CLASS 26 - PRINCIPAL STRESSES

OBJECTIVES:
1. REVIEW STRESS TRANSFORMATIONS
2. IDENTIFY PRINCIPAL STRESSES & MAX SHEAR STRESS
3. PROBLEMS

READ: PHILPOT 12.7-12.8

STRESS TRANSFORMATIONS (REVIEW)

\[
egin{align*}
\sigma_x' &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\
\sigma_y' &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \\
\tau_{xy}' &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta
\end{align*}
\]
A rectangular plate of dimensions 3.0 in. × 5.0 in. is formed by welding two triangular plates (see figure). The plate is subjected to a tensile stress of 500 psi in the long direction and a compressive stress of 350 psi in the short direction.

Determine the normal stress $\sigma_w$ acting perpendicular to the line of the weld and the shear stress $\tau_w$ acting parallel to the weld. (Assume that the normal stress $\sigma_w$ is positive when it acts in tension against the weld and the shear stress $\tau_w$ is positive when it acts counterclockwise against the weld.)
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\[
\begin{align*}
\sigma_x &= 500 \text{ psi} \\
\sigma_y &= -350 \text{ psi} \\
\tau_{xy} &= 0 \\
\theta &= \arctan \left( \frac{3 \text{ in.}}{5 \text{ in.}} \right) = \arctan 0.6 = 30.96^\circ
\end{align*}
\]

\[
\begin{align*}
\sigma_x &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\
&= 275 \text{ psi} \\
\tau_{x/y} &= \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\
&= -375 \text{ psi} \\
\sigma_y &= \sigma_x + \sigma_y - \sigma_n = -125 \text{ psi}
\end{align*}
\]

**Stresses Acting on the Weld**

\[
\begin{align*}
\sigma_w &= 125 \text{ psi} \\
\tau_w &= 375 \text{ psi}
\end{align*}
\]
From the stress transformations, there must be an angle ($\theta_p$) such that principal stress is $\text{MAX} \ (\sigma_{x_{\text{max}}} \ , \ \sigma_{y_{\text{max}}})$:

$$\frac{d\sigma_{x'}}{d\theta} = 0 \quad \rightarrow \quad \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

at $\theta_p$

**Principal Stress (maximum amount)**

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

No shear!

There is also an angle ($\theta_s$) such that shear stress is $\text{MAX} \ (\tau_{xy_{\text{max}}})$:

$$\frac{d\tau_{xy}}{d\theta} = 0 \quad \rightarrow \quad \tan 2\theta_s = \frac{- (\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

at $\theta_s$

$$\tau_{\text{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{x'} = \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} = \sigma_{\text{avg}}.$$