3. PREDICTION AND OBSERVATION OF EVENTS

3.1 Nature of Measurement

It is important to focus on quantities that can be observed in experiments.

3.1.1 Understanding What One Actually Measures

In physics we find that dramatically new insights can emerge from a careful interpretation of the process of measurement . For example, the theory of relativity came from a thorough analysis of the way one actually measures length and time. It started with the experimental evidence that observers in different inertial reference frames always measure the same numerical value for the velocity of light in vacuum. The theory that accommodated that basic observations also predicted the mass-energy relation, a totally surprising consequence of theory that began from almost purely kinematical considerations.

In a parallel manner, quantum mechanics develops from a close examination of the information that experiments give about the atomic world. What is truly observable? What is based on classical preconception? Again, as in our approach to relativity, we concentrate on operationally defined observables and discard unverifiable preconceptions. Then, having developed a theory that accommodates the basic observations, we ask what else the theory predicts: In the case of quantum mechanics, we find tunneling, resonance scattering, exchange forces, and many other surprising consequences that greatly enlarge our understanding of the microscopic world.

The Weak Flux Limit

We begin by discussing experiments where particles (or photons) are passed through fields, guided around obstacles, scattered from targets, and finally detected. It makes discussions easier, and contrasts clearer, if we analyze these experiments in the limiting case when there is, at most, one particle/photon in the apparatus at a time. We call this the *weak flux limit* , and the experimental finding that interference phenomena persist in this limit forces us to confront important issues in our theory of particle/photon behavior.

The weak flux limit is not very extreme. Many experiments in scattering and in spectroscopy are done with count rates well below the weak flux limit. A simple example of observation in this limit is when, on a dark night, you see a 60 watt incandescent lamp at a distance of 1 mile; the lamp is quite visible even though it puts no more than one photon in your eye at any one time.)

3.1.2 Basic Detection of Events on the Atomic Scale

Suppose we are scattering particles (or photons) from a target and that we want to detect those particles s that emerge from the target in a particular direction. This can be accomplished by placing a sensitive element [we will call this the *primary sensor*] at location x in the outgoing particle's path, with a collimator assuring that only particles from the selected direction will be counted:

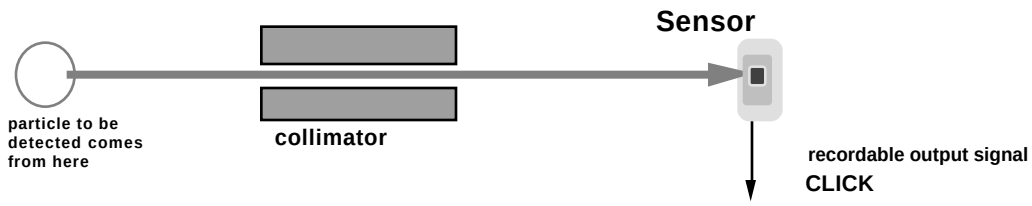


Fig. 3.1 Event (x): Basic particle (or photon) detection

There is a reasonable probability that the interaction of a particle with the sensor leads to a discernible output (denoted here as a "Click") that might be a surge of current from a photomultiplier or Geiger counter, a converted grain of silver halide in a photographic emulsion, or a newly-created ion that promotes nucleation in a bubble chamber.

Peripheral sensitivity

An individual sensor element inevitably covers a certain area, and its sensitivity usually extends over a finite region, the sensitivity falling off with distance in a manner that depends on sensor design. The diagrams here use shading of varying density to indicate relative sensitivity, with the sensor being

—most sensitive to particle paths through the small, dark region,
— less sensitive to paths farther away in the intermediate shaded area,
—only marginally sensitive to paths anywhere in the lightly shaded region.
—insensitive to paths that do not intersect a shaded region.

The detector output click, in itself, provides only minimal information

Fig 3.2: Paths leading to same event

Any of the paths (A, B, C, D) shown in Fig. 3.2 may lead to a click, although paths B and C seem more likely to yield a detector output than do A or D. But the click, once it has occurred, does not distinguish between A, B, C and D; in fact they all have the same event coordinate:

The location of a basic sensor element is specified by a number (coordinate x) that gives the nominal position of the element; it does ***not*** reflect the spatial extent of the element. The only information available from an individual, basic sensor is

(a) the nominal location of the sensor
(b) whether or not a particle arrived.
(c) and, in some cases, an indication of the particle's energy

One does not obtain a discrete click from an individual, incoming particle unless its energy exceeds the activation energy of the sensor. In some cases (e.g. with proportional counters) the size of the click gives a measure of the particle's energy.

In many experiments we employ an array of basic sensors. For example, consider a coarse photographic emulsion in which the basic sensors [small silver halide crystals with typical diam 10^{-4} cm) are dispersed over the surface of a glass plate. A photon incident on the emulsion may be absorbed by one of these crystals which then, upon being developed, turns to metallic silver. The pattern of photon arrival is then seen by experimenter as a pattern of metallic silver dots on the glass. This the available record of events for that experiment. The measured position of the center of each silver grain is the coordinate of the individual event. Note that this coordinate does ***not*** include the size of the grain.

Clicks

A *click* (i.e. a detector output signal) comes from a sensitive element that has spatial location x. A set of such clicks comprises the experimental data that must be sorted to define which of the clicks correspond to events of interest.

Events: When is a click significant?

Should we record all of the signals coming from a sensor? A common problem in doing an actual experiment is to tell whether a signal from the primary sensor really represents the arrival of a controlled particle and should be accepted as an event, or whether it is just system noise or trash background that should be rejected.

To accept a click as valid data event we demand that certain obvious preconditions be met, e.g. that the apparatus is turned on with the system parameters set to the appropriate values. But this informal screening of data is sometimes not enough to discriminate against irrelevant signals, so one often adds filters and auxiliary data counters in order to apply further formal criteria for the acceptance of clicks as actual events for analysis.

Given the availability of easy data storage and fast computers, experimentalists do not hesitate to seek correlations among a dozen or more variables in order to find and measure events from data in which the raw signal-to-noise ratio is 10^{-3} or even less. The extraction of meaningful signals from noise, and the interpretation of those signals involve sophisticated methods including autocorrelation and cross-correlation analyses, and these conceptually resemble the calculation of diagonal and off-diagonal operator matrix elements that we will do later in this study of quantum mechanics.

3.1.3 Detection may be preceded by State Preparation:

It is often the case that the particle (photon) of interest has a known energy, momentum (wavelength), or polarization. One can then reduce the unwanted background counts by adding a state preparation filter [**G**] to the system so that only those particles of interest reach the primary sensor:

Fig. 3.3 Event (x,G): Preparation device transmits only particles in selected state

We denote events from this setup with the notation (x,G).

