Class #9: Energy Storage Elements

Read: CH 7.1 - 7.7

OBJ: 1. Define Capacitors & Inductors
2. Energy Storage Calculations

01 Energy Storage Elements

- Circuits containing Capacitors & Inductors store energy
- Due to storing energy, circuits \( \rightarrow \) Dynamic \((f(t))\)
  \( \rightarrow \) Behavior modeled by
  differential equations
- In contrast, resistive circuits \( \rightarrow \) Static \((\#f(t))\)
  \( \rightarrow \) Behavior modeled by
  algebraic equations

02 Capacitor

- Charge held on plates by electric field, \(E\)
- Charge accumulated, \(Q\) is proportional to \(V\)
  \(Q(t) = CV(t)\)
  \(\uparrow\) Prop. const. called capacitance

\[ C = \frac{\varepsilon A}{d} \]
where \(\varepsilon\) = Permittivity of medium between plates (dielectric)

\(\frac{C}{N}\) or \(F, \text{farads}\)
\(\uparrow\)
Michael Faraday (1791 - 1867)
- Recall, \[ i = \frac{dQ}{dt} \]

\[ i(t) = C \frac{dV}{dt} \]

- Similarly,

\[ V(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau) d\tau \]

or

\[ V(t) = \frac{1}{C} \int_{t_0}^{t} i(\tau) d\tau + V(t_0) \]

- Capacitors are like batteries:
  - Stores energy
  - Finite capacity
    (Equivalent to bucket of water)

- Work done by charge:

\[ W_c(t) = \int_{-\infty}^{t} V(\tau) i(\tau) d\tau \]

\[ i = C \frac{dV}{dt} \]

\[ = \int_{-\infty}^{t} V(\tau) C \frac{dV}{d\tau} d\tau \]

\[ = C \int_{V(t_0)}^{V(t)} V dV \]

\[ = C \left[ \frac{1}{2} V^2 \right]_{V(t_0)}^{V(t)} \]

\[ E_c = \frac{CV^2}{2} = \frac{Q^2}{2C} \]

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EXAMPLE:

- 10 mF CAP is CHARGED TO 100V
- HOW MUCH ENERGY IS STORED BEFORE SWITCH IS OPENED

\[ E = \frac{CV^2}{2} = \frac{(10 \times 10^{-3} F)(100 V)^2}{2} \]
\[ E = 50 J \]

INDUCTOR:

- TURN OF COIL ON A CYLINDRICAL CORE

\[ V(t) = L \frac{di}{dt} \]
\[ L = \text{INDUCTANCE} \ (\text{H, HENRY'S}) \]

- IF CORE IS NOT FERROMAGNETIC,

\[ L = \frac{\mu_0 N^2 A}{l + 0.45d} \]

\[ \mu_0 = \text{PERMISIBILITY} \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{H/m} \]

- PROPERTY IS DUE TO MAGNETIC FLUX, \( \Phi \)
- Faraday proved this based on magnetic fields

\[ V = N \frac{d\phi}{dt} \]

\[ \because N \phi = Li \]

\[ \therefore V = L \frac{di}{dt} \]

- Essentially, energy is stored as a magnetic field

\[
L(t) = \frac{1}{t} \int_{t_0}^{t} Vd\tau + i(t_0)
\]

- Work done

\[
W = \int_{t_0}^{t} pd\tau = \int Vi d\tau = \int L \frac{di}{dt} i d\tau = L \int_{i(t_0)}^{i(t)} idi
\]

If \( t_0 = -\infty \) and \( i(-\infty) = 0 \)

\[ E_i = \frac{1}{2} Li^2 \]

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9. **Two Observations:**

- Capacitors & Inductors Have Memory
- Do Not Generate or Dissipate Energy → Only Store

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