2. THEORIES, MODELS AND OBSERVABLES

2.1 THE ROLE OF PHYSICAL THEORY

Truth and Phenomenology in Physical Theory

It is important to recognize that we need not label a physical theory "true" or "false" in the usual sense of those words: Practicing scientists judge a theory on the basis of its logical structure, practical utility, and aesthetic appeal: Does it offer economy in the sense of matching lots of data with only a few adjustable parameters? Is it logical and self-consistent? Is it easy to understand? Does it make interesting predictions? Is it intellectually satisfying?

Physicists, chemists, and astronomers often begin the analysis of experimental results by simply classifying and presenting the observed phenomena in arrangements that show interesting patterns. The scientists seek ad hoc mathematical expressions that yield similar patterns to form the basis of what is called a "phenomenological" theory. Sometimes the patterns from these theories have locations for which data seems to be missing, and often the searches to fill those voids in the pattern have led to interesting discoveries. Science is at its most exciting when researchers discover a fundamental principle that explains observations that had been described only in phenomenological ways.

Use of the adjective "phenomenological" is a recognition that the theory is basically descriptive and does not necessarily show how those patterns and regularities arise from a few deeper, underlying principles. Of course, a successful phenomenological theory is a powerful stimulus toward the longer range goal of developing a more fundamental theory for the situation at hand.

As already remarked, phenomenological theories often start with an empirical formula or pattern that reproduces a set of observed quantities or properties: Three famous but contrasting examples are the Bode-Titus "law" (1772-78) for the radius of the planetary orbits , the Mendelejev periodic table (1869) for the elements, and the Balmer expression (1885) for the wavelengths of the hydrogen spectrum.

Examples of the Evolution of Physical Theory
Kepler's Try: Geometric Patterns and Orbit Radii

To the naked eye, the sky presents a pattern of thousands of stars, and in this fixed pattern we find five points of light, the planets Mercury, Venus, Mars, Jupiter, and Saturn, that move in periodic fashion against the fixed background.

Kepler made careful studies of planetary motions and found that
(1)planetary trajectories are ellipses with the sun at one focus,
(2) a planet's radius vector sweeps out equal areas in equal times.
and
(3) the square of the period is proportional to the cube of the orbit radius

These empirical laws were later shown to be a consequence of the $1/r^2$ nature of the gravitational force in a Newtonian theory. So the Kepler Laws provide us with a very satisfying examples of phenomenological relationships later understood on the basis of fundamental principles.

But the actual sizes of the planetary orbits, and the ratios of those sizes to one another remained a puzzle because it seemed obvious that the Creator must have used a general principle to govern such fundamental properties of the natural world. There were numerous attempts to discover such a principle. For example, Kepler in his *Mysterium Cosmographicum* (1596) proposed to use the characteristic sizes that arise when one inscribes regular polyhedra within spherical surfaces. Then he tried to find the orbit sizes proportional to the dimensios of platonic solids inscribed one within one another. And later (1618), in his five-volume work *Harmonice Mundi* , he explored analogies between the solar system and the ratios found in musical tones, an extension of what Pythagoras had described as the "harmony of the spheres". This has obvious aesthetic appeal, but it has not yet led to much useful astrophysics.

The drive to understand the planetary orbit sizes continued

Bode's Law: Numerical Patterns and Orbit Radii

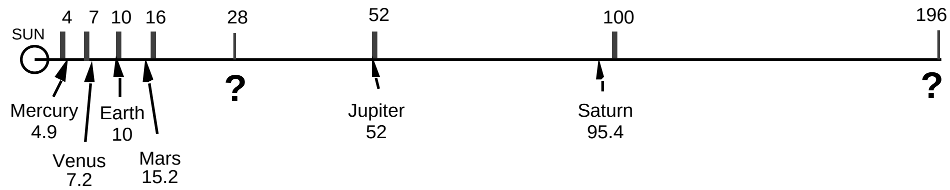
Bode and Titus [over the years 1772-78 in a way detailed in M. M. Nieto's *The Titus-Bode Law of Planetary Distances*, 1972]] proposed an empirical relationship between the radii of the planets and the numbers generated by the series:

$$4; \quad \underbrace{4+3}_{7}; \quad \underbrace{4+\left(3\times 2^{1}\right)}_{10}; \quad \underbrace{4+\left(3\times 2^{2}\right)}_{16}; \quad \underbrace{4+\left(3\times 2^{3}\right)}_{28} \quad \cdots$$

The next numbers in the series are 52, 100, 196, 388, 772, …
where the units are such that the earth-sun distance =10.