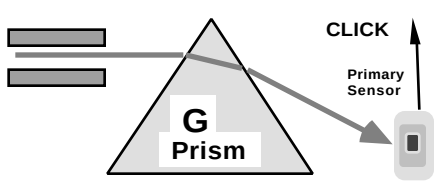
Preparation devices **G** can take various forms, for example

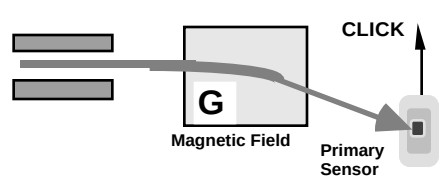
Magnetic fields provide momentum selection for charged particles.

Prisms or **gratings** provide wavelength discrimination for photons and particles

Inhomogeneous magnetic fields can sort particles according to spin polarization

Polaroid sheets assure that transmitted photons are polarized in selected direction.





3.1.4 Assuring validity with auxiliary signals: Clicks and Taps

In many experiments the detectors have substantial background count rates that are from particles other than those relevant to the experiment. In that case, we need to have more information to decide on which clicks to accept as useful data. In order to better discriminate against background, one often sets up auxiliary sensitive elements that will register other system signals (here called "Taps") that should accompany a valid count in the primary sensitive element. We then accept as event data only those Clicks that are coincident with an appropriate Tap:

Fig 3.4 Event (x,T,G): Click+Tap assures validity of data

The term "Tap" in this context refers to a recordable signal from an auxiliary sensor. As such, Taps are often of the same nature as Clicks; but we use the different words here to distinguish between the primary and auxiliary recordable signals in a given experiment. For example, if a beam of energetic electrons is being counted with an arrangement similar to the shown here, the Tap gives assurance that the subsequent Click can be attributed to a particle that emerged from the collimator and went through the state preparation **G** before entering the primary sensitive element. .

3.1.5 Observables as Coordinates of an Event:

In experiments like this, we denote an event by a set of numbers that are, in a generalized way, the coordinates of that event. These numbers are not at all abstract; they derive from sensors that we read or from experimental parameters that we adjust. These numbers, the coordinates, represent all that we really know about the event.

We might know only the spatial location of the click:	Event (x)
If we also demand occurrence of auxiliary tap signals,	Event (x,T)
If state preparation devices G is also employed then	Event (x,T,G)

Events are distinguishable only if they differ in

at least one of their observable coordinates.

3.2 Indistinguishable Events

3.2.1 Attempts to define a Trajectory

Suppose we want to specify the trajectory of a particle through the apparatus. To do this, we can use auxiliary counters positioned side-by-side as shown below, and we accept as events only clicks that are correlated with tap T_1:

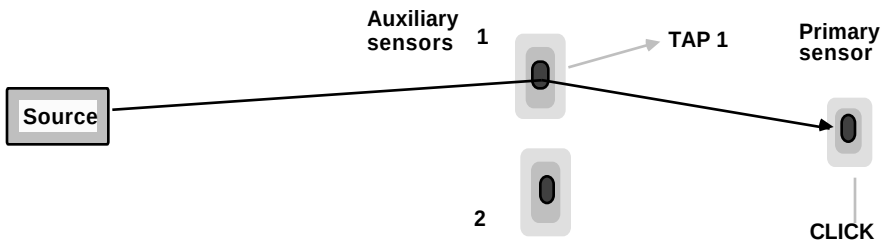


Fig. 3.5 Event (x,T_1): Path as a verified trajectory.

3.2.2 Alternate paths to the same event

Sometimes the auxiliary sensors may have some overlap in their regions of sensitivity, and in that case one cannot be as certain about paths and trajectories because there are many apparently different paths that could yield the same observable event (x,T_1):

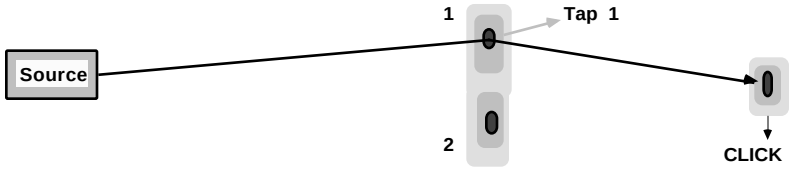
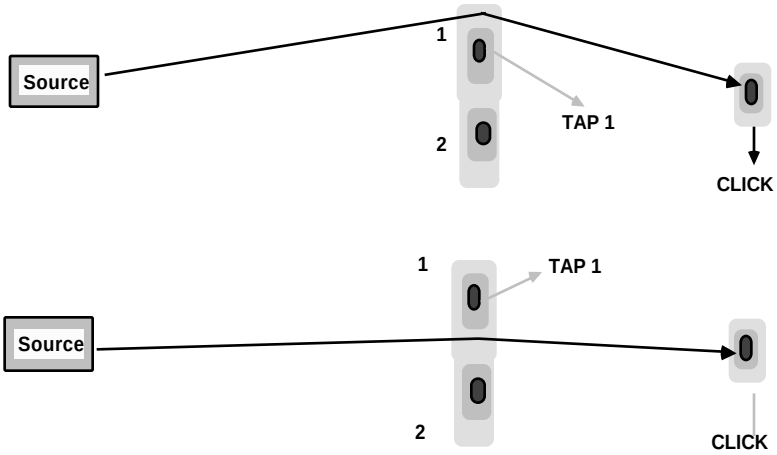


Fig. 3.6 As in Fig 3.5 but with overlap in sensors sensitivity



Figs 3.7, 3.8 Events (x,T_1): Paths near to but not "in" sensor #1 could still yield Tap 1

With overlapping regions of sensitivity, it might even be possible to record an event (x,T_1) even though the path would seem to make the other event (x,T_2) more likely:

(This is a significant possibility in many experiments, and we will later consider it in detail.)

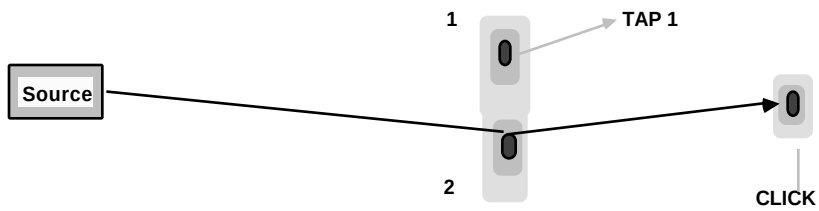


Fig 3.9 Event (x,T_1): Path much nearer to sensor #2 might actually yield Tap 1.

The events shown in Fig 3.7, 3.8 and 3.9 are
conceptually different
but
operationally indistinguishable
from the event in Fig 3.6. Having the same coordinates, they would all be recorded as valid events of the form (x,T_1) in the set of data from the experiment.

This apparent ambiguity about the behavior of the particle in the apparatus obliges us to consider more carefully the meaning of the terms *path* and *trajectory*..

In classical mechanics, the line showing the path of a projectile is taken to be a representation of experimentally verifiable history. The drawn line is a trajectory and it implies uniqueness: "The particle passed *this* way to the exclusion of other ways that it might have gone."

