8. THE QUANTUM THEORY OF RADIATION

To understand how an atom interacts with the electromagnetic field and finally how one atom interacts with another, we need to discuss the field in relationship to its sources. It is convenient to begin with the classical descriptions and then move to the quantum mechanical viewpoint from which the field is seen it its own right as a quantum system that can exchange energy with atoms.

There is a parallel between the oscillating, macroscopic currents found in antennas and the time-varying probability density for the electron in an excited atom. Considering the electromagnetic environment, there is a parallel between the vector of the macroscopic electric field and the probability amplitude for photons in a field of any strength.

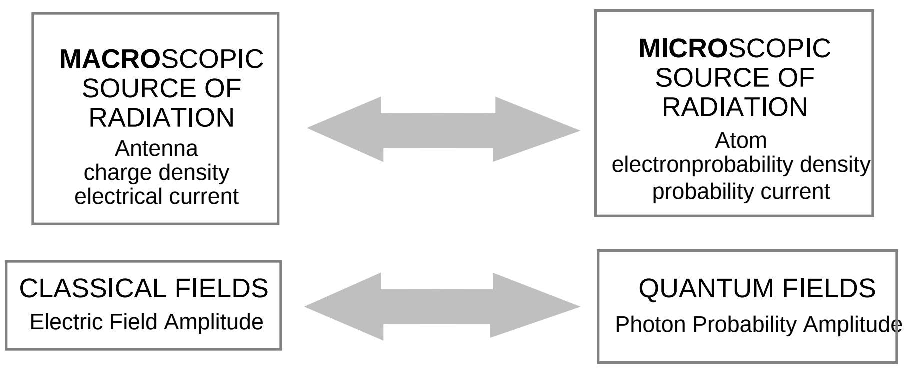

Fig. 8.1

The classical Maxwell equations describe how the periodic motion of a macroscopic electric charge (for example as with a current oscillating in a linear antenna) produces the electric and magnetic vectors of the radiation field. An individual atom can be influenced by the field, but the effect of the atom on the field is not clearly set forth.

In the quantum view, by contrast, the electromagnetic field itself is regarded as a system that can be described in terms of excitation of its normal modes, the occupation numbers for which describe the excited states of the system. The atom is also a system with normal modes, and atom-field coupling leads both the field and the atom to undergo transitions. We find that the quantum description, when used to express the expectation from a highly excited field coupled to many atoms, blends smoothly with the classical description.

8.1 Basic View of Radiation

In the classical view, charged particles that undergo accelerated motion produce electromagnetic radiation. Depending on the nature of the motion, the spectrum of this radiation can appear as narrow lines, as broad lines, or as a continuous distribution.

If the charged particle undergoes simple harmonic motion along a straight line, the resulting transverse field is a pure sinusoid at the frequency of the particle's oscillation; the radiation is said to be *monochromatic*. The particle in this motion comprises a dipole source, the radiation from which is *linearly polarized*. The radiation tends to propagate in a direction normal to the line of the charge's motion, the intensity being proportional to $\sin^2\theta$.

If the harmonically oscillating charged particle is not confined to the line but is allowed to move elliptically in a plane, then resulting radiation, which tends to propagate in a direction normal to the plane of the charge's motion, is still monochromatic but is perceived by a distant observer as being *elliptically polarized*.

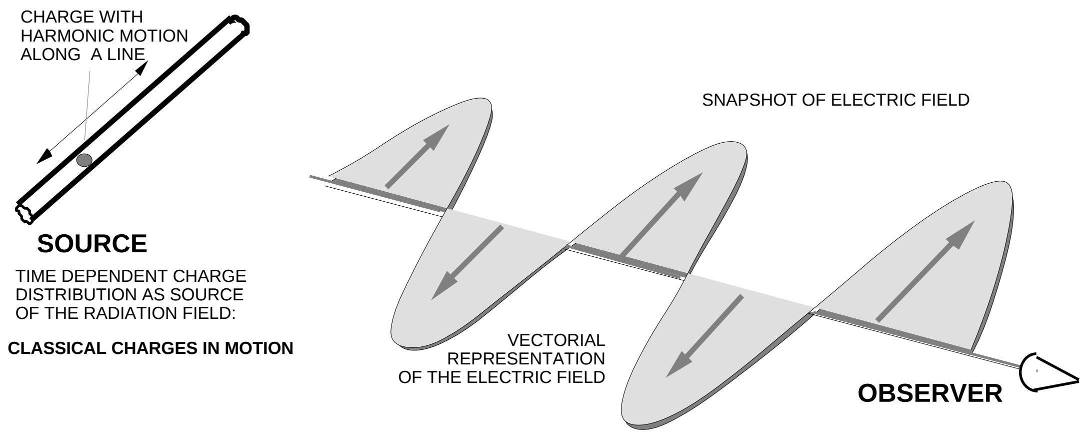

Fig. 8.2 Classical oscillating charge in linear motion produces linearly-polarized radiation

If the motion of the particle is still periodic but not simple harmonic (as, for example, when the restoring force that confines the particle is not linear) then the radiation field is no longer monochromatic but may include both the fundamental and some harmonics of the particle's basic frequency of motion.

If the motion of the particle is *aperiodic* then the radiation is correspondingly less regular. In actual situations we often do not have the ideal case of well-defined electron motion in isolated atoms. Real atoms have neighbors that perturb both the frequency and the polarization of radiation. As already described in Sec. 7, the limiting case is a that of a hot, dense aggregate of atoms in which the electrons have lost their identification with any particular atom and are, instead, undergoing essentially random motion in a way that needs only the kinetic temperature to describe it. In this limiting case the spectrum does not depend on the material with which the charges were associated but only on temperature. We call this *thermal* (or *black body*) radiation and it is described by the Planck distribution (Fig. 7.5.); it has Fourier components over a very wide range of frequencies.

Moving Charges Radiate (Classical Example): Radio Antenna:

A purely classical theory is sufficient to describe the manner in which radiation is generated by the driven motion of charges in metals. In an antenna wire, for example, large numbers of nearly free electrons move in synchronism under the influence of the RF voltage from a transmitter. This charge motion is described as though it were a current.

Moving Charges Radiate (Quantum Example): Electron in an Atom.

Quantum mechanics is able to describe how an atom, with its "stationary" states, can participate in a radiation process. We will show (in chapter 12) that transitions between quantum states involve probability currents in atoms that are quite analogous to the ordinary current in a classical antenna. We will show that during a transition:

a) The function that describes the atom's electron distribution oscillates systematically at the frequency associated with the emitted/absorbed radiation.

b) The amplitude of the oscillations dies away until eventually the electron distribution function is a stationary pattern in the form associated with the final state of the transition.

c) The motion in the atom's electron distribution arises from coupling with the radiation field.

