Sensors Lecture #4: Resistivity-based Sensing - Strain, Temperature and Cracks

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Reading and Objectives

• **Readings:**
  – Chapter 3.5 and 8 in Fraden

• **Objectives of today’s class:**
  – Define piezo resistivity
  – Describe the operational principles of strain gages
Resistivity

- **Resistivity, \( \rho \), is a material property:**

\[
\rho = \frac{E}{j} = \frac{Electric \ Field}{Current \ Density}
\]

\[
j = \frac{i}{A}
\]

\[
\rho = [\Omega \cdot m]
\]

- **Inverse of resistivity is conductivity, \( \sigma \):**

\[
\sigma = \frac{1}{\rho}
\]

\[
\sigma = [(\Omega \cdot m)^{-1} = S / m]
\]
**Resistance**

- **Resistance is a characteristic of a physical device:**
  - Based on a materials resistivity
  - Based on the device volume

\[
R = \rho \frac{L}{A}
\]
Variations in Resistivity

• **Resistivity (and hence, resistance) vary with environment:**
  – Temperature
  – Strain (piezoresistivity)

• **Temperature variations:**

\[
\rho = \rho_o \left(1 + \alpha (T - T_o)\right)
\]

\[
R = R_o \left(1 + \alpha (T - T_o)\right)
\]

  – Temperature coefficient of resistance, \( \alpha \)
  – \( T_o \) is a reference temperature of known resistance/resistivity
  – Basis of temperature sensors
Thermistors

• **Constructed of a polymeric or ceramic material:**
  – Designed to have high thermal accuracy (typically -100 to 150 C°)

• **First-order approximation:**
  \[ \Delta R = k \Delta T \]
  – If \( k > 0 \), positive temperature coefficient (PTC) type thermistor
  – If \( k < 0 \), negative temperature coefficient (NTC) type thermistor

• **Over a broader range, 1st-order approximation is insufficient:**
  – Third-order approximation is needed (Steinhart–Hart equation)
  – Device has defined \( a \) and \( b \) constants
  – Temperature is defined in Kelvin
  \[
  \frac{1}{T} = a + b \ln(R) + c \left(\ln(R)\right)^3
  \]

Piezoresistivity

- **Piezoresistivity:**
  - Variation in resistance due to strain in the device

- **Sources of piezoresistivity:**
  - Geometric
  - Material
If incompressible, volume must remain constant:

- Density and mass will not change and hence volume doesn’t

\[
R = \rho \frac{L}{A} = \rho \frac{L^2}{V}
\]

\[
\frac{dR}{dL} = 2 \frac{\rho L}{V} = 2 \frac{R}{L}
\]
Gage Factor

• **Gage factor is a measure of sensitivity of strain sensor:**
  - Percent change in resistance per unit strain

  \[ S_e = \frac{\Delta R}{R} \frac{R}{\varepsilon} = \frac{\Delta R}{\Delta L} \frac{R}{L} \]

  \[ S_e = \frac{dR}{R} \frac{R}{dL} = \frac{2 R}{L} \frac{R}{L} \]

  \[ S_e = 2 \]
Liquid Strain Gage

- **Based on volumetric change in an incompressible liquid:**
  - Mercury is an incompressible fluid so it is widely used
  - Resistance of mercury changes as a function of change in length
  - Widely used in medical industry for pressure measurements but now being phased out by *solid-state alternatives*


Source: Hokanson Inc. (2013)
Metal Piezoresistors

- **Metal wires are also very good piezoresistors:**
  - Metal wires are not incompressible
  - Material piezoresistivity now contributes \( i.e., \Delta \rho = f(\epsilon) \)

\[
R = \frac{\rho L}{A}
\]

\[
dR = \frac{L}{A} d\rho + \frac{\rho}{A} dL - \frac{\rho L}{A^2} dA
\]

\[
\frac{dR}{R} = \left( \frac{d\rho}{\rho} \right) + \frac{dL}{L} - \frac{dA}{A}
\]