In quantum mechanics, paths are representative of possibilities, and the constraint of uniqueness is applied only to the extent demanded by coordinates of the event being observed. Most remarkable is that the probability for the event has contributions, via amplitudes, from *all* the possible paths to the event

As an example, we consider the event when a photon emitted from a source reaches a detector by way of being reflected from a mirror. (The argument given here follows the approach given by Feynman in his book *QED*)

3.3 Paths to a Given Event

3.3.1 Analysis of Reflection Process

We consider the process whereby a photon leaves a source and arrives at a detector by reflection from a mirror. We sketch various possibilities (paths A, B, and C being among the many) for the passage. We do *not* assume a law of equal angles at reflection, and although the light paths are indicated as sequences of straight lines, this is for ease of discussion: Straight line propagation of light in a homogeneous medium is also a consequence of the general principles employed here.

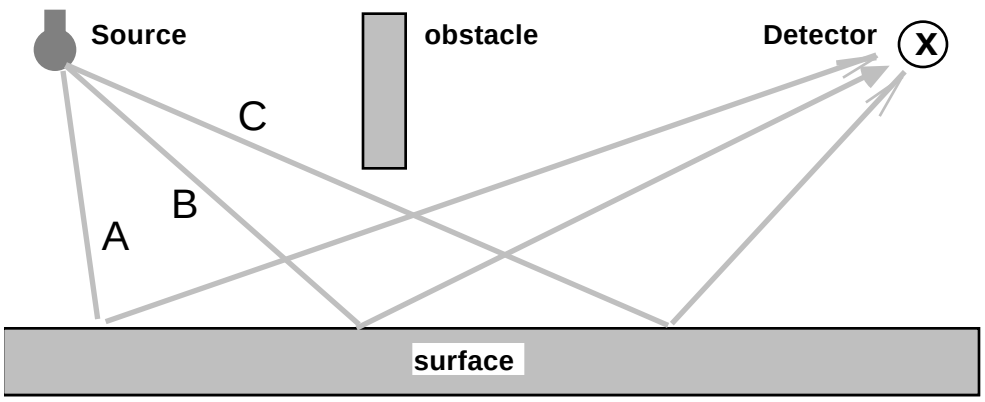


Fig. 3.10 Various paths for reflected photon

We are interested in the events where a photon leaves the source and registers at the detector at location x. We represent such an event with the notation (x), and we express the net probability for the event as P(x) Associated with each point on a given path is a **probability amplitude** $\underline{\mathbf{A}}$ that has both magnitude and phase. An individual probability amplitude is $\underline{\mathbf{A}}_k(\xi)$, where the subscript k indicates an amplitude as reckoned for the k-th path, where ξ is the position on that path, $\underline{\mathbf{A}}_k(x)$ is the k-th amplitude evaluated at the detector location x.

The vector sum of all amplitudes for a given event comprises the net amplitude;
The absolute value, squared, of that net amplitude yields the probability for the event.

$$\underbrace{P(x,T,G)}_{\substack{\textit{probability for event at } x \\ \textit{with conditions } T \textit{ and } G}} = \left|\overbrace{\sum_k \vec{A}_k(x,T,G)}^{\textit{net amplitude}}\right|^2 \quad \text{¾¾¾¾¾¾} \xrightarrow[\text{abbreviated}]{\text{commonly}} \quad \underbrace{P(x)}_{\textit{probability}} = \left|\sum_k \vec{A}_k(x)\right|^2 \qquad (3.1)$$

The event in question almost always involves accepting the detector click at x subject to confirmation signals ("taps") T and state preparation G and should thus be designated as (x,T, G), but for convenience we often abbreviate the event just with (x).

In making the (vector) sum of amplitudes indicated in Eq. (3.1) we must take into account all the indistinguishable paths that might lead to the event of interest.

In our example there are many paths, each with its probability amplitude, each associated with reflection from a particular location on the mirror. At the detector, each of the amplitudes will have roughly the same magnitude but the relative phase depends on the physical length of each path. If we are reflecting from an arbitrary

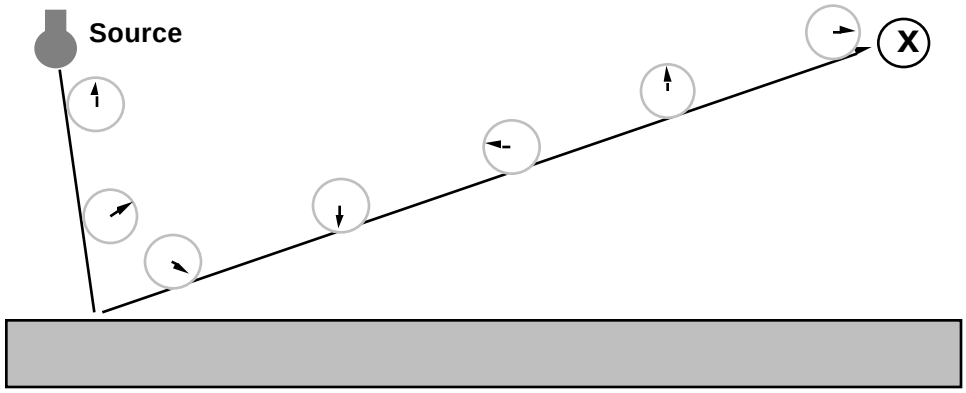


Fig. 3.10 Showing evolution of phase over arbitrary path

location on the mirror (Fig. 3.10), then a small shift in that location usually results in a large change in phase (Fig 3.11), so the vector sum of amplitudes from reflections in that region may involve significant cancellations:

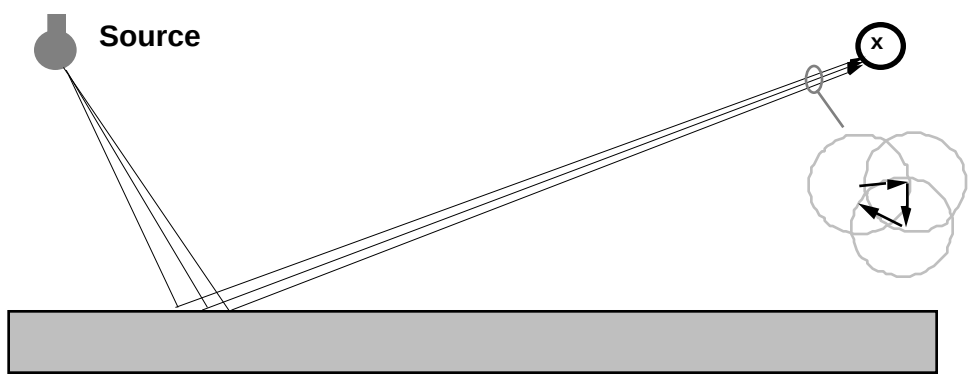


Fig. 3.11 Slight shift of path leads to large changes in phase at detector

The relatively rapid variation of phase with reflection location means that this portion of the mirror ordinarily contributes very little to the probability amplitude for photon arrival at the detector location x.