The Two-Level Model for the Atom

It is sufficient to regard the atom as a microscopic entity that has only a ground state (#1) and a single excited state (#2); such a "two-level" atom absorbs or emits only near its intrinsic frequency $\omega = \Delta E_{12}/\hbar$. An aggregate (i.e. one or more) of identical two-level atoms is a system that can exchange energy with the surrounding field.

8.2 The Field Regarded as a Quantum System

An elementary view is that an electromagnetic field is produced by some charges that are considered the source of the field, but that the behavior of those source charges are essentially unaffected by the field. In this view the field is considered as an entity that is subordinate to its source.

The more useful view, necessary if one is going to understand modern experiments, regards the source and the field as being able to exchange energy with one another.

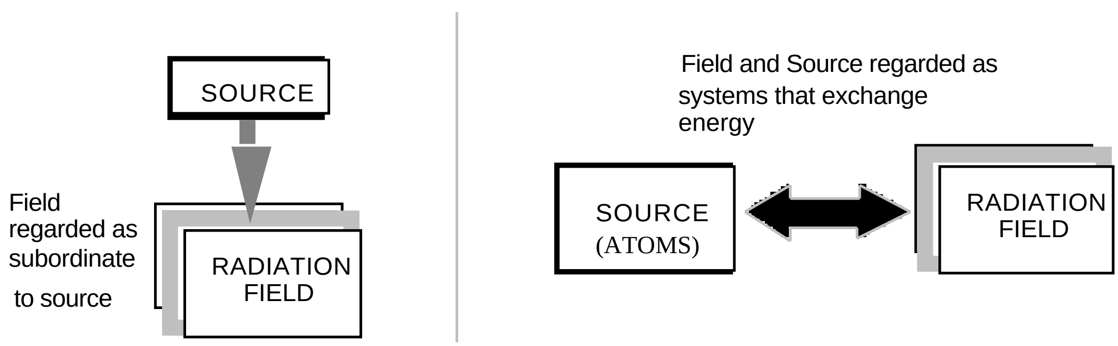

Fig. 8.3 Classical and Quantum Views of the Source–Field Relation

The simplest field is that of radiation propagating between two mirrors, with the dimensions of the mirrors being small compared to their separation, so that only modes along one direction need be considered.

The simplest model atom is a structure that has only a ground state $|0\rangle$ and an excited state $|1\rangle$. It responds significantly to perturbations only when the frequency of the perturbation is very near to $\omega = \Delta E_{01}/\hbar$

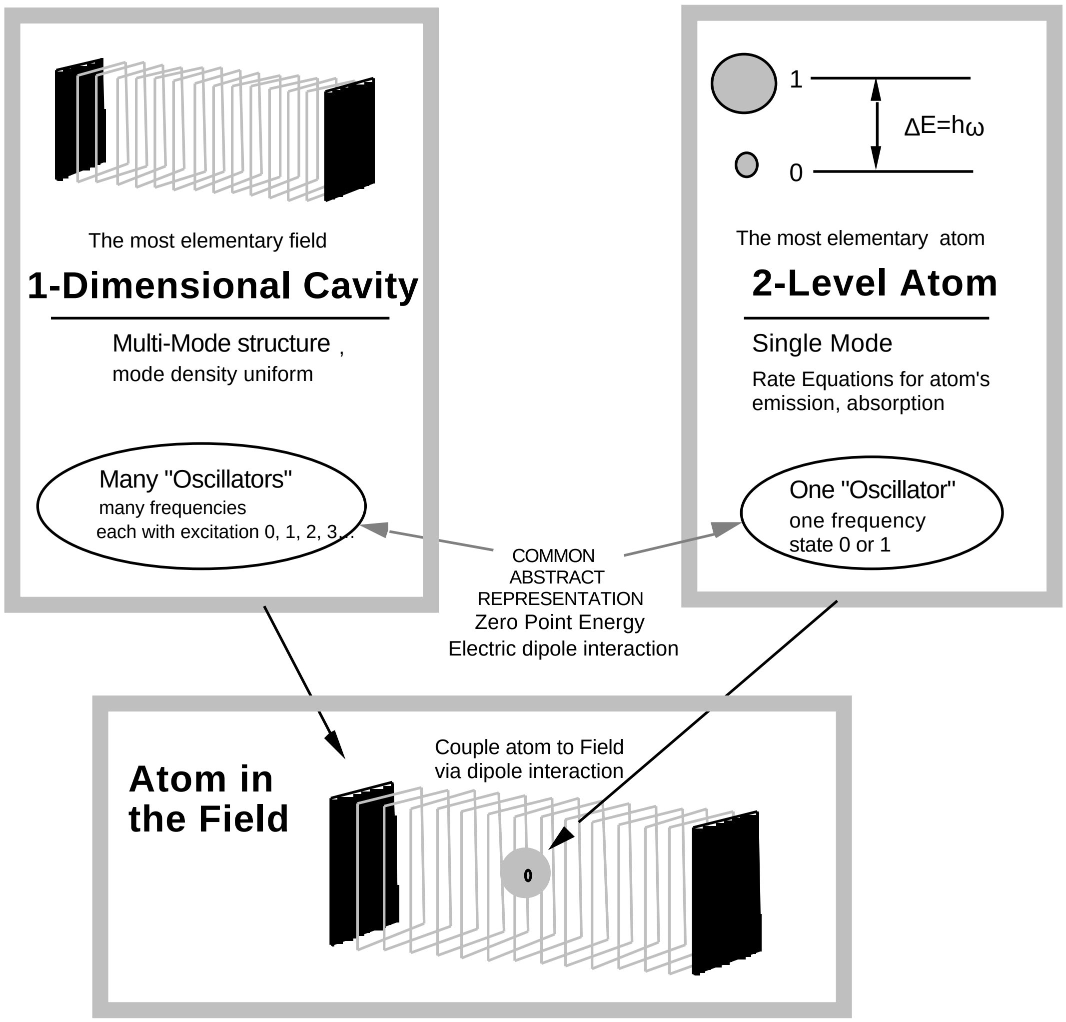

Fig. 8.4 Elementary quantum model for a 2-level atom in a one-dimensional field.

We now make a conceptual transition to a more general view in which the electromagnetic field is itself regarded as a quantum mechanical system that has its own eigenstates and energy eigenvalues. We describe the field with a generalized representation of oscillating systems, a representation that can also be used for the atoms. With both the field and the atom described in the same representation, we have a unified quantum theory for the behavior of light and matter in actual experiments. This more general view shows spontaneous emission to be a consequence of equipartition in the atom+field system. It shows us how the properties of stimulated emission underlie the operation of lasers. It explains the distribution of radiant energy within a field that is in thermal equilibrium.

8.3 THE FIELD AS A SUPERPOSITION OF NORMAL MODES

The theory outlined here is presented in more detail by Sargent, Scully, and Lamb in chapter 14 (pp. 222-241) of their book *Laser Physics* (Addison Wesley, Reading, Mass. 1982).

