CLASS #6: REALIZATION OF DYNAMICAL SYSTEMS

OBJECTIVES:
1. INTRODUCE BLOCK DIAGRAMS
2. DEFINE BUILDING BLOCKS IN ANALOG DOMAIN
3. SHOW BUILDING BLOCK IN CIRCUIT FORM
4. EXAMPLES

0. ANALOG COMPUTING:
- IN OLD DAYS, REQUIRED MEANS OF SIMULATING DYNAMICAL SYSTEMS
- COMPUTERS NOT AS COMMON (1950-1970's)
- ENGINEERS SIMULATED SYSTEMS USING ELECTRICAL CIRCUITS
  → "ANALOG COMPUTER"

- USED "OPERATIONAL AMPLIFIERS" IN CIRCUIT (TRANSISTOR-BASED INTEGRATED CIRCUIT)

Rules of Operational Amplifier:
1. CAN SOURCE NO CURRENT ON INPUT
2. VOLTAGE AT POSITIVE & NEGATIVE INPUT MUST BE EQUAL

FOR EXAMPLE:
\[ V_{\text{out}} = 0 - i \frac{R_2}{R_1} \]
\[ V_{\text{out}} = -\frac{R_2}{R_1} V_{\text{in}} \]

"INVERTING AMPLIFIER" OF GAIN \( R_2/R_1 \)

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- Analog computers are still used in many control systems for simple & fast (low delay) execution.

2 Building Blocks

Continuous-Time Systems $\rightarrow$ Nth Order Differential Equation

$$\frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1 \frac{dy}{dt} + a_0 = u$$

\[ \text{SUMMATION} \quad y(t) = x_1(t) + x_2(t) + \cdots + x_i(t) \]
\[ \text{GAIN (MULTIPLICATION)} \quad y(t) = Gx(t) \]
\[ \text{DIFFERENTIATION (L-E INTEGRATION)} \quad y(t) = \int_0^t x(t) \, dt + y(0) \]

i) Integration:

\[ x(t) \quad \int \quad y(t) \]

\[ y(t) = \int_0^t x(\tau) \, d\tau + y(0) \]

\[ V_y = -\frac{1}{C} \int_0^t V_x(\tau) \, d\tau + V_y(0) \]

\[ V_y(0) = -\frac{1}{RC} \int_0^t V_x(\tau) \, d\tau + V_y(0) \]

If \( RC = 1 \)

\[ V_y(0) = -\int_0^t V_x(\tau) \, d\tau + V_x(0) \]

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- You can build a differentiator:

\[ \frac{dy}{dt} = \frac{dx}{dt} \]

- Integrators more common

(ii) Summation:

\[ V_y(t) = -\frac{R_1}{R} V_{x1} - \frac{R_2}{R} V_{x2} - \cdots - \frac{R_m}{R} V_{x_m} \]

Includes gain for "free"!
8. DIRECT REALIZATION OF CONTINUOUS-TIME SYSTEMS:

CONSIDER DYNAMIC SYSTEM:

\[ \frac{d^3 y}{dt^3} + 10 \frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 5y = u(t) \]

i) CREATE "OUTPUT PIPELINE":

\[ \frac{d^3 y}{dt^3} \rightarrow \frac{d^2 y}{dt^2} \rightarrow \frac{dy}{dt} \rightarrow y(t) \]

ii) ISOLATE HIGHEST ORDER:

\[ \frac{d^3 y}{dt^3} = u(t) - 10 \frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} - 5y \]

iii) ADD GAIN BLOCKS

\[ \frac{d^3 y}{dt^3} \rightarrow \frac{d^2 y}{dt^2} \rightarrow \frac{dy}{dt} \rightarrow y(t) \]

iv) SUM ALL PARTS

\[ u(t) \rightarrow \rightarrow \frac{d^3 y}{dt^3} \rightarrow \frac{d^2 y}{dt^2} \rightarrow \frac{dy}{dt} \rightarrow y(t) \]

**NOTE:** DYNAMICS OF SYSTEM ARE DRIVEN BY FEEDBACK LOOPS

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- Consider case of higher order inputs \(u(t)\):\
\[
\frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 2 y = 3 \frac{du}{dt} + 4 u
\]

Let us put in operator form:
\[
Y(t) = \frac{3p + 4}{p^2 + 8p + 2} u(t) = L(p)
\]
\[
Y(t) = \frac{N(p)}{D(p)} u(t)
\]
\[
D(p)Y(t) = N(p)u(t)
\]

Introduce "\(x(t)\)" by \(N(p) = 1\)
\[
D(p)x(t) = u(t) \quad \rightarrow \quad (p^2 + 8p + 2)x(t) = u
\]
\[
p^2 x = u - 8px - 2x
\]
\[
\rightarrow (3p + 4)x = y
\]

Create pipeline of \(x(t)\):

\[
D(p) = \text{characteristic equation} \quad \text{"} x \text{"} \quad N(p) = \text{observation of system to } y \text{ mapping}
\]
Realization of Discrete-Time Systems

Same procedure as continuous-time systems but replace integrator with delay operator, $q^{-1}$

$$\frac{y(k)}{q^{-1}} y(k-1)$$

Example

$$y(k+3) - 8y(k+2) + 37y(k+1) - 50y(k) = 3u(k+1) + 5u(k)$$

$$(q^3 - 8q^2 + 37q - 50)y(k) = (3q + 5)u(k)$$

$$y(k) = \frac{3q + 5}{q^3 - 8q^2 + 37q - 50} u(k)$$

$$y(k) = \frac{N(q)}{D(q)} u(k)$$

$D(q)\times x(k) = u(k) \quad \rightarrow \quad (q^3 - 8q^2 + 37q - 50) x = u$

$N(q)\times x(k) = y(k) \quad \rightarrow \quad (3q + 5) x = y$

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