But it is another story for paths near the location where angle of incidence equals angle of reflection.

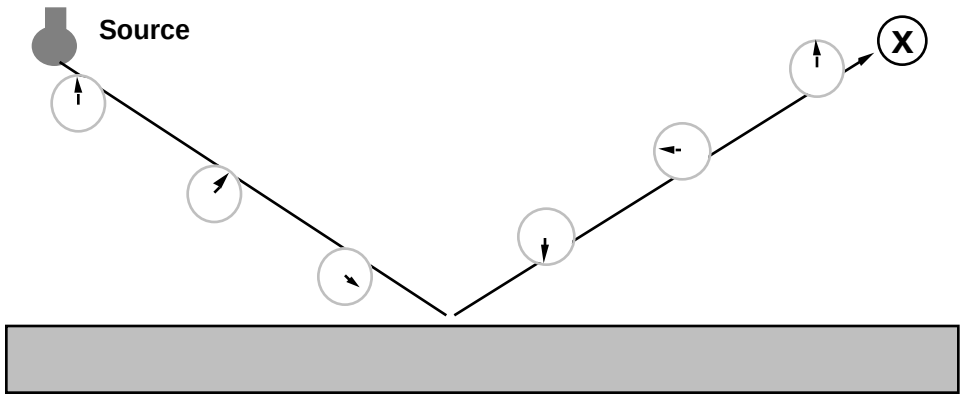
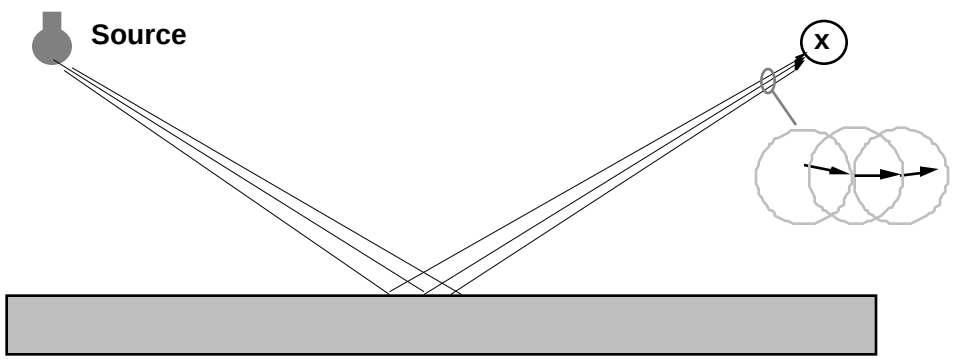


Fig. 3.12 Path where angles are equal

It is readily shown that a slight shift of the path does not influence phase very much when one is near the equal-angles location. (Fig. 3.13) (One sometimes refers to this as a region of stationary phase.) So the contributions from this region of reflection tend



3.13 Slight change of path causes only small phase shift

to be in phase with one another and thus contribute strongly to the net probability of the event (x). This is in harmony with the geometric optics law of reflection as used when drawing classical ray diagrams to describe the behavior of light at the boundaries of a homogenous medium.

[**Exercise**: show that the point of stationary phase is also the point of minimum time]

[**Exercise:** Suppose we are working with λ=3 cm radiation, 45 deg incidence, source and detector are 20 meters away from a reflector. How large is the region of stationary phase on the mirror ("stationary" means less than $\pi/6$ phase shift from one end to the other).

In the figure below we show, schematically, the sum from 20 different regions on the reflector. The slight variation in path lengths results in significant changes in phase, with a minimum of phase shift occurring for regions 8-9-10-11 which then make the principal contribution to the net amplitude

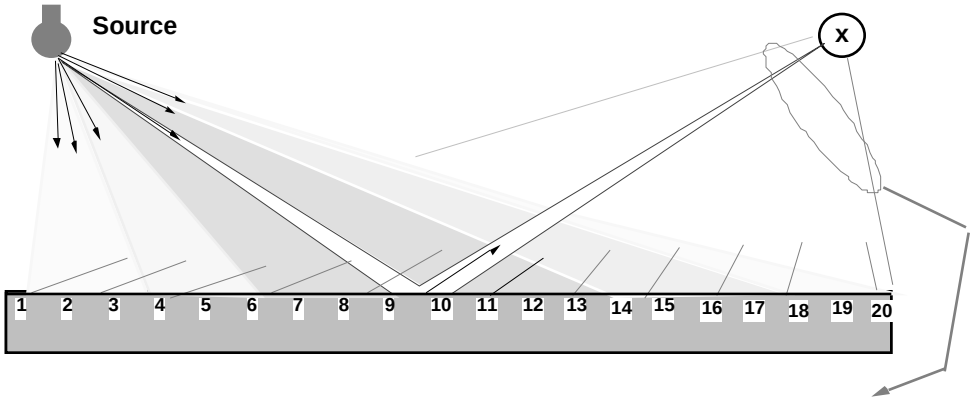


Fig 3.14 Showing reflection from 20 different regions on surface

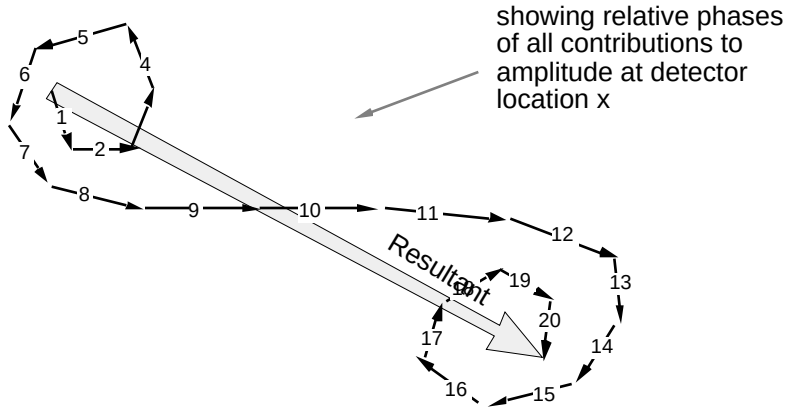


Fig 3.15 Addition of probability amplitudes from 20 different regions;
This is the *Cornu Spiral* of classical wave optics

Is this just talk? Do the other parts of the surface actually contribute? One can selectively coat those parts of the mirror that give destructive contributions (in the sketch above we gain net amplitude by removing contributions 1, 3,4,5,6 and 13, 14, 15, 16, 17, 20) so that one retains only those that give constructive additions to the resultant.... so one finds that selectively *reducing* the area of the reflecting surface *increases* the amplitude of the resultant significantly.

