 11. The spectrograms are plotted for each of the twenty accelerometers on the bridge deck with each bridge span grouped separately. Because the first four modes of response of the bridge fall between 2.25 and 4.0 Hz, each spectrogram spectral range is limited to 1 to 5 Hz to enhance the visual readability of the spectrogram.

![Fig. 9. Power spectral density (PSD) function of the measurement data: (a) PSD of span accelerations during the forced-vibration response of the bridge in Fig. 7; (b) PSD of the truck response during the forced-vibration response period in Fig. 8.](image1)

![Fig. 10. First four mode shapes of the Yeondae Bridge identified by a previous modal analysis study [13]: (a) 2.25 Hz; (b) 2.64 Hz; (c) 3.35 Hz; (d) 4.00 Hz.](image2)
Fig. 11. Spectrograms (from 1 to 5 Hz) of the measured bridge accelerations of the Yeondae Bridge for the truck driven at 65 km/h: (a) span 1; (b) span 2; (c) span 3; (d) span 4.
Depending on the moving truck location, the dominant modes of the bridge in the frequency-domain changed. Even though the fourth mode of the bridge at 4.00 Hz would normally have a lower participation factor in the overall system behavior, this mode contained significant energy as was evident by the high amplitudes in the spectrograms; the fourth mode amplitude was often as large as the first mode amplitude at 2.25 Hz. Since the natural frequency of the truck sprung mass coincides with the fourth mode, it can be concluded that the bridge is undergoing forced-vibration by the bouncing motion of the moving truck body.

A number of preliminary conclusions can be drawn from the analysis of the time- and frequency-domain data presented:

- While the axes of the truck at the bridge–truck interface are characterized by high frequency content, this frequency response is poorly matched to the low frequency global behavior of the bridge. As a result, the high frequency content does not effectively induce the global response of the bridge. Rather, bridge components (e.g., bridge deck) defined by lower effective mass are more prone to localized excitation by the higher frequency content of the truck axes.
- The large mass of the truck body (i.e., sprung mass) coupled with its low frequency vibratory behavior represents a more influential source of global excitation for a bridge defined by primary modes within the same frequency regime. This conclusion is consistent with the literature where evidence of truck bouncing-induced excitation has been widely reported [32,16].
- The speed of the truck has a strong influence on the vehicle–bridge interaction [16]. At low speeds (20 km/h or less), the truck acts like an unsprung mass exciting the bridge with the bridge first mode dominant in the vertical response of the truck body. However, as the speed of the vehicle increases, the sprung mass of the vehicle drives more of the system dynamics with the sprung mass natural frequency dominant in both the truck vertical acceleration PSD function (as was the case in Fig. 9b) and in the bridge response PSD functions (as was the case in Fig. 11).

4.2. Effect of the truck on the extraction of bridge modal properties

Based on the bridge drawings, the weight of the Yeondae Bridge superstructure was calculated to be 2800 metric tons. The additional weight added to the bridge by the 20.9 t truck was only 0.75% of the bridge superstructure weight. Even though this additional mass is expected to be trivial, the effects of the additional mass on the bridge modal properties should be checked prior to system identification. To assess this potential influence, two separate modal analyses were conducted using forced and free-vibration response data of the bridge. Modal analysis of forced-vibration data would contain the influence of the vehicle (i.e., additional mass and truck dynamics). In contrast, free-vibration response data would not be affected by the vehicle. To make a fair comparison, the same system identification technique was applied to the bridge forced- and free-vibration response data. In this study, the output-only stochastic subspace identification (SSI) system identification was applied to all 11 data sets during forced and free-vibration, resulting in 22 sets of estimated modal parameters for the Yeondae Bridge (as tabulated in Table 1).

The modal frequencies extracted from the free-vibration response data were consistent with the modal frequencies previously identified for the Yeondae Bridge [13]. This result was expected because the frequency content contained in the free-vibration response is independent of the truck. In contrast, there was notable variation in the modal frequencies identified from the forced-vibration data. While strong agreement existed for the first two modes of the bridge, the third and fourth modes extracted from the forced-vibration data differed significantly from those extracted from the bridge free-vibration response. In addition, notable variability existed in the identified forced-vibration modes for the differing truck speeds, especially in the third and fourth modes. These observation lead to the conclusion that:

- Discrepancies observed in the modal frequencies of modes 3 and 4 extracted from forced-vibrations imply different truck bouncing behavior affecting the results of the output-only SSI method. Thus, output-only system identification with forced-vibration data is not appropriate.
- The similarities in the modal frequencies of mode 1 and mode 2 for both the free and forced-vibration imply that the additional truck mass is trivial and that the truck dynamics do not strongly influence these modes. Thus, the mass of the truck can be reasonably ignored during the system identification analyses.