General Approach

We regard the electromagnetic field as existing within a large, empty volume [often referred to as a *cavity*] assumed bounded by a perfectly conducting enclosure. An arbitrary field within the cavity can be described, in the Fourier manner, as a superposition of excitations of the cavity's normal modes.

For simplicity we first consider a field propagating back and forth along only one direction (this will sometimes be referred to as a *one-dimensional* radiation field) with reflecting end boundaries. An analysis of this simple field shows many of the salient features, and the examples it provides are easily (well, fairly easily) generalized to two and three dimensional fields.

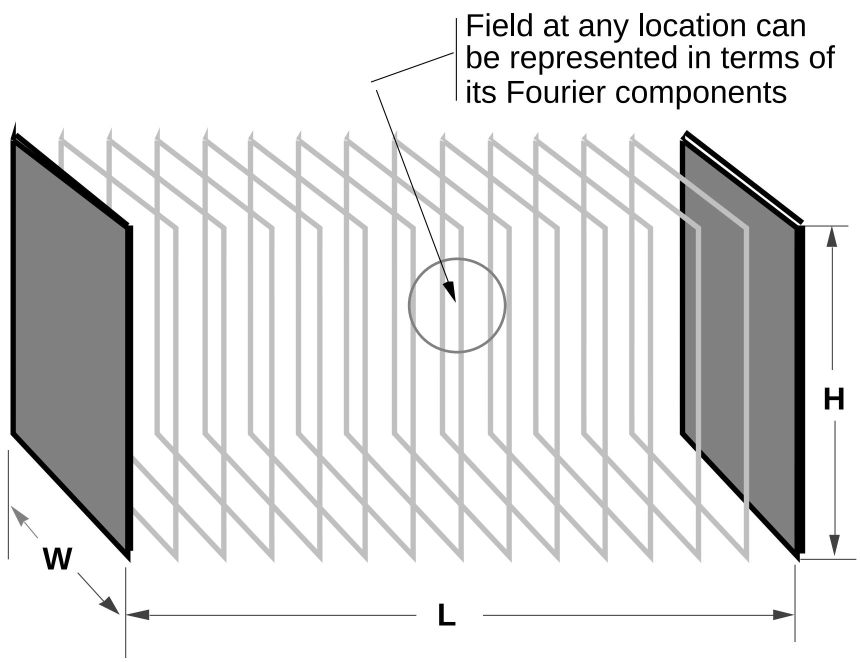

Fig. 8.5 Electromagnetic Field in Basic Resonator

Modes in a simple, one-dimensional resonator

Two parallel reflectors spaced L apart will support standing waves at selected frequencies, and for this reason the configuration is called a resonator (if the structure uses highly reflecting surfaces to work with microwave, infrared, or optical radiation, it is sometimes called a *Fabry-Perot* resonator). The preferred modes are those associated with sinusoidal standing waves that vanish at the boundaries 0 and L; they are called the *normal modes* of the resonator and their frequencies are integer multiples of the reciprocal of the time it takes a signal to go from one reflector to the other and then return.

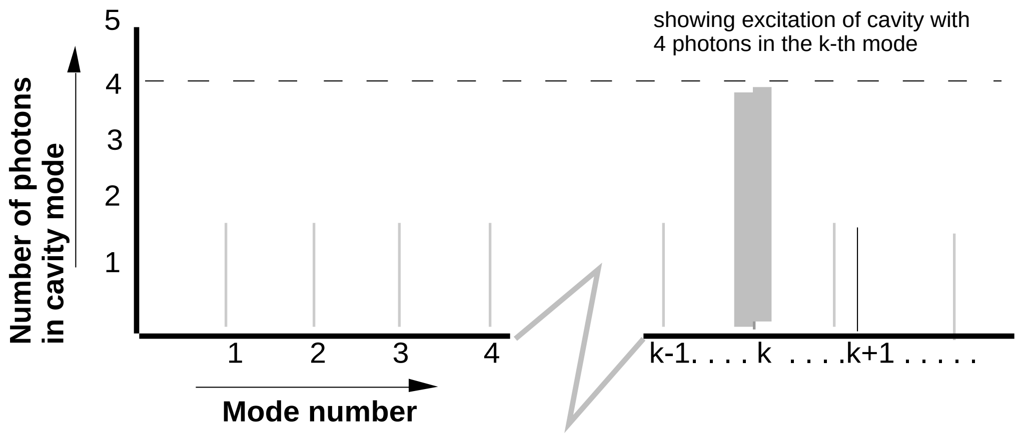

Fig. 8.6 Field described by a single excited mode

As is true for acoustic resonators of analogous construction (closed end organ pipe or stretched string), this structure can have several, or many, modes excited at the same time. Indeed, by using Fourier methods, we can represent any field that exists within the cavity with a superposition of normal modes. The mode frequencies along any given axis form a set $\{\omega_m\}$, the members of which are integer multiples of the fundamental: $\omega_m = m \times \omega_1$. When representing a particular field within the cavity, one finds the relative contribution from each of the modes by the methods of Fourier analysis.

$$\vec{E}(x,t) = \sum_m E_m^o \; e^{i\left(k_m x - \omega_k t\right)}, \tag{8.1}$$

where the mode frequencies are:

$$\omega_m = m \left(\frac{\pi c}{L}\right) = m \quad \omega_1 \tag{8.2}$$

Photon Numbers Represent Available Energy in a Modes

To discuss the state of the resonator in terms of its energy content, we specify the number of photons as distributed over the various resonator modes. The energy *available* in the m-th mode of the cavity can be represented by n_m photons, each with energy $h\omega_m$. The integer n_m specifies the photon number (or occupation number) of the m-th mode. The total *available* energy of the field is then given by

$$W_{AVAILABLE} = \sum_m n_m \quad \hbar\omega_m \tag{8.3}$$

We distinguish between the available energy and the total energy because (as we will find in Sec. 17) the quantum mechanical analysis of harmonic oscillators shows that each field mode has a *zero point energy* of $\hbar\omega_m/2$ in addition to its available energy $m \times \hbar\omega_m$ ascribed to the photons. The t*otal* energy in the field is then:

$$W_{TOTAL} = \sum_m \left(n_m + \tfrac{1}{2}\right) \hbar\omega_m \tag{8.4}$$

Zero-Point Energy

The existence of the zero-point energy, is an unavoidable consequence of the current (1994) formulation of quantum theory, and it implies that no mode of the cavity is ever completely quiet. Although this energy is not available in the usual sense, it nevertheless has observable effects, notable among which are the limiting of excited state lifetimes [e.g. "spontaneous" decays] and the displacement of atomic energy levels [e.g. the Lamb shift].