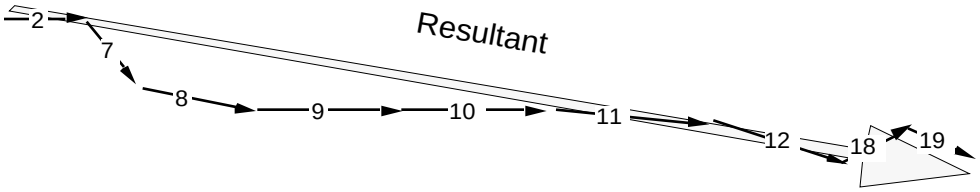


Fig 3.16 Increase of resultant by selective reflection

The description given here applies to many devices that enhance the propagation of a selected wavelength in a particular direction. These devices include diffraction gratings multi-coated optical surfaces, directional microwave antennas, and interferometers

We repeat: Interference effects persist in the weak flux limit, and this gives a mortal blow to the notion that paths, drawn as lines between source and detector, represent a trajectory that was actually taken. We cannot reasonably explain the weak flux experiments with any theory that envisions the quantum entity (whether a photon or an object with finite mass) constrained to a single classical path.

Instead we adopt a quantum theory in which the probability for a detectable event (arrival of a photon or of a particle) can be predicted by

(a) considering all the indistinguishable ways (paths) that could lead to the event,

(b) representing the contribution of each of those paths with a probability amplitude that has both a magnitude and a phase

(c) adding the individual probability amplitudes to get a net resultant

(d) squaring the total probability amplitude to get a net probability for the event.

This approach explains not only geometric optics (the law of reflection is but one example) but also wave optics (e.g. diffraction gratings). Moreover, this approach provides us with the basis of a unified theory that predicts the behavior not only of photons but also electrons and other particles on the atomic scale.

3.3.2 More on the Concept of Path

We speak of *paths* and draw them as lines that indicate some possible passage for the particle to go from the source to the final detector. "Possible passage" means that if we move an auxiliary detector along the path, then there is a finite probability that the particle could be detected at any place along that path.

In a two-slit experiment where the event (x) is defined as a photon leaving source S and arriving at detector location x, there are two paths that are usually drawn.

Fig. 3.17 Alternative paths from source S to detector at location x

But the paths drawn in Fig 3.17 are oversimplifications.
A detector placed almost anywhere in the region between the source and the plane of the slits could register particles .

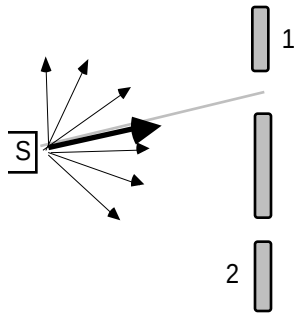


Fig. 3.18 Particles might be found anywhere

So the probability amplitude anywhere along the straight line between S and #1 has contributions over many paths, and we show only a few of them here:

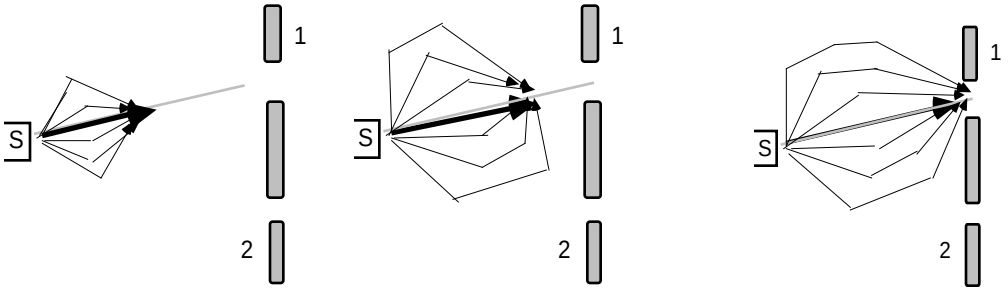


Fig. 3.19 A, B, C Contributions to phase maximum along classical trajectory

But just as in our discussion of reflection in the previous section, the overwhelming contribution to the net probability amplitude comes from those paths that do not shift phase very much with respect to each other, and those paths lie very close to the classical trajectory. This is a close parallel to the wave-optics proof that light in a homogenous medium tends to travel in a straight line.

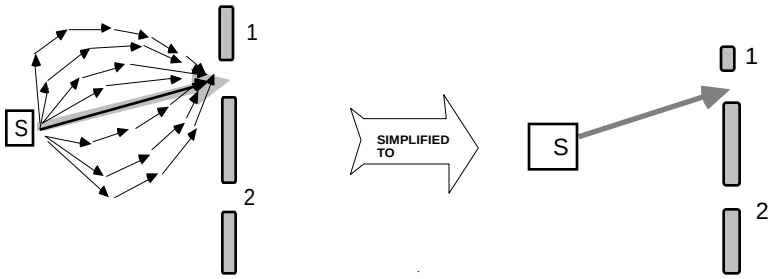


Fig. 3.20 Classical trajectory can represent sum over many amplitudes

3.4 Functional Analysis Of Interference Experiments

Events, in our theory, are binary constructs: Either the counter clicks or it does not. Either the preconditions (themselves binary) are fulfilled or they are not. (Analog signals from experiments actually turn out to be a summation of many binary events.)

Events are regarded as *distinguishable* if they differ in a way that is potentially measurable: (x,T,) ≠ (x,T') . Events can be regarded as distinguishable even if one does not actually test all the preconditions for acceptance; it is enough that one *could* do the testing.

[note that we had been designating events as (x,T,G), but to reduce the algebraic clutter here we omit the specifier G]

There may be conceptually different paths that lead to an event (x,T), and we can indicate the amplitudes associated with those paths with an additional index k, as in $\underline{\mathbf{A}}_k(x,T)$. Then, as already stated earlier, the probability $P(x,T_1)$ of getting a count at x correlated with auxiliary signal T_1 is found by squaring the vector sum of amplitudes over all paths

$$\underbrace{P(x,T_1)}_{\substack{probability\ for\ event\,(x)\\ correlated\ with\ T_1}} = \left|\overbrace{\underbrace{\sum_k \vec{A}_k(x,T_1)}_{k\ different\ paths}}^{net\ amplitude}\right|^2 \qquad (3.2)$$

We may, of course, be interested in the probability of event (x) irrespective of any other correlation. If T has only possible values ,T_1 and T_2, for example, then there are two distinguishable events that lead to the detector click of interest, and the total probability is the arithmetic sum of probabilities for those two distinguishable events:

$$\underbrace{P(x)}_{\substack{probability\\ for\ event(x)}} = \underbrace{P(x,T_1)}_{\substack{probability\ for\ event(x)\\ correlatedwith\ T_1}} + \underbrace{P(x,T_2)}_{\substack{probability\ for\ event(x)\\ correlated\ with\ T_2}}$$

formally, we write this ↑ as ↓ (3.3)

$$P(x) = \left|\sum_k \vec{A}_k(x,T_1)\right|^2 + \left|\sum_{k'} \vec{A}_{k'}(x,T_2)\right|^2$$

where k and k' are the indices for the paths leading to events (x,T_1) and (x,T_2), respectively. If there are N possible values for the correlation (tap) signal T, then the total probability for the click at detector location x is an obvious generalization of (3.3), an *arithmetic* sum of N squares of *vector* sums: :

$$P(x) \quad \underset{\substack{for\ N\ distinguishable\\ ways\ of\ gettingevent\ (x)}}{\longrightarrow} \quad \sum N\left|\sum_k \vec{A}_k(x,T_N)\right|^2 \qquad (3.4)$$

3.5 Two-Slit and N-Slit Diffraction Experiments

The Basic Experiment

Consider electrons passing from a source through a two-slit grating to a detector. We want a theory that will predict the distribution of electrons over the detector region.

