CLASS #9: HARMONIC BASE MOTION

HARMONIC BASE MOTION: (i.e. EARTHQUAKE)

INSTEAD of APPLIED LOAD, SHAKE BASE

\[ u(t) \]

\[ U_g = U_0 \sin \omega t \]

\[ \ddot{u} + k(u - U_g) + c(\dot{u} - \dot{U}_g) = 0 \]

\[ \ddot{u} + c\ddot{u} + ku = \ddot{U}_0 \sin \omega t \]

\[ = \frac{C}{\omega} u_0 \cos \omega t + ku_0 \sin \omega t \]

\[ = U_0 \sqrt{k^2 + (c\omega)^2} \sin (\omega t + \phi) \]

where \( \tan \phi = \frac{c\omega}{k} = 2\sqrt{\beta} \)

\[ m\ddot{u} + c\ddot{u} + ku = p_0 \sin (\omega t + \phi) \]

where \( p_0 = U_0 \sqrt{k^2 + (c\omega)^2} \)

\[ U_p = U_{0r} \cdot D \cdot \sin (\omega t + \phi + \theta) \]

\[ = \frac{p_0}{k} \sqrt{1 - \beta^2} + (2\sqrt{\beta} \theta) \cdot \sin (\omega t + \phi + \theta) \]

\[ = U_0 \sqrt{k^2 + (c\omega)^2} \cdot \sin (\omega t + \phi + \theta) \]

\[ = U_0 \sqrt{1 + (2\sqrt{\beta} \theta)^2} \cdot \sin (\omega t + \phi + \theta) \]

\[ = U_0 T_r \sin (\omega t + \phi + \theta) \]

\[ T_r = \text{TRANSMISSIBILITY} = \frac{\sqrt{1 + (2\sqrt{\beta} \theta)^2}}{\sqrt{(1 - \beta^2)^2 + (2\sqrt{\beta} \theta)^2}} = D \sqrt{1 + (2\sqrt{\beta} \theta)^2} \]

Very Useful
VIBRATION ISOLATION SYS.
Transmissibility for harmonic excitation

\[ p(t) = p_0 \sin \omega t \]

\[ \zeta = 0.01 \]

\[ \zeta = 0.05 \]

\[ \zeta = 0.1 \]

\[ \zeta = 0.2 \]

\[ \zeta = 0.7 \]

\[ \zeta = 1 \]

Damping decreases transmitted force only if \( \frac{\omega}{\omega_n} < \frac{1}{\sqrt{2}} \)

\[ \sqrt{2} \]

Frequency ratio \( \frac{\omega}{\omega_n} \)

Transmissibility, \( TR = \frac{f_{T}}{f_0} \), \( f_{T} = \frac{\dot{u}}{\dot{u}_0} \)

Figure 3.5.1 from *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, by Anil K. Chopra, Prentice-Hall, 2001.

So transmitted force is less than applied then make \( \omega_n \) small enough so that \( \frac{\omega}{\omega_n} > \frac{1}{\sqrt{2}} \), no damping.
Define relative motion: $U_r = U - U_g$

$$m \ddot{U}_r + c \dot{U}_r + kU_r = -m \ddot{U}_g$$

$$m \ddot{U}_r + c \dot{U}_r + kU_r = mU_0 \ddot{w}^2 \sin \omega t$$

Solution:

$$U_r(t) = \frac{mU_0 \ddot{w}^2}{K} \frac{\sin(\omega t - \theta)}{\sqrt{(1-\beta^2)^2 + (2\delta \beta)^2}}$$

$$U_r(t) = U_0 \frac{\beta^2}{\sqrt{(1-\beta^2)^2 + (2\delta \beta)^2}} \sin(\omega t - \theta)$$

$D$ Dynamic Magnification Factor for ground motion

2. $D$ serves as foundation of accelerometer:

Device widely used in structural engineering to measure building responses

Device will provide as an "output" a measure of $U_r$

$$m \ddot{U}_r + c \dot{U}_r + kU_r = 0$$

$$m \ddot{U}_r + c \dot{U}_r + kU_r = -m \ddot{U}_g \sin \omega t$$

$$U_r = \frac{-m \ddot{U}_g}{k} \cdot D \cdot (\sin \omega t + \theta)$$

$$D = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\delta \beta)^2}}$$
\[ U_r = -\frac{\ddot{U}_g}{\omega_n^2} \cdot D \cdot \sin(\omega t + \theta) \]

\[ \text{If } \zeta = 0.7 \]
\[ \bar{\omega} < 0.6 \omega_n \]

Then \( D \approx 1 \)

\[ U_r = -\frac{\dddot{U}_g}{\omega_n^2} \sin(\omega t + \theta) \]

\[ U_r \propto \dddot{U}_g \]

\[ U_{\text{AMP}} \propto U_g \]

\[ \therefore \text{Design Accelerometer} \rightarrow \text{pick } M_p, K, C \]

\[ \text{such that } \zeta = 0.7 \]
\[ \& \omega_n > 1.6667 \bar{\omega} \]

\[ \text{but we will need to know something about ground motion.} \]
\[ \text{Namely, } \bar{\omega} \]
CEE511 Structural Dynamics:

Accelerometers

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Accelerometers

- 3 accelerometer types used in civil structures:

  ✓ Force balance accelerometers (FBA)
    - Greatest market share
    - Expensive at approximately $1000 per FBA

  ✓ Piezoelectric accelerometers
    - Popular in dynamic settings
    - Moderate pricing at approximately $400 per accelerometer

  ✓ Microelectromechanical system (MEMS)
    - Represents the future
    - Demand driven by car manufacturer
    - Small and accurate
    - Inexpensive $5-50 per accelerometer
Force Balance Accelerometer

- Feedback current that balances internal mass is proportional to acceleration

- California Building Code (2001) requires at least 3 accelerometers (usually FBAs) installed in structures:
  - Floor areas greater than 5,500 m² or over 6 stories
Piezoelectric Accelerometer

- Piezoelectrics are a crystalline material that:
  - Generate current when strained
  - Strain when voltage is applied

- Can be used with a proof mass to generate an electrical current proportional to acceleration
  - Can not sense static (DC) acceleration fields (gravity)

![Piezoelectric Accelerometer Diagram]

2.5 cm
MEMS-Based Accelerometers

- **Microelectromechanical system (MEMS) accelerometers**
- Creation of mechanical structures only micrometers in size on silicon wafers (same process as CMOS integrated circuits)
- Cost advantage derived from using well developed CMOS fabrication process
- MEMS - more accurate and sensitive sensors in form factors and unit costs not previously possible
- Cost advantage of MEMS - integration of sensors and digital circuitry (like A/D conversion) all on one die
- Car industry (accelerometers for air bag deployment) has driven market demand
  - GM will only buy sensor when they are less than $5!
Analog Devices ADXL210

Completed ADXL210 Device in Silicon

Springs

Proof Mass = 0.7 µg

Capacitive Readout Mechanism

Packaged Device

Springs

Proof mass = 0.7µgram

1.3 Micron Gap

3 Microns Thick

125 Micron Overlap
Analog Devices ADXL210

- Low cost, low power 2-axis accelerometer - 10 g

- Balanced differential capacitors measure acceleration of silicon proof mass

- Variable bandwidth and resolution
  - Frequency range 0 - 50 Hz (Bandwidth)
  - Noise floor 4 mg (Resolution)
  - Range of linearity (+/- 10g)
  - Dynamic Range (in dB):
    - \( DR(dB) = 20\log(\text{Max}/\text{Min}) = 20\log(10/4E-3) = 68 \text{ dB} \)