CLASS 25 - PLANE STRESS TRANSFORMATION

OBJECTIVES:
1. EXPLAIN PLAIN-STRESS TRANSFORMATION
2. DERIVE GENERAL EQUATIONS OF PLANE STRESS

READ: PHILPOT 12.1 - 12.6

1) CONSIDER STRUCTURE UNDER LOAD:

6 INDEPENDENT STRESSES:
\( \sigma_x, \sigma_y, \sigma_z, T_{yz}, T_{xz}, T_{xy} \)

ENGINEERS USUALLY SIMPLIFY ANALYSIS & CONSIDER BEHAVIOR ON A PLANE OF STRUCTURE

"PLANE STRESS" - IGNORE ONE DIMENSION.

SIGN CONVENTION

(C) JEROME P. LYNCH, 2015
NOTE:
The state of plane stress at a point is uniquely represented by three components acting on an element that has a specific orientation.

Orientation of cube is arbitrary. Therefore if we change its orientation, element is still in equilibrium.

Equivalent representations

2 Plane stress transformations:
How do we go from \((\sigma_x, \sigma_y, \tau_{xy})\) to \((\sigma_x', \sigma_y', \tau_{xy}')\)?

Stress

(c) Jerome P. Lynch, 2015
"AREAS":

\[ \Delta A \cos \theta \]
\[ \Delta A \sin \theta \]

"FORCES":

\[ \Delta A \cos \theta \overrightarrow{O_y} \]
\[ \Delta A \sin \theta \overrightarrow{T_{xy}} \]
\[ \delta_{x} \overrightarrow{O_{x}} \]
\[ \delta_{y} \overrightarrow{O_{y}} \]

"EQUILIBRIUM":

\[ \sum F_{x} = 0: \quad \delta_{x} \Delta A - (T_{xy} \Delta A \sin \theta) \cos \theta - (\sigma_{x} \Delta A \sin \theta \sin \theta) \sin \theta \]
\[ - (T_{xy} \Delta A \cos \theta) \sin \theta - (\sigma_{x} \Delta A \cos \theta) (\cos \theta) = 0 \]

\[ \delta_{x} = \sigma_{x} \cos^{2} \theta + \sigma_{y} \sin^{2} \theta + T_{xy} (2 \sin \theta \cos \theta) \]

\[ \sum F_{y} = 0: \quad T_{xy} \Delta A + (T_{xy} \Delta A \sin \theta) \sin \theta - (\sigma_{y} \Delta A \sin \theta) \cos \theta \]
\[ - (T_{xy} \Delta A \cos \theta) \cos \theta + (\sigma_{x} \Delta A \cos \theta) \sin \theta = 0 \]

\[ T_{xy} = (\sigma_{y} - \sigma_{x}) \sin \theta \cos \theta + T_{xy} (\cos^{2} \theta - \sin^{2} \theta) \]