The existence of the zero-point energy implies that every possible mode of the cavity has at least $\hbar\omega_m/2$ of energy associated with it.. This is awkward because the sum in (8.3) is over all possible modes, and there are infinitely many of them. This means that the expression for total energy, as written, is divergent. This is a theoretical awkwardness , a crack in the gleaming mirror of quantum electrodynamics that remains even 70 years after its development.. And since it remains, we work around it by noting that observables are associated with differences in energies, and in computing those differences , the infinities fall away:

$$\Delta W_{OBS} = \underbrace{W_{TOTAL}}_{\infty} - \underbrace{W'_{TOTAL}}_{\infty} = \underbrace{\sum_m \left(n_m - n'_m\right) \hbar\omega_m}_{finite} \tag{8.5}$$

In principle the sum Σ_m runs over all possible modes, but we can argue that the highest value of m that needs to be considered is set by the maximum value of available energy in the system, If all the available energy were in a single photon, $\hbar\omega_{MAX}$ then

$$\omega_{MAX} = \frac{W^{MAX}_{AVAILABLE}}{\hbar} \longrightarrow m_{MAX} = \frac{\omega_{MAX}}{\omega_1} = \frac{W^{MAX}_{AVAILABLE}}{\hbar\omega_1} \; . \tag{8.6}$$

8.4 Distribution Of Available Energy In The Modes Of A Cavity

Observables only involve changes in the available energy, so we will shift notation by omitting the subscript label $W_{AVAILABLE} \rightarrow W$ for available energy in the discussion that follows.

The energy within a given cavity of length L may be apportioned in different ways,

–A possible but unlikely distribution would be one of Eq. 8.5 above in which the entire available energy is tied up in a single photon with wavelength λ'=hc/W and mode number m'=W/[hc/2L].

–At the other extreme, the cavity energy might all be concentrated in photons of the lowest mode (λ=2L), so that the occupation number of the lowest mode would be n1=W/(hc/2L).

Between these two extremes there are many other possibilities, each described by a set of integers $\{n_m\} = n_1, n_2, n_3$ …that are constrained only by the condition that all of the available energy [Eq. (8.3)] be accounted for:

Monochromatic Distribution: [See Fig. 8.7] The distribution of energy among the modes, described by the set $\{n_m\}$, depends on the manner in which energy is supplied to the field. If the cavity is undergoing excitation at a well defined frequency ω' then only modes very near in frequency to that excitation ($\omega_m \approx \omega'$) will have occupation numbers n_m of significant size.

If the spacing between cavity modes is large compared to the spectral width of the exciting radiation, then we expect that only one mode of the cavity will be excited. This is often seen when the cavity dimensions are not much larger than the wavelength of the exciting radiation.

If the intermode spacing is about the same as, or smaller than the linewidth of the exciting radiation (common in optical resonators where cavity dimensions are usually much larger than the wavelength) we expect to excite several modes simultaneously, but these will all be at frequencies quite near to that of the exciting radiation.

Where found

Monochromatic and single mode fields are found in radiofrequency systems (AM, FM transmitters) in microwave resonators (radars, microwave communication links, maser frequency standards) , in infrared systems (spectrometers, lasers), and in the regime of visible light (lasers, interferometers).

The application of monochromatic fields at higher frequencies is less common because we have yet to learn how to adequately control this radiation, and because the dimensional tolerances on the resonant structures are progressively harder to maintain as the wavelengths get shorter and shorter.

Nearly Monochromatic: This is a very common distribution in which only a few, closely-spaced modes of the resonator are excited. (Indeed many distributions advertised as "monochromatic" (e.g. in usual Helium-Neon laser). An aggregate of excited atoms will, because they have finite lifetime, produce a spectral line of finite width; if this line is used to excite an optical resonator then it is likely that several modes of the resonator will have energy. Indeed the operational distinction between "monochromatic" and "nearly monochromatic" depends on the finesse of the measuring instruments. Indeed radiation sources that were taken to be standards of monochromaticity several years ago are now regarded as being inadequate in that respect. (see the discussion about *cavity Q factors* adjoining).

Fig 8.7: Monochromatic and nearly monochromatic spectra

Fundamental plus Harmonics: If there are any non-linearities in a cavity being supplied with energy at frequency ω', then harmonics (energy at integer multiples of ω') are likely to be present in the field. Non-linear responses are employed for harmonic generation in systems designed to generate sub millimeter radiation from microwave power. Michigan physicists were the first to generate harmonics at optical frequencies.

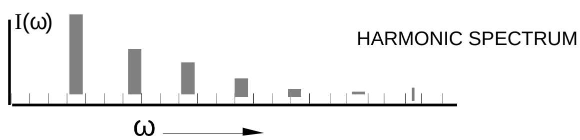

Fig 8.8: Fundamental with harmonic overtones

Polychromatic, with Sum and Difference Frequencies:

A cavity can be excited at two or more distinct frequencies, ω' and ω", and the power spectrum will show two peaks. Again, however, if non-linearities are present in any part of the system, one can get not only harmonics of ω' and ω", but also sums (ω' + ω") and differences (ω'-ω") in the spectrum of radiation energy from the cavity.

Where found: A cavity that sustains two different oscillations can be used to provide a reference resonance at one frequency can have its dimensions stabilized against thermal drift by locking its length (via feedback loops and piezoelectric controllers) to a standard spectral line. A cavity with non-linear elements can generate difference frequencies that allow one to demodulate information that is on an optical carrier wave.

Very Broad, but non-Thermal Distribution: If the cavity is excited by a non-monochromatic source (perhaps a sodium vapor lamp operating at pressures so high that the lines are very broad) then the field will have a broad distribution that is often far from a line spectrum but still not describable in thermodynamic terms.

Where found: Radiofrequency energy from a faulty ignition spark system, cheap microwave heating systems, and infrared illuminators. Fluorescent lamps, mercury and sodium vapor lamps are designed to save energy by producing visible light without generating all the infrared that a thermal source of comparable brightness would entail.

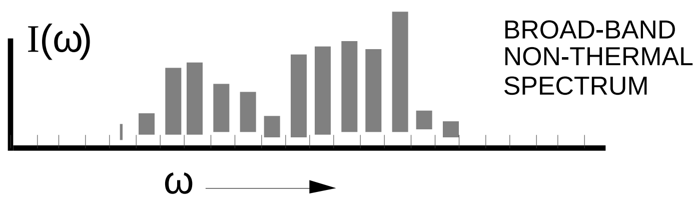

Fig 8.9 Broad spectrum from non-thermal source

Thermal Distribution: If the cavity acquires random energy in a thermodynamic process, for example by heating with a flame, then any set $\{n_m\}$ of occupation numbers consistent with the known value of total energy is equally likely. In that case it can be shown that the output spectrum follows the Planck distribution which has only temperature and total energy as its descriptive parameters.

Where found: The radiation from deep space appears to follow a Planck distribution quite closely with a peak in the microwave region; the anisotropies in this radiation, although quite small, provide important information about clustering of objects in the early universe. The thermal radiation from biological objects peaks in the far infrared and can be used for diagnostics. The temperature of rolled steel is controlled by optical pyrometry. Ordinary incandescent and quartz-halogen lamps produce a thermal spectrum. And our knowledge of temperature of astrophysical objects is deduced entirely from analysis of their broad, thermally distributed radiation.

