8.2: Applications to Geometry

Problems

Consider the region $R$ which is bounded by the curves $y = x^2$ and $y = x^4$, between $x = 0$ and $x = 1$. Write an integral for each of the following quantities.

1. The area of $R$.
2. The perimeter of $R$.
3. The volume of the solid obtained by rotating the region $R$ around the $x$-axis.
4. The volume of the solid obtained by rotating the region $R$ around the line $x = -1$.

For the next two problems use the table of values of $f(x)$ and $f'(x)$ provided below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5.5</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>2</td>
<td>1.5</td>
<td>1.2</td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

5. Estimate the length of the arc defined by the graph of $f(x)$ between $x = 0$ and $x = 2$.

6. Let $R$ be the region bounded by the $x$-axis, the graph of $f(x)$, and the lines $x = 0$ and $x = 2$. Estimate the volume of the solid obtained by rotating the region $R$ around the $x$-axis using the trapezoidal rule.

7. Find the volume of the region bounded by $y = e^{-x/3}$, the $x$-axis, the $y$-axis, and the line $x = 5$ and whose cross sections perpendicular to the $x$-axis are equilateral triangles.

8. Consider a solid $S$ whose base is the region bounded by the circle $x^2 + y^2 = 4$ and the $y$-axis with $0 \leq x \leq 2$ in the $xy$-plane, and whose cross-sections perpendicular to the $x$-axis are half ellipses. The major and minor axes of the ellipses satisfy $a = \frac{1}{4}b$. The area of an ellipse is $A = \pi ab$. Write a definite integral of the solid $S$. 
Answers

1. $\int_0^1 x^2 - x^4 \, dx$  
2. $\int_0^1 \sqrt{1 + 4x^2} + \sqrt{1 + 16x^6} \, dx$  
3. $\pi \int_0^1 x^4 - x^8 \, dx$  
4. $\pi \int_0^1 (\sqrt{y} + 1)^2 - (\sqrt{y} + 1)^2 \, dy$  
5. 3.228044  
6. 103.084  
7. $\frac{3\sqrt{3}}{8} \left( 1 - e^{-10/3} \right)$  
8. $\frac{\pi}{8} \int_0^2 4 - x^2 \, dx$