Problem 1: Dynamics of Car Platoons

Cars on the road in steady state traffic have a predictable dynamical model that is based on one car following the other. The model is best described by a dynamical model defined as \( \ddot{z}_f = \lambda [\dot{z}_l(t - T) - \dot{z}_f(t - T)] \) where \( \dot{z}_f \) is the velocity of a follower car, \( \dot{z}_l \) is the velocity of a leader car, \( \lambda \) is a constant (with \( \lambda > 0 \)), and \( T \) is a time delay associated with the follower response.

(a) Write a state-space model of the system where the system output is the velocity of cars 2 and 3 and the input is the velocity of car 1. Clearly denote \( A \), \( B \), \( C \), and \( D \). [Hint: Define the state vector as the displacements and velocities of cars 2 and 3 but don’t be afraid to include the time delay, \( T \).]

(b) Write the system in the frequency domain through a Laplace transform of the system.

(c) Write the solution of the system as an analytical expression in the time domain (namely, \( x(t) \) and \( y(t) \)).

(d) If \( T = 0 \) is the system stable? Yes or no and why.

(e) If \( T = 0 \), re-write the analytical expressions for \( x(t) \) and \( y(t) \).

(f) Again with \( T = 0 \), plot the displacements of all three vehicles when the first car slows down exponentially with a rate of \( 0.1/s \) (i.e., \( e^{-0.1t} \)) for two cases: \( \lambda = 0.1 \) and \( \lambda = 0.5 \). Assume \( z_1(0) = 0 \) m, \( z_2(0) = -100 \) m, \( z_3(0) = -200 \) m, and \( \dot{z}_1 = \dot{z}_2 = \dot{z}_3 = 25 \) m/s. [Hint: use MATLAB and the \texttt{lsim} command to simulate the system.]

(g) Which \( \lambda \) value (0.1 or 0.5) is more realistic and why?
**Problem 2: Building Environmental Systems**

You are given a two-room building for which you seek to analyze the thermodynamics of its space (perhaps to design a new HVAC system). Assume the outside ambient temperature of a building is $T_a$ while the interior temperature is $T_i$ for each room (hence, room 1 is $T_1$). Furthermore, the heat capacity of the interior space of the building is $C_1$ and $C_2$ for rooms 1 and 2, respectively.

The envelope of the building will conduct heat through based on the thermal resistance of the envelope, $R_{wa}$. Room 1 is enclosed by three equal sized walls (each with $R_{wa}$) and one window while room 2 is enclosed by four walls; the window has an area, $A_w$, and thermal resistance of $R_{win}$. The solar radiation on room 1 is modeled as energy flux, $\Phi_s$. Assume $C_1 = 100 C^o / J$, $C_2 = 80 C^o / J$, $R_{wa} = 0.01 C^o / J \ast s$, $R_{win} = 0.1 C^o / J \ast s$, $A_w = 10 m^2$.

(a) Write an analytical model of the two-room building thermal environment. Write the model in terms of the system output as temperature of room 1 and 2 (i.e., $T_1$ and $T_2$).

(b) Is the system linear? Why or why not?

(c) Rewrite the analytical model as a linear time invariant (LTI) state space model.

(d) Given the initial temperature of the interior rooms are both $20^o C$ and the ambient temperature outside the building is $T_a = 20^o C$, write an analytical expression for the temperature time histories of the two rooms when the solar radiation is a step function with magnitude 0.5. Plot the temperature of each room as a function of time with $t \in [0, 10]$.

**Problem 3: Tuned Mass Dampers for Structures**

One strategy for controlling structures exposed to dynamic lateral loads is the use tuned mass dampers (TMD). As shown, a structure with mass, $m$, damping, $c$, and stiffness, $k$, has a TMD installed consisting of its own mass, $m_p$, attached to the structure through a spring with stiffness, $k_p$, and damper with a damping coefficient of, $c_p$. The displacement of the structure is $y$ and the relative displacement of the TMD mass (relative to the structure) is $y_r$. The structure is exposed to a lateral force, $f(t)$. In this problem, assume $m = 20,000kg$, $k = 2.5 \times 10^6 N/m$, $c = 0 Ns/m$, $m_p = 2,000kg$, $k_p = 2.0 \times 10^5 N/s$, and $c_p = 7.2 \times 10^3 Ns/m$. 
(a) Write a state space model for the structure for the given input, \( f(t) \), and output, \( y(t) \).  
[Hint: use \( y \) and \( y_r \) in the state].

(b) Determine the eigenvalues and eigenvectors of the system; identify the poles of the system.

(c) Re-write the state space model with the system decoupled through a transformation of the state.

(d) Determine the response of the system analytically from the first mode (i.e., the first set of conjugated poles with the lowest frequency) when \( f(t) = 1 \times 10^5 e^{-100t} \) and assuming the system is initially at rest.