CLASS #7: MATERIAL STRAIN ENERGY & POISSON'S RATIO

READ: CH 3.4 PHILPOT

OBJECTIVES:
1. DEFINE STRAIN ENERGY
2. PRACTICE USING STRESS- STRAIN DIAGRAMS
3. DEFINE POISSON'S EQUATION

0 CONSIDER A SPRING

Hooke's Law States: \( F = k \delta \)

How much energy is stored in spring?

\[
\frac{1}{2} \delta_i F_i = U = \frac{1}{2} k \delta_i^2
\]

0 CONSIDER SOLID

As material deforms, it stores energy "strain energy"

- Force: \( \Delta F = \sigma_z A = \sigma_z \Delta x \Delta y \)
- Distance: \( \Delta \delta = \epsilon_z \Delta z \)
- Energy: Based on spring analogy:
  \[
  \Delta U = \frac{1}{2} \Delta \delta \Delta F = \frac{1}{2} \sigma_z \Delta x \Delta y \epsilon_z \Delta z
  \]

\[
\Delta U = \frac{1}{2} \sigma_z \epsilon_z \Delta V
\]

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- Normalize by Volume:

\[ U = \frac{\Delta U}{\Delta V} = \frac{1}{2} \sigma \varepsilon \]

"Strain Energy Density"

- If material is linearly elastic:

\[ \sigma = E \varepsilon \]

\[ U = \frac{1}{2} \sigma \varepsilon \rightarrow U = \frac{1}{2} \sigma^2 \left( \frac{\varepsilon}{E} \right) \]

\[ = \frac{1}{2} E \varepsilon^2 \]

(3) Modulus of Resilience:

Consider steel in tension:

Modulus of Resilience:

\[ U_R = \frac{1}{2} \sigma_{PL} \varepsilon_{PL} = \frac{1}{2} \sigma_{PL}^2 \left( \frac{\varepsilon}{E} \right) \]

"Intuitively": Ability of material to absorb energy w/o permanent damage

Modulus of Toughness: Entire area up to fracture

"Intuitively": Provides sense of how much warning material give

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**Poisson’s Ratio**

When material is loaded, it will deform (assume elastic)

![Diagram showing deformation](image)

2 Deformations:
- Get deformation in direction of load
- Get deformation perpendicular to load

The Poisson effect measured by Poisson’s ratio:

\[ \nu = -\frac{\varepsilon_{lat}}{\varepsilon_{axial}} \]

Dimensionless number

Usually

\[ \nu \to \frac{1}{3} \to \frac{1}{4} \]

\[ \nu < \frac{1}{2} \text{ (MAX)} \]
6. **Shear Stress - Strain**

If plot $T_{xy}$ versus $\gamma_{xy}$,

\[ T = G \gamma \]

where $G$ is shear modulus of elasticity.

(c) Jerome P. Lynch, 2015
A circular bar of magnesium alloy is 800 mm long. The stress-strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 5.6 mm, and then the load is removed.

(a) What is the permanent set of the bar?
(b) If the bar is reloaded, what is the proportional limit?
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(Hint: Use the concepts illustrated in Figs. 1-18b and 1-19.)

Magnesium bar in tension

\[ \sigma \]

\[ (\sigma_{PL})_1 \]

\[ (\sigma_{PL})_2 \]

\[ \epsilon_R \]

\[ \epsilon_E \]

\[ \epsilon_B \]

\[ L = 800 \text{ mm} \]

\[ \delta = 5.6 \text{ mm} \]

\[ (\sigma_{PL})_1 = \text{ initial proportional limit} \]

\[ = 88 \text{ MPa} \text{ (from stress-strain diagram)} \]

\[ (\sigma_{PL})_2 = \text{ proportional limit when the bar is reloaded} \]

**INITIAL SLOPE OF STRESS-STRAIN CURVE**

From \( \sigma-\epsilon \) diagram:

At point A: \( (\sigma_{PL})_1 = 88 \text{ MPa} \)

\[ \epsilon_A = 0.002 \]