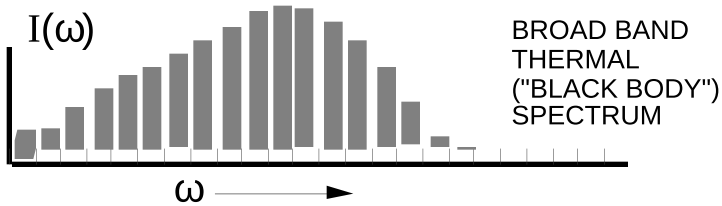

Fig 8.10 Showing many modes with thermal distribution of excitation

Comments on Color Temperature of Non-Thermal Sources

As mentioned earlier, the visible radiation from a broad band source is often described with a color temperature T_c to indicate that the distribution of radiation from that source creates a good color match (in the human eye) to the sensation of color obtained from an ideal thermal source that is at a thermodynamic temperature T numerically equal to T_c.

The correspondence between color temperature and thermodynamic temperature does not hold for all broad-band sources. For example, the rationale for fluorescent lamps is that they produce a power spectrum that somewhat approximates that of daylight within the visible region, but they produce *much* less infra-red than an incandescent lamp of the same visual brightness. The incandescent lamp, being just a hot tungsten wire, is a thermal radiator and puts less than 25% of its power into the visible ["lumens per watt" rating is a measure of this] , and a fluorescent may exceed 50% conversion of input power to useful and relatively pleasant illumination.

There is a huge difference between a fluorescent lamp's color temperature and its apparent thermodynamic temperature. : For example, the 40 watt fluorescent lamp tubes sold for shop lamps have a typical color temperature of 5200K, but the lamp tube is only just warm to the touch, and the thermodynamic temperature of the hottest object in that lamp is much less than 2000 K. Moreover, a 40 watt, 5200K fluorescent lamp runs cooler than a 40 watt incandescent lamp that has a 3000K color temperature.

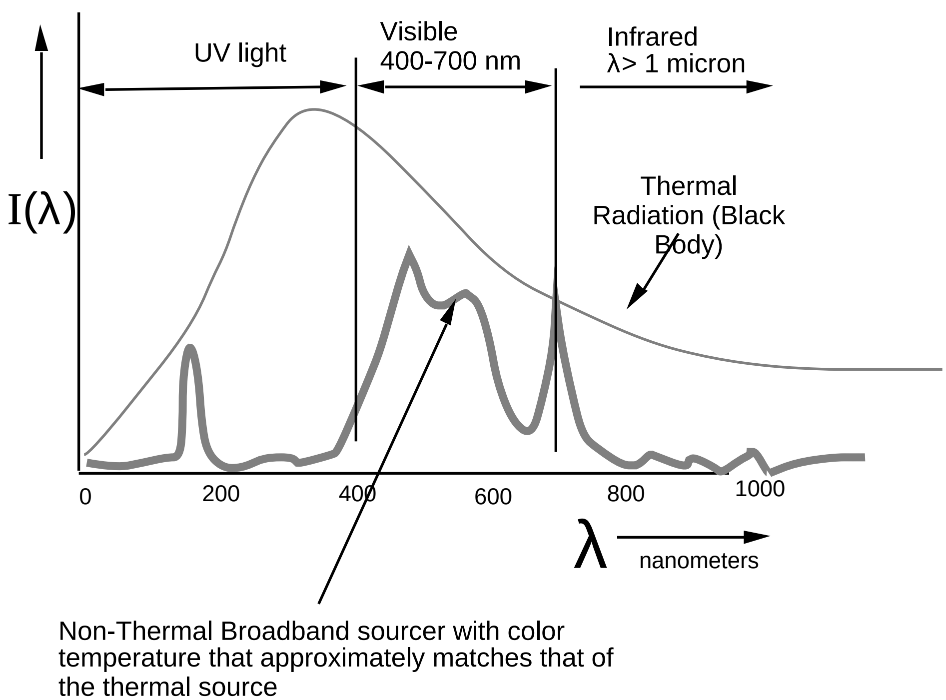

Fig 8.11 Comparison of Thermal and Fluorescent Lamp Spectra

The luminous efficiency of a fluorescent lamp is exceeded by sodium vapor lamps and mercury vapor lamps. These orange and blue lamps are used for streets, parking structures, and in other applications where their economical operation makes up for their harsh, unpleasant spectra.

8.5 MODELS and REPRESENTATIONS for FIELDS

The traditional representation for the electromagnetic field is of plane waves propagating between plates. The early studies on with visible water waves lead us to this, and we are reinforced in our view by experience with acoustic waves. Our conclusions were confirmed by many experiments on electromagnetic radiation in the radio, microwave, infrared, and optical regimes. The photoelectric effect and the persistence of interference in the weak flux limit brings us to a re-examination of the traditional views as discussed in Sec. 2

Our observable, in the simplest case, is that an electromagnetic field mode changes its energy in a stepwise fashion:

$$\Delta W_{mode\ m} = \Delta n_m \quad \hbar\omega_m \tag{8.7}$$

But we also know that a quantized mechanical harmonic oscillator (as represented by a spring & ball, or a pendulum, or torsion pendulum, or …) has the same energy level structure as a single mode of the electromagnetic field in a simple resonator.

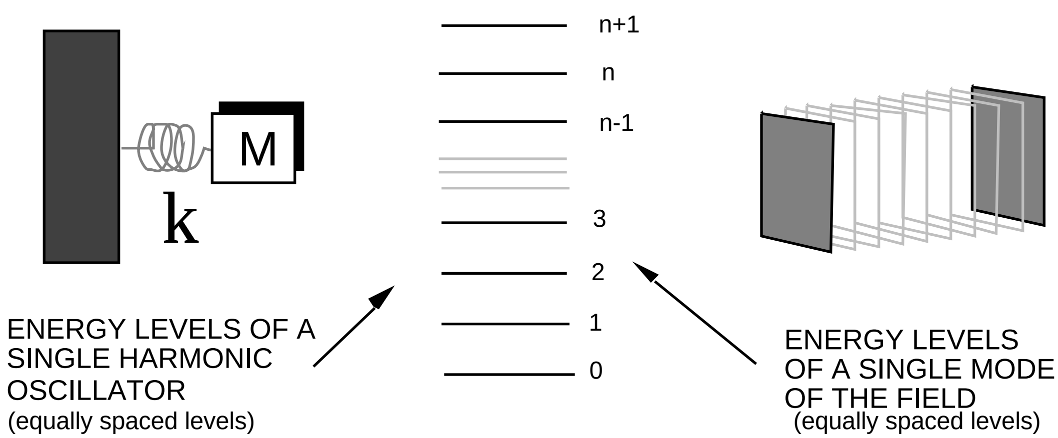

Fig. 8.12 Mechanical Oscillators and Electromagnetic Field Modes both have equally-spaced energy levels

Since the oscillator and the field mode have similar observables, their theories should have much in common. But finding this commonality is difficult if we insist on a highly pictorial view of either system. We need a representation that is sufficiently general to encompass them both.