4.3. System identification of vehicle–bridge interaction

Using the aforementioned two-stage system identification methodology, identification of the vehicle–bridge interaction on the Yeondae Bridge was conducted using the experimentally measured data sets collected for each of the 11 tests. Before execution of the two-stage system identification process, data polishing of the measured truck bouncing and bridge response was conducted as advocated by Ljung [17]. Since the global bridge vibration induced by the moving sprung mass motion was intended to be identified, low-pass filtering (10 Hz cutoff frequency) was applied to input and output data. Then, considering the frequency range of the filtered data, the data was down-sampled to 25 Hz for system identification.
The filtered vertical bouncing acceleration of the truck body during the 65 km/h tests is presented in Fig. 12. In addition, the position of the truck using the Kalman estimation framework proposed in [11] was used. Using the truck bouncing motion, $u(k)$, and the vehicle position-load effect kernel function, $\Phi(k)$, derived from the truck position, the two-stage system identification procedure was carried out. The system output predicted for the forced- and free-vibration response of the Yeondae Bridge is plotted in Fig. 13. The predicted response (thick line) is superimposed over the low-pass filtered measured bridge output (thin line) for all 20 sensor locations.

Excellent agreement was found in the system identification results. The results validate that the proposed two-stage system identification strategy under a position-changing input was a suitable model when considering the bridge response to the bouncing motion of a sprung mass truck model. While the model does an excellent job in predicting the bridge response, some notable discrepancies were encountered in the time history plots of Fig. 13. For example, when the truck and sensors were on the same span, the differences between the measured and predicted system response were notable (i.e., between 34 and 36.5 s in Fig. 13a; between 36.5 to 39 s in Fig. 13b; between 39 to 41.5 s in Fig. 13c; between 41.5 and 44 s in Fig. 13d). This discrepancy may be attributed to some local nonlinearities in the truck (e.g., the leaf spring suspensions, visco-elastic behavior of the tires, energy dissipation in the shock absorbers) as well as in the bridge system when the truck is over a given span.

5. Conclusions

In this study, an effective strategy for the system identification of vehicle–bridge interaction was proposed. A data-driven mathematical model was derived from the experimental data collected from a heavy truck and highway bridge using wireless sensors contained within the same wireless monitoring system architecture. Based on the preliminary data analysis of the measured bridge and truck response in the time- and frequency-domains, the reaction force of the bouncing truck body was revealed to be the primary exciter of the bridge. The bouncing motion of the truck sprung mass worked as

<table>
<thead>
<tr>
<th>Truck velocity</th>
<th>Free-vibration</th>
<th>Forced-vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 1</td>
<td>Mode 2</td>
</tr>
<tr>
<td>30 km/h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 1</td>
<td>2.26</td>
<td>2.68</td>
</tr>
<tr>
<td>Test 2</td>
<td>2.26</td>
<td>2.67</td>
</tr>
<tr>
<td>Test 3</td>
<td>2.25</td>
<td>2.68</td>
</tr>
<tr>
<td>Mean</td>
<td>2.26</td>
<td>2.68</td>
</tr>
<tr>
<td>50 km/h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 4</td>
<td>2.26</td>
<td>2.67</td>
</tr>
<tr>
<td>Test 5</td>
<td>2.23</td>
<td>2.66</td>
</tr>
<tr>
<td>Test 6</td>
<td>2.23</td>
<td>2.67</td>
</tr>
<tr>
<td>Mean</td>
<td>2.25</td>
<td>2.67</td>
</tr>
<tr>
<td>65 km/h</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 7</td>
<td>2.25</td>
<td>2.67</td>
</tr>
<tr>
<td>Test 8</td>
<td>2.25</td>
<td>2.66</td>
</tr>
<tr>
<td>Test 9</td>
<td>2.25</td>
<td>2.66</td>
</tr>
<tr>
<td>Mean</td>
<td>2.25</td>
<td>2.66</td>
</tr>
<tr>
<td>70 km/h</td>
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<td></td>
</tr>
<tr>
<td>Test 10</td>
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<td>2.66</td>
</tr>
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<td>Test 11</td>
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<td>2.69</td>
</tr>
<tr>
<td>Mean</td>
<td>2.25</td>
<td>2.67</td>
</tr>
</tbody>
</table>

Fig. 12. Low-pass (10 Hz cutoff frequency) filtered bouncing vertical acceleration of the truck body during the 65 km/h test.
a fundamental input to an input–output system model formulated for the modeling of vehicle–bridge interaction. To identify the key parameters of a state-space input–output model, a two-step identification strategy was explored in detail. In the first stage, the time-invariant properties of the bridge are estimated using the free vibration response of the bridge.

Fig. 13. Two-stage system identification of the Yeondae Bridge during 65 km/h truck test with thick line denoting the predicted system output and the thin line the low-pass 10 Hz filtered measured response: (a) span 1; (b) span 2; (c) span 3; (d) span 4.
In the second stage, the vehicle bouncing motion and the truck position history were combined with the bridge forced vibration response to estimate the time variant load matrix of the system model. Excellent system identification results were obtained with the model predicting closely the measured system response.
Future work is aimed towards improving the wireless monitoring system to allow for ad-hoc wireless interaction between vehicle-based wireless sensors and those permanently installed on a bridge for structural health monitoring purposes. In addition, the proposed two-stage system identification model is being expanded using different vehicle position-load effect kernel vectors. In addition, the simple sprung mass model proposed in Fig. 5 is being expanded to include the rocking motion of the vehicle and the use of more than one axle to impart dynamic load into the bridge structure. Finally, an alternative approach to the estimation of the system input is being explored that does not require the assumption of a load effect kernel vector; rather, vehicle horizontal position and vertical body acceleration could potentially serve as inputs to the state space model with a time invariant system input matrix, B.

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