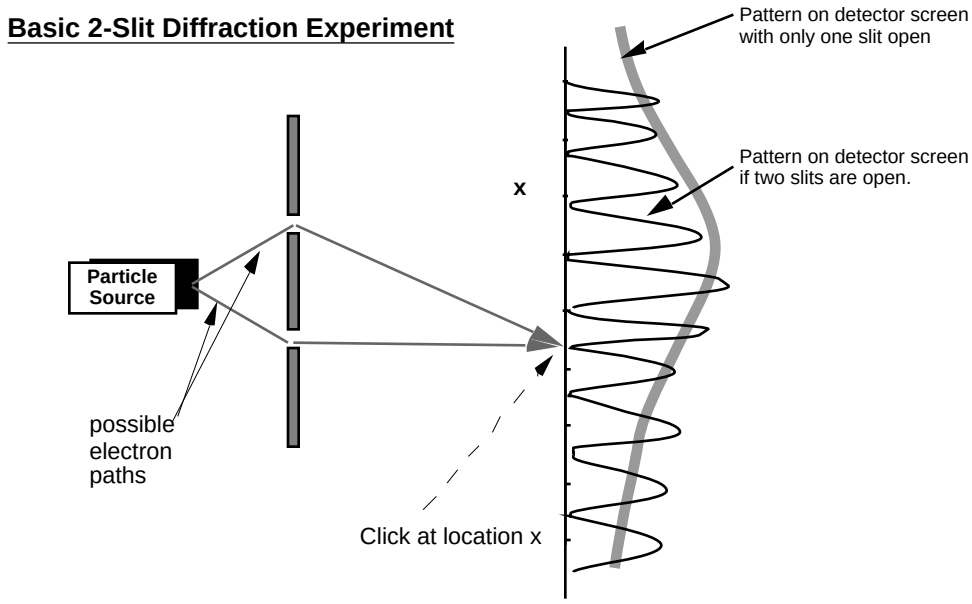


Fig 3.21 2-Slit Experiment demonstrates interference.
Denote the slit width as δ, the slit separation as S, and the wavelength as λ. With one slit closed and the other open, we get a broad pattern with an angular spread on the order of δ/λ. With both slits open, we get a pattern with interference pattern maxima having angular separations on the order of S/λ.

The counting of an electron at detector location x is denoted as event (x). The contributions to this are over two possible paths shown, and the amplitudes for these paths are denoted $\underline{\mathbf{A}}_1(x)$ and $\underline{\mathbf{A}}_2(x)$. The probability for the event (x) is then:

$$P(x) = \left|\vec{A}_1(x) + \vec{A}_2(x)\right|^2 = A_1^2(x) + A_2^2(x) + 2A_1A_2 \cos\phi_{12} \quad (3.5)$$

The resultant probability undergoes major fluctuations as the phase of the two contributions controls the last term in this sum which, we note, can be *either positive or negative*. This last term, sometimes called the *interference term* or *cross term*, often

reveals very interesting information so experiments are often deliberately designed so that both amplitudes $\underline{\mathbf{A}}_1$ and $\underline{\mathbf{A}}_2$ are of reasonable size.

Fig 3.22 Addition of amplitudes in a two-slit experiment

Thus experiments can give the superficially surprising result in which opening additional pathways for the occurrence of an event may in fact *decrease* the probability of an event for some observers. For example, suppose slit #1 is open, slit #2 is closed and the observer is measuring P(x); suddenly someone opens slit #2. The rate of electron arrival at location x may increase, stay the same, decrease, or even vanish completely!

As we have emphasized, the addition of amplitudes involves keeping track of relative phases. This is relatively simple in the two-slit situation.

For N slits, we find that the interference maxima can be quite pronounced if all N contributions are coherent ; on the other hand, if the contributions have random phases, the interference terms may tend to average to zero and the time-averaged probability becomes the arithmetic sum of the N individual probabilities.

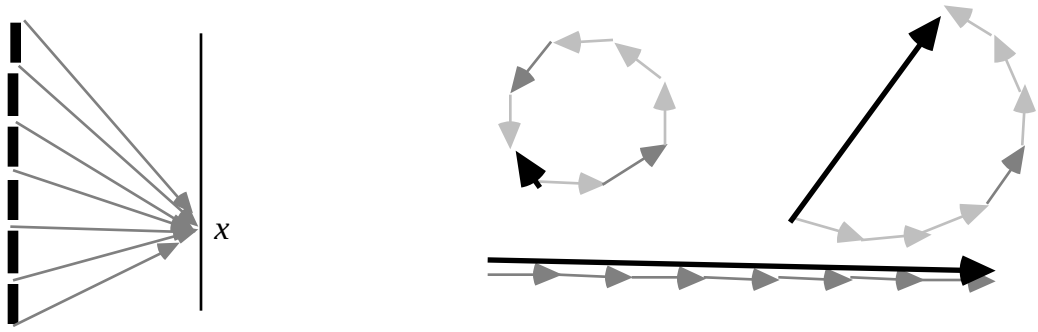


Fig. 3.23 Addition of amplitudes in an N-slit experiment
resultant depends strongly on relative phase angles

The statement of probability deriving from the addition of N amplitudes is:

$$\left|\sum_k \vec{A}_k\right|^2 = \sum_k A_k^2 + \sum_{j\neq k} \vec{A}_j \cdot \vec{A}_k \;\; \underset{\substack{\textit{display}\\ \textit{relative}\\ \textit{phases}}}{\longrightarrow} \;\; \sum_k A_k^2 + \sum_{j\neq k} A_j A_k \; e^{i(\phi_k - \phi_j)}$$

$$= \sum_k A_k^2 + \frac{1}{2}\sum_{j\neq k}\left[A_j A_k \; e^{i(\phi_k - \phi_j)} + A_k A_j \; e^{-i(\phi_k - \phi_j)}\right] \tag{3.6}$$

where to simplify we write merely A instead of A(x). The last term can be recognized as a sum, over all combinations, of cosines of relative phase angles of the different amplitudes:

$$P(x) = \left|\sum_{k=1}^{N} \vec{A}_k(x)\right|^2 = \overbrace{\sum_{k=1}^{N} A_k^2(x)}^{N\ \textit{terms here}} + \overbrace{\sum_{j\neq k}^{N} A_j(x) A_k(x) \; \cos\left(\phi_k - \phi_j\right)}^{N(N-1)\ \textit{terms here}} \tag{3.7}$$

The last term can easily dominate P(x) if the phase angles are stable over the time of observation. (This question of phase stability is very important and we will discuss it in detail somewhat later in this volume.)

