CLASS #21: ALIASING AND OTHER SIGNAL ISSUES

OBJECTIVES: 0 REVIEW of DISCRETE FOURIER TRANSFORM
0 DESCRIE ALIASING
0 PRESENT OTHER DFT PITFALLS

0 REVIEW of DFT:

\[ X_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-i \frac{2\pi kn}{N}} \]

- \( N = \) # of SAMPLES
- \( X_n = \{x_1, x_2, \ldots, x_N\} \)
- \( k = \) DISCRETE-FREQ
- \( v = \) DISCRETE-TIME

- UNIQUE SPECTRA of \( X_k \) \( k \in \{0, \frac{N}{2}\} \)
- SYMMETRY PROPERTIES: \( X_1 = X^*_2, \) CONJUGATE

- \( \frac{1}{2} W_s = \) NYQUIST FREQUENCY
- MUST SAMPLE AT LEAST TWICE (IF NOT 3-5x) HIGHEST DYNAMIC
- D.F.T is $F(z)$ at $z = e^{-i\frac{2\pi}{N}k} \text{ (unit circle)}$
- Nyquist frequency defines half of unit circle
- Explains repeating spectra from $k=0 \rightarrow \frac{N}{2}$

![Diagram of D.F.T. and Nyquist frequency]

- **Aliasing**

$\omega(\tau) = \sin \omega \tau$

![Continuous time sinusoidal signal]

- Now, sample at $\omega_s > \bar{\omega}$

- Now, sample at $\frac{\omega_s}{2} \approx \bar{\omega}^+$
NOW LET $w_s \approx \bar{w}$

- CONSIDER: $X_{n+L} = \frac{1}{N} \sum_{k=0}^{N-1} x_n e^{-i \frac{2\pi}{N} (n+L)k} = \frac{1}{N} \sum_{k=0}^{N-1} x_n e^{-i \frac{2\pi}{N} } e^{-i \frac{2\pi}{N} \frac{Lk}{N} }$

  $= \frac{1}{N} \sum_{k=0}^{N-1} x_k e^{-i \frac{2\pi}{N} \frac{Lk}{N} }$

  $X_{n+L} = X_L$

- SIMILARLY:

  $X_N = X_0$

  $X_{2N} = X_N = X_0$

  $X_{\frac{N}{2}} = X_{N+\frac{N}{2}} = X_{2N+\frac{N}{2}}$
ALIASING (DISTORTION of $X_k$ COMPARED TO TRUE $X(\omega)$)

WHY?

ADDITIVE B/C HARMONICS EXISTING ABOVE $W_s/2$
ADD TO HARMONICS BELOW $W_s/2$

AS SAMPLE FASTER, ALIASING LESS & LESS SINCE ALIASING IS MINIMAL

MINIMAL DISTORTION DUE TO NEAR ZERO $X(\omega)$
3. **Anti-Aliasing**:  

- MUST ELIMINATE ALIASING IN HARDWARE

- **Anti-Alias Filters**
  
  L "Low Pass" Filter (LPF)

- **In Frequency Domain**:
  
  \[ X(\omega) = \mathcal{F}(x(t)) \]
  
  \[ L(\omega) = \mathcal{L} \]
  
  \[ \overline{X}(\omega) = L(\omega)X(\omega) \]
  
  \[ \mathcal{F}_c \]

- **Nyquist Frequency**:
  
  Set \( w_c = \frac{w_s}{2} \)

- **Corresponding Frequency**:
  
  \( w_c \)
LEAKAGE:

- Discrete Fourier Transform is based on Fourier Series
  - Treats $x(t)$ as periodic

- Due to periodic signal assumption:

  - Discontinuity at boundaries ($x(1) \neq x(N)$)
  - Get Leakage (artificial harmonics introduced due to artificial "step"

- Use windowing:
- **COMMON WINDOWS**:

  i) BARTLETT: \( W(n) = 1 - \frac{|n - N/2|}{N/2} \)

  \[ \text{Graph of Bartlett window} \]

  ii) HANNING: \( W(n) = \frac{1}{2} \left( 1 - \cos \left( \frac{2\pi n}{N-1} \right) \right) \)

  \[ \text{Graph of Hanning window} \]

  iii) HAMMING: \( W(n) = 0.54 - 0.46 \cos \left( \frac{2\pi n}{N-1} \right) \)

  \[ \text{Graph of Hamming window} \]
**5. Transients**

- **Discrete Fourier Transform** based on "Periodic" signal (from Fourier series)

- Periodic signals have no transients (since they damped out)

- Careful when using D.F.T. for dynamic analysis in frequency domain.

![Diagram showing the process from time domain to frequency domain and back]

**Time Domain**

- Excit. $U(t)$
- System $h(t)$
- Response $y(t)$

**Frequency Domain**

- $U_k$ (D.F.T.)
- $H(w)$ (D.F.F.)
- $y_k = H(w)U_k$ (I.D.F.F.)

**Arbitrary Initial Condition**

$y(0) \& y'(0)$ [if wrong, must perform free vibration response with $y(0) \& y'(0)$ and remove from $y(k)$]