CLASS 27 - INTRODUCTION TO MOHR'S CIRCLE

OBJECTIVES: 1. SUMMARIZE PREVIOUS FINDINGS
2. DEVELOP CONCEPT OF MOHR'S CIRCLE
3. ILLUSTRATE WITH EXAMPLE

READ: PHILPOT CH 12.8-12.11

1) WHERE ARE WE?
- CONSIDERED STRESS ON PLANE ELEMENTS ("IN-PLANE STRESS")

\[
\begin{align*}
\sigma_x &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + T_{xy} \sin 2\theta \\
\sigma_y &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - T_{xy} \sin 2\theta \\
T_{xy}' &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + T_{xy} \cos 2\theta
\end{align*}
\]
TRANSFORMATION EQUATIONS TO GET EQUIVALENT STRESSES

2) INTRODUCE SOME TERMS:

\[
\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2}
\]
\(\sigma_{\text{avg}}\) AVERAGE OF AXIAX
STRESSES \(\sigma_x \& \sigma_y\)

\[
\sigma_{1.2} = \sigma_{\text{avg}} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + T_{xy}^2}
\]
PRINCIPLE STRESSES

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3. ALGEBRAIC "TRICK"

\[ \sigma' = \frac{\sigma - \sigma_x}{2} \cos 2\theta + T_{xy} \sin 2\theta \]
\[ \tau' = -\left( \frac{\sigma - \sigma_x}{2} \right) \sin 2\theta + T_{xy} \cos 2\theta \]

**SQUARE BOTH EQUATIONS & SUM**

\[ \left( \sigma' - \sigma_{\text{avg}} \right)^2 + \tau'^2 = \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + T_{xy}^2 + T_{xy} \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \frac{\sigma_x - \sigma_y}{2} \sin 2\theta \]

**KNOWN (GIVEN)**

\[ \therefore \text{LET } R^2 = \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + T_{xy}^2 \]

**WHAT TYPE OF EQUATION IS THIS?**

**CIRCLE**

\[ \sigma_x' - \sigma_{\text{avg}} \]
\[ T_{xy} \]

\[ \sigma_x = \sigma_{\text{avg}} + \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + T_{xy}^2} \]

**MOHR'S CIRCLE**

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Method to Draw Mohr's Circle:

1) Known \( \sigma_x, \sigma_y, \tau_{xy} \) - Plot on \( \sigma - \tau \) coordinates:

\[ \sigma_{\text{max}} = \frac{\sigma_x + \sigma_y}{2} \quad \{ \text{Center of Circle} \} \]

2) Draw Full Circle:

\[ R = \sqrt{\left( \frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau_{xy}^2} \]

3) Find Principle Stress = Max Shear Stress:

\[ \sigma_1 \quad \sigma_2 \]

\[ \tau_{\text{max}} = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} = R \]
A rectangular plate of dimensions 3.0 in. × 5.0 in. is formed by welding two triangular plates (see figure). The plate is subjected to a tensile stress of 500 psi in the long direction and a compressive stress of 350 psi in the short direction.

Determine the normal stress $\sigma_w$ acting perpendicular to the line of the weld and the shear stress $\tau_w$ acting parallel to the weld. (Assume that the normal stress $\sigma_w$ is positive when it acts in tension against the weld and the shear stress $\tau_w$ is positive when it acts counterclockwise against the weld.)

**USE MOHR'S CIRCLE TO SOLVE PROBLEM. PLUS, FIND MAXIMUM SHEAR STRESS.**