3.5.1 An Analysis of "Which Way" Experiments

A popular question in discussion of this sort asks
"which slit did the electron (or photon) *really* go through?" [German:*welcher Weg*?]
By now we know that such a question must be answered in an experimental manner if one wants to come to meaningful conclusions.

We install counters near slits 1 and 2 of the apparatus. The counters are intended to click when the particle passes nearby. Unfortunately, the counters are not perfect, so there is ambiguity. For example, if we want be sure that an event (x,1) corresponds to an electron path through slit 1, we are impeded by the possibility that the path through slit 2 might still coincide with a click in counter 1

We show two paths consistent with (x,1)

Indistinguishable contributions to event (x,1)

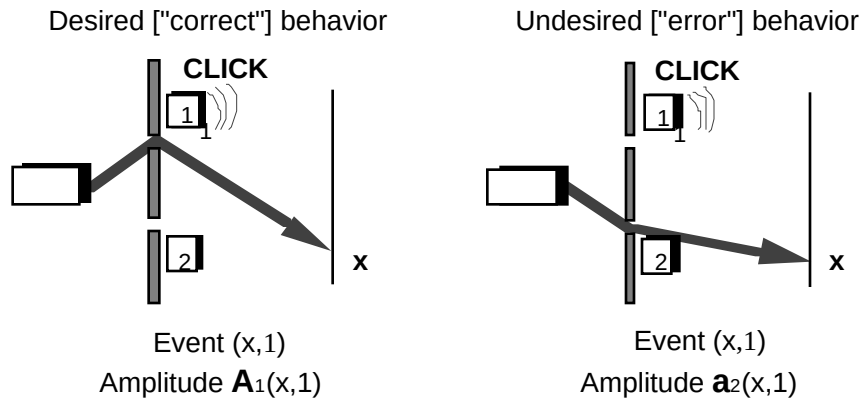


Fig 3.24: Both slits provide paths to event (x,1)

where the two amplitudes are for **in**distinguishable events. We use lower-case "a" for the "undesired" $\mathbf{a}_2(x,1)$ to symbolize our hope that it is smaller than the amplitude for the desired "correct" path amplitude $\mathbf{A_1}(x,1)$

If we want to assign an event (x,2) to a path through slit 2, we have a similar difficulty because the path through slit 1 might still coincide with a click in counter 2.

Indistinguishable contributions to event (x,2)

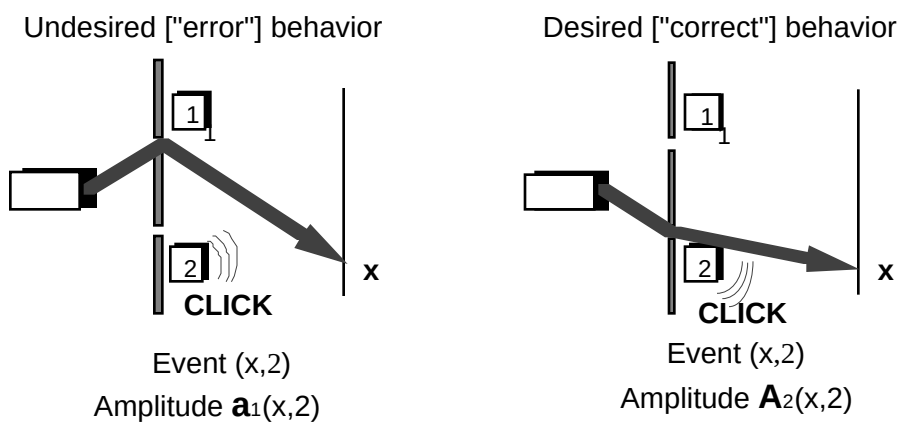


Fig 3.25: Both slits provide paths to event (x,2)

Suppose we want to determine the trajectory of the particle through the apparatus. How carefully should we construct the slit counters 1 and 2? In particular, how small should we strive to make the "error" amplitude **a**?

$\mathbf{A}_1$(x,1) represents the path amplitude for an electron to arrive at position x associated with path over slit 1 and a click in counter 1.

$\mathbf{a}_2$(x, 1) represents the amplitude for the electron at position x associated with path over slit 2 but still with a click in counter I. [For the moment we use a small "a" here to remind us that this amplitude should be small "in a well designed experiment", although to be consistent with our notation we should use **A** for both.]

These two amplitudes, $\mathbf{A}_1$(x,1) and $\mathbf{a}_2$(x, 1) , are for **in**distinguishable events.

As shown in Fig. 3.24, the probability P(x,1) of an electron arriving at x with a concurrent click in 1 is:

$$P(x,1) = \left| \underbrace{A_1(x,1)}_{large} + \underbrace{a_2(x,1)}_{small} \right|^2 = A_1^2(x,1) + a_2^2(x,1) + 2\vec{A}_1(x,1) \cdot \vec{a}_2(x,1) \qquad (3.8)$$

or, suppressing some of the arguments to reduce the clutter, we have

$$P(x, I) = \underbrace{A_1^2}_{large} + \underbrace{2\vec{A}_1 \cdot \vec{a}_2}_{\substack{modest \\ interference \\ term}} + \underbrace{a_2^2}_{\substack{very \\ small}} \qquad (3.9)$$

In a "very well designed experiment" we would have $\mathbf{a}_2$ -> 0 and the interference term would disappear completely. On the other hand, the maximum interference is seen when $\mathbf{A}_1$=$\mathbf{a}_2$, that is when one has given up trying to answer "which way?"

The probability P(X,2) of an electron arriving at x with a concurrent click in 2, ostensibly to verify the passage of the electron through slit 2, is described in a similar way (Fig. 3.25).

Again the experiment is "well designed" so that the amplitude $A_2(x,2)$ is larger than $a_1(x,2)$; we find the probability is::

$$P(x,2) = \left| \underbrace{a_1(x,2)}_{small} + \underbrace{A_2(x,2)}_{large} \right|^2 = \underbrace{a_1^2}_{\substack{very\\small}} + \underbrace{2\vec{a}_2 \cdot \vec{A}_2}_{modest} + \underbrace{A_2^2}_{large} \qquad (3.10)$$

And as before the interference term is small because we designed a_1 to be small.
If we have a "well-designed" experiment in which A>>a , we then have a net probability of getting electrons at point x which is a sum:

$$P(x) = P(x,1) + P(x,2) \quad \underset{small\ a}{\rightarrow} \quad P(x) \approx A_1^2 + A_2^2 \qquad (3.11)$$

this is, of course, the classical result in which the probability of getting the electron at location x is just the probability via slit 1 added to the probability via slit 2. Interference is not evident.