> The construction of theories spanning apparently diverse mechanisms has been done in many disciplines. Gambling provides us with a simple example: There is a theoretical model that predicts the outcomes of games played with very different physical objects (dice, roulette wheel, cards, electronic random number generators, etc.). That model concerns itself only with observables and relevant parameters; it uses an abstract formalism and it ignores most of the actual physical characteristics of the game apparatus itself. Our approach to a common theory for fields and atoms is on this pattern.

Why bother to look for a theory that handles both atoms and fields?

The traditional semi-classical theories for atoms and for fields have the advantage that their elements are easily pictured: a mass whizzing back and forth; a wave propagating in space. But the difficulty arises when one tries to describe the interaction of the two in a manner consistent with observations. Our literal wave picture does not have any mechanism to generate energy quantization, and our atom has no way to decay spontaneously. So we work on the more abstract theory that will encompass both atom and field. We have a correspondence between an oscillator and a field mode

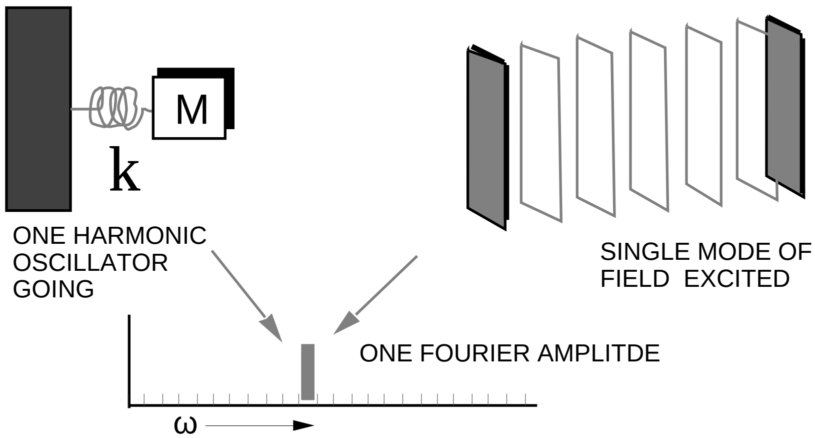

Fig 8.13 Emission-absorption of 2-level atom coincident with a single field mode

And an arbitrary excitation of the field can be regarded as a collection of excited oscillators:

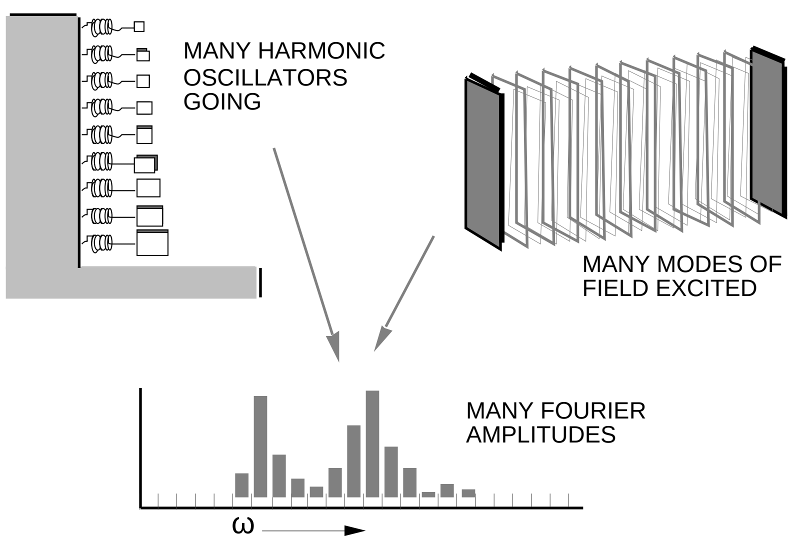

Fig 8.14 Multimode excitation of one-dimensional field

Of course, someone may look into the resonant cavity and say "I don't see the oscillators", to which you must reply "I don't see the waves either"

The observation that field modes and mechanical oscillators both have equally-spaced energy levels leads to a deeper underlying theory that encompasses both of these apparently dissimilar systems. This is more than just a philosophical exercise because using the harmonic oscillator representation for the field enables us to understand that an atom can never be considered as completely decoupled from its surroundings. Even if the occupation number of the k-th mode is zero we find that an average of measurements of E for that mode will be non-zero. This is the zero point energy, and a semi-classical wave theory of the field does not easily show the existence or magnitude of this significant contribution to the physics of free atoms. Spontaneous emission is one consequence of the zero point energy: An atom in its excited state is a large, delicate structure that will collapse to its smaller ground state upon slight simulation ... and we find that the giant zither of the electromagnetic field, its modes never truly quiet, will stimulate that collapse sooner or later.

Later we will find that the abstract representation used to describe the electromagnetic field and the simple atom is in fact capable of being extended to cover many other physical systems. A powerful unity is gained by recognizing that a given formalism is able to describe a wide variety of entities that seem, at first glance, to have nothing in common.

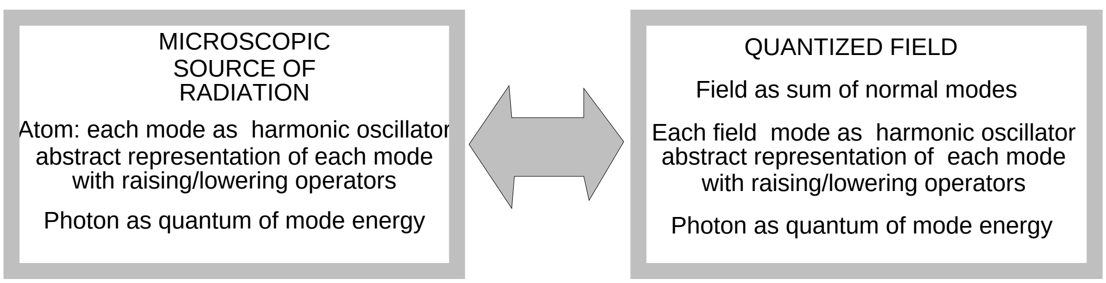

Fig. 8.15 Field and Atom considered on equal footing
Quantized source and quantized field are most likely to exchange energy when the mode of the field is nearly in resonance with the mode of the atom.