**Slope**

\[ \frac{(\sigma_{PL})_1}{\epsilon_A} = \frac{88 \text{ MPa}}{0.002} = 44 \text{ GPa} \]

**STRESS AND STRAIN AT POINT B**

\[ \epsilon_B = \frac{\delta}{L} = \frac{5.6 \text{ mm}}{800 \text{ mm}} = 0.007 \]

From \( \sigma-\epsilon \) diagram: \( \sigma_B = (\sigma_{PL})_2 = 170 \text{ MPa} \)

**ELASTIC RECOVERY** \( \epsilon_E \)

\[ \epsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{(\sigma_{PL})_2}{44 \text{ GPa}} = 0.00386 \]

**RESIDUAL STRAIN** \( \epsilon_R \)

\[ \epsilon_R = \epsilon_B - \epsilon_E = 0.007 - 0.00386 = 0.00314 \]

(a) **PERMANENT SET**

\[ \epsilon_R L = (0.00314)(800 \text{ mm}) = 2.51 \text{ mm} \]

(b) **PROPORTIONAL LIMIT WHEN RELOADED**

\[ (\sigma_{PL})_2 = \sigma_B = 170 \text{ MPa} \]

A wire of length \( L = 4 \text{ ft} \) and diameter \( d = 0.125 \text{ in.} \) is stretched by tensile forces \( P = 600 \text{ lb} \).

The wire is made of a copper alloy having a stress-strain relationship that may be described mathematically by the following equation:

\[ \sigma = \frac{18,000 \epsilon}{1 + 300 \epsilon} \quad 0 \leq \epsilon \leq 0.03 \quad (\sigma = \text{ksi}) \]

in which \( \epsilon \) is nondimensional and \( \sigma \) has units of kips per square inch (ksi).

(a) Construct a stress-strain diagram for the material.
(b) Determine the elongation of the wire due to the forces \( P \).
(c) If the forces are removed, what is the permanent set of the bar?
(d) If the forces are applied again, what is the proportional limit?
A steel bar of length 2.5 m with a square cross section 100 mm on each side is subjected to an axial tensile force of 1300 kN (see figure). Assume that $E = 200$ GPa and $v = 0.3$.

Determine the increase in volume of the bar.
A steel bar of length 2.5 m with a square cross section 100 mm on each side is subjected to an axial tensile force of 1300 kN (see figure). Assume that $E = 200$ GPa and $\nu = 0.3$.

Determine the increase in volume of the bar.

**Square bar in tension**

Find increase in volume.

Length: $L = 2.5 \text{ m} = 2500 \text{ mm}$

Side: $b = 100 \text{ mm}$

Force: $P = 1300 \text{ kN}$

$E = 200 \text{ GPa}$ \hspace{1cm} $\nu = 0.3$

**AXIAL STRESS**

$\sigma = \frac{P}{A} = \frac{P}{b^2}$

$\sigma = \frac{1300 \text{ kN}}{(100 \text{ mm})^2} = 130 \text{ MPa}$

Stress $\sigma$ is less than the yield stress, so Hooke's law is valid.

**AXIAL STRAIN**

$\varepsilon = \frac{\sigma}{E} = \frac{130 \text{ MPa}}{200 \text{ GPa}}$

$= 650 \times 10^{-6}$

**DECREASE IN SIDE DIMENSION**

$\varepsilon' = \nu E = 195 \times 10^{-6}$

$\Delta b = \varepsilon' b = (195 \times 10^{-6})(100 \text{ mm})$  

$= 0.0195 \text{ mm}$

**FINAL DIMENSIONS**

$L_1 = L + \Delta L = 2501.625 \text{ mm}$

$b_1 = b - \Delta b = 99.9805 \text{ mm}$

**FINAL VOLUME**

$V_1 = L_1 b_1^2 = 25,006,490 \text{ mm}^3$

**INITIAL VOLUME**

$V = L b^2 = 25,000,000 \text{ mm}^3$

**INCREASE IN VOLUME**

$\Delta V = V_1 - V = 6490 \text{ mm}^3$