We see that maximum interference in this case (found when the slit counters provide no discrimination whatever) has the same magnitude as when no slit counters are mounted in the apparatus.

$$P(x) = P(x,1) + P(x,2) \quad \underset{\substack{equal\\amplitudes}}{\rightarrow} \quad P(x) \approx 2A^2\left(1 + \cos\theta_{12}\right) \qquad (3.10)$$

where Θ_{12} represents the relative phase difference of the two paths.

> The relative sizes of probability amplitudes depend on the apparatus design, principally on the proximity and sensitivity of the slit sensors, so the experimenter can make a smooth, operational transition between the "SLIT KNOWN" and the "SLIT UNKNOWN" situations.

In particular, we find

(1) that the interference terms are at their largest (of the same size as the direct terms) when all attempt at "which slit" discrimination is abandoned, and

(2) that the price of absolute certainty about the electron path slit is the complete disappearance of the interference term.

3.5.2 Is It The Sensor That Destroys The Interference?

The argument is often made that the disappearance of the interference in a "which way" experiment is caused by an interaction of the particle with the sensors: It is alleged that a count in sensor #1 is inevitably accompanied by a significant shift in the phase of A(x,1), a shift that varies randomly for each electron as the experiment acquires data, and that it is this phase randomness, somehow induced by the sensors #1 and #2, that washes out the time-averaged interference pattern.

But loss of the interference pattern follows automatically from our original rules for computing the probabilities of events. We do not need to hypothesize some special effect of the sensor on the electron to describe the loss of interference when the electron trajectory is determined more closely. So we should follow Newton's exhortation:

Make no unnecessary hypotheses.

3.6 Is there a Wave/Particle Duality?

Experiments on electron diffraction, on optical interference, and on the photoelectric effect can be done under so-called *weak flux* conditions when no more than one photon/particle is in the apparatus at any one time. The data from such an experiment consists of a sequence of discrete events, and the experimenter accumulates events until a statistically significant pattern is obtained. Without exception, the individual events are point-like in that they can be highly localized to an arbitrarily small region of space by after-the-fact measurements.

In this context, "after-the-fact" (*a postoriori* in more classical language) means that the measurement of event coordinates is made well after the particle/photon has traversed the apparatus. We note that such an after-the-fact measurement is not limited by a Heisenberg dx>h/dp principle.

> When we describe (*a postoriori*) the observable events that have been created by an individual photon (electron, neutron …), we find that the photon in this respect showed the behavior that we ordinarily associate with a particle.

But if we want to make "before-the-fact" (*a priori* in classical language) predictions about the events that will be caused by an individual photon (electron, neutron…), then it is clear from particle diffraction experiments that one *cannot* conceptualize the photon as always being continually localized in a well-defined (i.e. point-like) place. Our calculations are done with the aid of amplitudes that allow for the possibility of simultaneous presence, albeit unobserved, over a wide range of space. Thus the completely classical notion of a trajectory does not extrapolate successfully to the microscopic regime.

> When we make predictions (*a priori*) for the location of events yet to be created by an individual photon (electron, neutron, …), we find it impossible to assign it a trajectory and in this respect it does not have all of the properties that we ordinarily associate with particles. Instead, we use continuous functions that resemble those used for discussion of electromagnetic and acoustic wave phenomena.

Thus the distinction between particle emphasis and wave emphasis depends on whether one is talking about the past or the future.

Relation to Interference in Classical Electromagnetic Theory

Classical wave theory describes interference phenomena (as seen in water waves, for example) in a very satisfactory manner, so that theory has been adapted to electromagnetic radiation and to particles. But the water-wave interference demonstrations are somewhat misleading because they suggest that interference depends on one tangible entity (a wave from source #1) adding to or subtracting from another tangible entity (wave from source #2).

Classical electromagnetic theory associates $|\mathbf{E}(x,t)|^2$ with the energy of the field at location x. And because optical interference effects persist in the weak flux limit, we must regard $|\mathbf{E}(x,t)|^2$ as expressing the net probability for the arrival of photons at a particular location. We form the total field from a vector sum of components $\mathbf{E}_k(x)$, where the $\mathbf{E}_k$ can be regarded as probability amplitudes from the individual paths. Maxwell's equations and the wave equation enable us to relate the **E**'s to their sources. The highly-developed formalism of electromagnetic theory can be adapted to predict the behavior of particles.

Weak Flux Interference as *experimentum crucis*

We know that experiments with particles (or photons) show interference effects even if they are done with fluxes so feeble that there is no more than one tangible entity (particle or photon) in the apparatus at one time. In this weak flux limit, a short exposure of the detector may show only several scattered dots where the few electrons have landed. When the detector is exposed for a longer time, the integrated pattern of dots on the detector plane begins to reveal the spatial distribution that is the signature of interference.

This persistence of interference in the weak flux limit shows that interference does not depend on the interaction of two tangible entities, in this instance a particle from slit #1 interacting in some way with a particle from slit #2.

Instead we regard the wavefunction as a representation of expectations for the future, not as a physical entity in the usual sense.

The weak flux limit experiments show (as Dirac has emphasized):

Each particle (or photon) interferes only with itself.

3.7 Building a Quantum Theory of Light and Matter

The approach here follows the direction shown by Feynman and his associates who described the behavior of light and matter in a symbolic way in their introductory books. [*Lectures in Physics*, vol. III (Addison Wesley, 1965); also the elegant, economical 1988 paperback entitled ***QED*** (Princeton U. Press, 1988)].

Feynman emphasizes that quantum mechanics depends on a careful definition of what one means by "observable event". It also depends on an appropriate account of conceptually distinct but operationally indistinguishable paths as contributors to a given event.

Feynman stresses that probabilities for indistinguishable events are combined with phases in a way quite different from the simple addition-of-scalars used for distinguishable events. His elementary presentations are expressed in a symbolic way that removes the mathematical clutter from the expression of physical ideas. The manner of description is not only accessible to the beginner but also a source of deeper insight for the more advanced student. In this respect, the Feynman treatment contrasts with many other quanta-for-the-layman explications that must eventually be unlearned if the student wants to go further.

But having started with a Feynman overview to get a sense of the method and its interpretation, we want to develop a usefully quantitative theory. So we borrow from the formalism of classical acoustics and classical electrodynamics in order to calculate numbers that apply to actual experiments. The result will be the formulation of quantum theory known as "wave mechanics" in which Schrödinger's differential equation and its solutions $\Psi(r,t)$ are central concepts. The wave-mechanical approach evolves from a classical, point-particle preconception of the microworld to an newer and somewhat more general image in the mind's eye, an image almost but not quite free of the preconception of Piaget's child.

There is another approach, usually associated with Heisenberg, that downplays the use of literal models and instead focuses on observables. This more abstract approach seeks mathematical constructs and systems with operational behaviors that are somehow isomorphic to that of the micro system. The abstract approach is extraordinarily broad in its compass and fruitful in its results. It has often revealed unexpected symmetries and relationships between apparently disparate fields. We will get an exposure to the abstract method this term as we explore theories for the harmonic oscillator, for the radiation field, and for angular momentum.