8.6 When is a Cavity "One Dimensional"?

Consider a cavity in the form of a hollow rectangular box with length L along x, width W along y, and height H along z, We make the usual assumption that waves may propagate independently along any one of the natural axes (x,y,z). The velocity of propagation in any direction is c. We impose the boundary condition that the wave amplitude F(x,t) vanishes at the walls. The normal modes in the x direction have amplitudes of an obvious form and have frequencies that are integer multiples of a fundamental:

$$F_n(x,t) = F_n^o \sin\left(\frac{n\pi}{L}x\right)\sin(\omega_n t) \qquad \textit{frequency} \quad \omega_{nx} = \left(\frac{2\pi\ c}{L}\right) n \qquad (8.7)$$

with similar expressions for waves along y and z. The fundamental frequencies along the three directions are:

$$\omega_{1x} = \left(\frac{2\pi\, c}{L}\right) \qquad \omega_{1y} = \left(\frac{2\pi\, c}{W}\right) \qquad \omega_{1z} = \left(\frac{2\pi\, c}{H}\right) \tag{8.8}$$

If the dimensions L, W, and H are roughly the same, then an excitation of the cavity could lead to waves propagating along any of the three directions. However if the length L exceeds the height or width, then low frequency excitations are limited to waves propagating along x. For example if our cavity is in the form of a pipe of square cross section (H=W and L=20×H) then we can excite the first 19 longitudinal modes of the cavity before the transverse modes even enter the picture. At frequencies below $20\times\omega_{1x}$, therefore, the cavity can be considered as a system with only one relevant dimension.

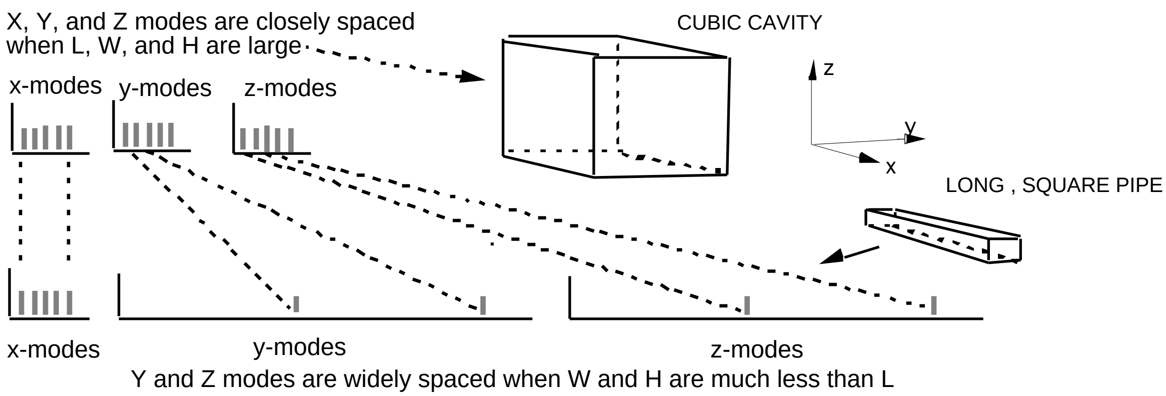

Fig. 8.16 Compression of height and width spreads out the moves of a cavity

Examples of this are plentiful. Organ pipes, flutes and other wind instruments are acoustic resonators that operate on longitudinal excitations; their transverse modes are far above the usual range of hearing.

A resonant cavity responds when one perturbs it near one of its normal mode frequencies, so the long thin tube responds only to its x- mode frequencies until the perturbation gets to very high frequencies (i.e. high energies) that resonate with one of the y- or z-modes. It is interesting to consider the conceptual transition between a "two-dimensional" and a "one-dimensional" resonant cavity.

If we have the situation where the cavity length and width are comparable, but the cavity height is much smaller (for example, a shallow, square box L=W » H), then x- and y- excitations will dominate. The x- and y- modes are spaced by an amount that depends on $(1/L_x)$, and $(1/L_y)$, respectively. (the very small z-dimension assures that the z-modes are at a very high frequency and will not be excited). The state of the field depends on which x and y modes are excited and how many photons are in each mode..

At lower frequencies, therefore, the cavity will show up with two sets of normal mode frequencies from which the characteristic x and y dimensions can be found. The compression of the y dimension causes the y-mode spacing to increase. With sufficient compression even the lowest y-mode will be at a very high frequency, This implies (for an electromagnetic field resonator) that y-mode photons will have such high energy that ordinary excitation will not provide them. In that case, the cavity behaves more and more like a system with only one available direction for propagation. Operationally speaking, the cavity has evolved from a two-dimensional to a one-dimensional system. .

From the standpoint of an experimenter who limits the probing radiation to low frequencies, then, the response of the long tube suggests that it only has one dimension. Most of these arguments apply also to acoustic resonators and, as we shall discuss later, to the quantum mechanical description of particles confined in potential wells. (e.g. quantum wells, quantum wires, and quantum dots). In addition, the experimenter who uses thermodynamic methods may find that the specific heat of the system is now different because the degrees of freedom have changed.

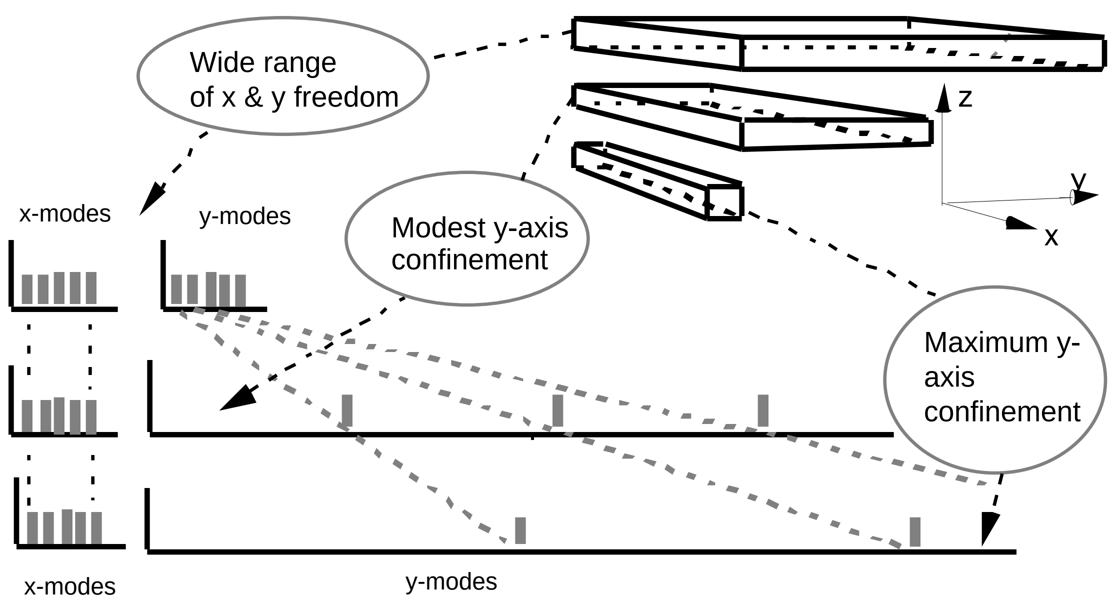

Fig 8.17 If width is increased to match length then both x and y modes are closely spaced.
the small value of H makes the z-modes so widely spaced that they are not shown here.

