CLASS 16: FLEXURAL PROPERTIES OF BEAMS

OBJECTIVES:
1. Describe deformation of beams
2. Define change in strain
3. Derive flexural formula

READ:
CH 8.1 - 8.4 PHILPOT

0. Consider bending beam:

Assume homogeneous symmetric cross section

Bend

Compression

Top surface "contracts"

Bottom surface "expands"

Tension

Plane sections remain plane

Neutral surface - no change in axial strain is 0

Ignore deformation of cross sectional area

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Clearly have axial strain:

\[ \epsilon = \lim_{\Delta x \to 0} \frac{\Delta s' - \Delta s}{\Delta x} \]

\[ \Delta x = \rho \Delta \theta \]

\[ \Delta s' = (\rho - y) \Delta \theta \]

\[ \therefore \epsilon = \lim_{\Delta \theta \to 0} \frac{(\rho - y) \Delta \theta - \rho \Delta \theta}{\rho \Delta \theta} = -\frac{y}{\rho} = \epsilon \]
**FLEXURAL FORMULA:**

Hooke's Law: \( \sigma = E \varepsilon \)

\[ \therefore \text{if strain is linearly proportional so must stress} \]

\[ \sigma_{\max} = -E \frac{\varepsilon}{p} = -\frac{Y}{C} \]

\[ \sigma = -E \frac{\varepsilon}{p} = \left( -\frac{Y}{C} \right) \sigma_{\max} \]

**Consider Equilibrium:**

\[ \sum F_x = 0 \]

\[ 0 = \int_A dF = \int_A \sigma \, dA \]

\[ = \int_A -\frac{Y}{C} \sigma_{\max} \, dA \]

\[ 0 = -\frac{\sigma_{\max}}{C} \int_A Y \, dA \]

\[ \Rightarrow \int_A Y \, dA = 0 \]

**First Moment About Cross Sectional Area Must Be Zero.**

N.A. is horizontal to centroidal axis.
\[ M = \sum y dF = \int y (\sigma dA) = \int y \left( \frac{y}{E} \sigma_{\text{max}} \right) dA \]

\[ M = \frac{\sigma_{\text{max}}}{c} \int y^2 dA \]

\[ \sigma_{\text{max}} = \frac{Mc}{I} \]

\[ \sigma = -\frac{My}{I} \]

**FLEXURE FORMULA**