Geometric change

Resistive change
Metal Piezoresistors

• **Gage factor ($S_e$) of metal materials:**

$$S_e = \frac{dR/R}{dL/L} = \frac{d\rho/\rho}{dL/L} + \frac{dA/A}{dL/L}$$

$$S_e = \frac{d\rho/\rho}{dL/L} + 1 + \frac{dA/A}{dL/L}$$

$$S_e = \frac{Ld\rho}{\rho dL} + 1 + 2v$$

• **$S_e$ for some common materials used in strain sensors:**
  – 2 for nickle-copper alloys (55%Cu – 45%Ni – Constanan)
  – 6 for platinum alloys
Metal Foil Strain Gages

- **Serpentine wire deposited on an elastic (flexible) carrier:**
  - Strain sensitivity based on direction of long-run of serpentine wire
  - Gage length – length over which strain is averaged
  - Exhibits piezoresistivity (w/gage factor) but also thermal sensitivity
  - Low cost devices costing approximately $1 to $10 per gage

**STRAIN GAGE TECHNICAL DATA**

**STRAIN GAGE MEASUREMENT**

The most universal measuring device for the electrical measurement of mechanical quantities is the strain gage. Several types of strain gages depend for their operation on the proportional variance of electrical resistance to strain: the piezoresistive or semiconductor gage, the carbon resistive gage, the bonded metallic wire, and foil resistance gages. The bonded resistance strain gage is by far the most widely used in experimental stress analysis. This gage consists of a grid of very fine wire or foil bonded to a backing or carrier matrix. The electrical resistance of the grid varies linearly with strain. In use, the carrier matrix is bonded to the surface, force is applied, and the strain is found by measuring the change in resistance. The bonded resistance strain gage is low in cost, can be made with a short gage length, is only moderately affected by temperature changes, has small physical size and low mass, and has fairly high sensitivity to strain. In a strain gage application, the wiring is located in a time-varying magnetic field.

Magnetic induction can be controlled by using twisted lead wires and forming minimum but
Strain Gage Bridge Circuit

- **Wheatstone Bridge circuit:**
  - Convert change in resistance to a voltage reading
  - Various configurations associated with strain instrumentation
    - Reduce/eliminate thermal influence
    - Enhance measurement resolution
Rosettes are multiple (>2) strain gages on same substrate:

- Designed to determine principle strain in structural element
- Necessary condition for determining principle strain but not sufficient
- Extrapolation of three measurements to principle strains performed by Mohr’s circle

Source: Omega (2013)
Source: Vishay (2013)
Rosette Types

• **Three major Rosette types:**
  - Tee:
  ![Tee Image]
  Source: Measurement Specialties 2014
  - 45° Rectangular:
  ![45° Rectangular Image]
  Source: Measurement Specialties 2014
  - 60° Delta:
  ![60° Delta Image]
  Source: Omega 2014
Rosettes and Principle Strain
Thermal Effects on Strain Gages

- **Temperature impacts the gage factor of the sensor:**
  - If not accounted for, would be interpreted as a change in strain
  - Manufacturers minimize thermal sensitivity by compensating for the structural material to which the gage is mounted (e.g., steel vs. Al)

Thermal Effects on Strain Gages

• **Seek materials with flat thermal dependency (-45 to 100°C)**
  
  - Some gage alloys have been designed so that the temperature effects on the gauge resistance itself cancel out the change due to the thermal expansion of the object under test
  
  - Termed self-temperature compensation or S-T-C
  
  - $4^{th}$ order polynomial function tabulated in the data sheet

Source: Vishay (2013)
Other Temperature Issues

• **High bridge voltages result in high current:**
  – Current induced local heat effect in the gage
  – Minimize bridge voltage (but lower bridge voltages = lower resolution)

• **Lead wires, if long, can introduce thermal dependencies:**
  – Minimize length of lead wires
  – Ensure wire feeds in the Wheatstone bridge are same length
    • Termed “3-wire bridge” configuration
    • Three wires, A, B, and C made to be same length so that thermal influence cancel on right side of the Wheatstone bridge
Crack Gages

**Thin Film Crack Gages**

- **Crack gages are used to track the evolution of cracks:**
  - Crack propagation can be tied to limit states of the structure
  - Fabricated using similar thin film methods used for strain gages
  - As crack grows, the film traces break resulting in a change in resistance

Source: Vishay (2013)