The energy associated with he field quantum (whether photon or phonon) is proportional to the frequency $\omega_{n,}$ and taken together with the above discussion, we have the interesting principle:

> *Structures that appear to have only a few degrees of freedom when explored with low energy experiments often exhibit additional degrees of freedom when the experiments are extended to higher energy.*

We have been discussing the cavity modes as though their frequencies were defined to within arbitrarily small limits. In fact the modes have a finite width that arises from dissipation of the mode energy. A mode, once excited and then left alone, tends to die away at a rate that depends on the reflectivity of the cavity walls, the interaction of the mode radiation with atoms within the cavity, and similar loss effects

The parameter Q ["Quality"] is often used to characterize the spectral line.... it is approximately equal to the number of oscillations that occur before the amplitude has gone to 1/e of its original value.

$$Q \approx \frac{f}{\delta f} \qquad F(t) = \underbrace{F_0 \cos(\omega t)}_{\text{steady component}} \times \underbrace{e^{-\gamma t}}_{\text{decay factor}} \qquad \underbrace{\gamma}_{\substack{\text{decay}\\ \text{rate}}} = \frac{1}{\underbrace{\tau}_{\text{lifetime}}} \tag{8.9}$$

The spectral lines associated with atoms have finite widths (and are also characterized by Q numbers) that arise from finite lifetimes of the excited states. The usual limiting factor is the occurence of spontaneous transitions, and these will be discussed in the next section.

8.7 Mode Densities in 1, 2 and 3 dimensional cavities

[Calculations for this section not available for this printing of the notes]

In a three-dimensional cavity, the available energy is a sum over the modes in each of the three directions:

$$W_{AVAILABLE} = \sum_{j=1}^{\overbrace{3}^{\substack{\text{sum over}\\ \text{directions}}}} \underbrace{\sum_{k=1}^{\infty} \overbrace{n_{jk}}^{\substack{\text{occupation}\\ \text{number of}\\ k-\text{th mode}}} \overbrace{\hbar\omega_k}^{\substack{\text{mode}\\ \text{photon}\\ \text{energy}}}}_{\text{total enegy in } j-\text{th direction}} \tag{8.10}$$

The occupation numbers n_{jk} specify the energy distribution. They are determined by the way one excites the cavity. Laser excitation may lead to population of just a few modes, while heating the system with a blowtorch will yield a thermodynamic distribution in which all of the n_{jk} are relevant.

The atom will respond only if its resonant frequency has significant overlap with one or more of the natural modes of the cavity. In some situations where the physical dimensions of the cavity are on the order of the wavelength of the atomic transition (e.g. the NH_3 maser or the atomic hydrogen maser) only one mode of the cavity is relevant. In other cases, including most situations involving optical radiation, the atomic linewidth spans many modes of the cavity, and the density of cavity modes (number of modes per frequency interval) is then an important parameter for analysis of the situation in which an atom is within the cavity.

8.8 The Concept of "Elementary" Particle

A particle is said to be "elementary" when it does not exhibit internal structure as it interacts with its surroundings. In this sense, a helium atom is elementary for processes involving energies less than the 19 eV it takes to reach the lowest excited state of its electronic structure. Other atoms and molecules have excited states at lower energies, but for each species there is an energy beneath which the behavior can be described as though the atom or molecule were just a mass point undergoing elastic collisions.

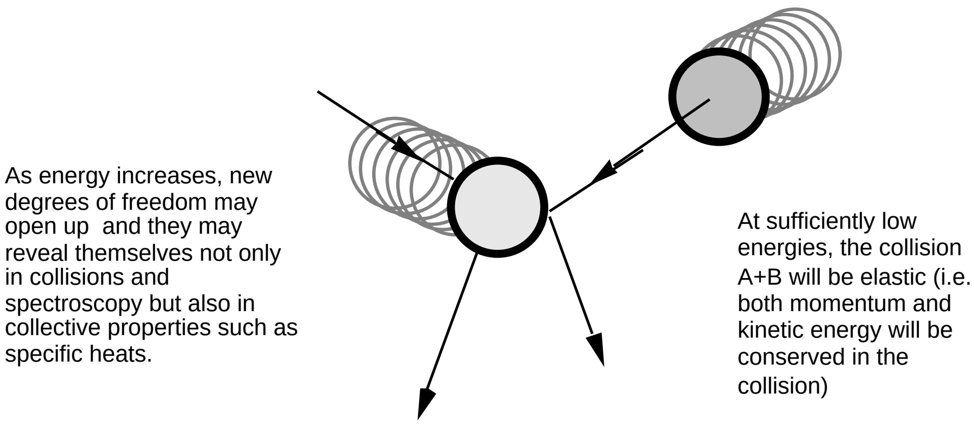

Fig 18.18

Processes involving tens or hundreds of eV will show the electronic structure of the target, but until one goes beyond 250 keV or so, one can still regard the nucleus as an elementary structure characterized by the static properties of mass, spin, magnetic dipole moment and electric quadrupole moment.

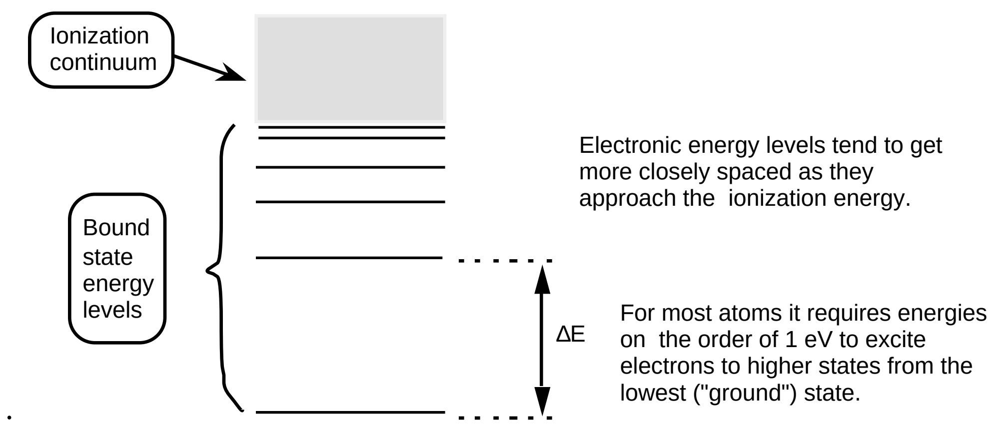

Fig 18.19: Processes with energies less than ΔE will not reveal internal properties of the target

The concept of "elementary" is thus somewhat more applicable to electrons and muons .(still not exhibiting internal structure) than to protons and neutrons whose interesting behaviors motivate the construction of the highest energy accelerators.