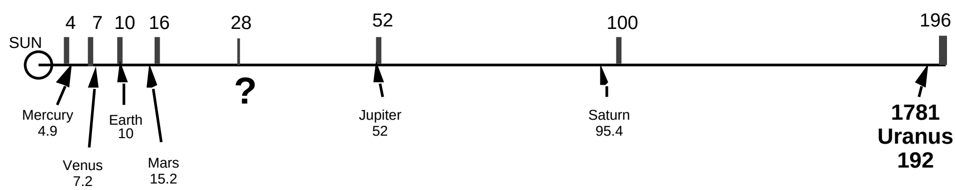
At the time of theTitus and Bode work, the situation was as shown here:

1776: Radii of the Classical Planets and the Bode Law Predictions



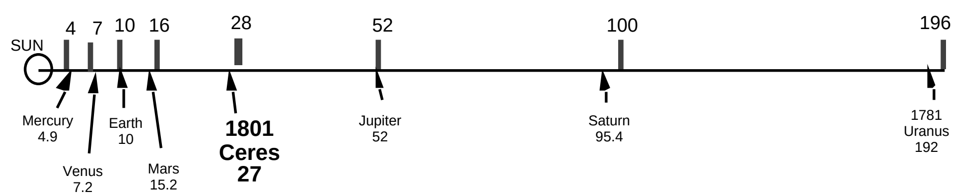
Less than a decade passed before observations revealed the planet Uranus very near to the predicted radius beyond Saturn:

1781: Uranus Discovered



And soon thereafter the large asteroid Ceres was discovered at the radius value between Mars and Jupiter. So the Bode-Titus relationship had clearly passed one of the major tests for acceptance of a theory: it made significant predictions.

1801: Asteroids Discovered



But the Bode-Titus successes were not to continue. Analysis of planetary orbits with Newtonian theory showed that the motionof Uranus could not be understood on the basis of the then-known planets, and 1845 Adams and Leverrier, independently, showed that the existence of a new, more distant planet would explain the data. In 1846 Neptune was discovered; its orbit radius was 301 units. The prediction was for an orbit with a radius of 388 units, and this unexplainably large discrepancy raised serious questiona about the soundness of the tbe Bode-Titus relation.

Finally, when Tombaugh discovered Pluto in 1932, the planet was found at a distance of 395 units instead of at the predicted value of 772 units. This huge discrepancy was a fatal blow to the theory, such as it was. Now, with no known physical basis, the Bode-Titus relationship (sometimes even called a "law") remains today as a curious and purely phenomenological expression for the ratios of the radii of the inner planets.

The Periodic Table: A Pattern of Chemical Properties

The work of Mendelejev on the periodic table of elements (1869) is another notable example of phenomenological analysis that not only organized the information already available but also made significant predictions of new elements. It was not until 1924 that the periodic table was understood to be a consequence of the exclusion principle that governs the electronic structure of atoms.

The Balmer Formula: Numerical Patterns and Spectral Lines

The spectrum of atomic hydrogen was first studied in 1853 by Anders Ångstrom but it was not until 1885 that Balmer put forth his purely empirical but astonishingly successful relationship between the wavelengths of the spectral lines in the visible spectrum of atomic hydrogen.

$$\underbrace{\lambda = A\frac{m^2}{m^2-4}}_{\text{Balmer's original}} \rightarrow \underbrace{\lambda = A\frac{m^2}{m^2-2^2}}_{\text{suggestive change}} \rightarrow \underbrace{\lambda = A\frac{m^2}{m^2-n^2}}_{\text{generalization}}$$

where m is an integer greater than n.

Using only one adjustable parameter (A), Balmer was able to get spectactular agreement with the known spectrum of hydrogen. He promptly noted the possibility of writing the relation in more general form, and he confidently predicted the discovery of hydrogen spectral lines associated with values of n different from 2. Moreover, his empirical expression was later useful for the analysis of spectra from the helium ion that was observed in 1896 by Pickering.

The reciprocal of the wavelength is a quantity that tells one how many waves per centimeter are in the radiation; this is generally called the *wave number*. The Balmer expression for wave numbers in the spectrum of atomic hydrogen can be written as the difference of two terms:

$$\frac{1}{\lambda_{nm}} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

and from this Ritz extracted the combination principle that any wave number can be represented as the difference between two terms. That is to say, the wave numbers associated with the spectrum from a given atomic species form a simple pattern under addition and subtraction, a feature that we now know represents energy conservation as the atom undergoes transitions between its energy levels.

We note that both the Balmer and the Ritz results were obtained even in absence of a basic atomic theory.... an absence not surprising since it was not until the late 1890's that the properties of the electron began to be known in quantitative detail.

In 1912 Bohr was able to predict the hydrogen spectrum by quantizing angular momentum in his planetary model and still later (1924-26) the hydrogen levels could be derived from a quantum mechanical model.

[The reader should consult Chapter 10 of Pais's wonderfully readable book *Inward Bound* (Oxford University Press, 1986)

Theories Old and New

Older Theories have their uses since newer and more comprehensive physical theories may be difficult to understand or complicated to apply. So the older theories, although formally obsolete, may still be useful in spite of their limitations. For example, most of our situations of everyday life seem easiest to describe in non-relativistic terms as though were happening on a flat earth. In the same vein, many technically sophisticated people are still happy to describe atomic processes with the concepts and language of the Bohr model because the conceptual simplicity of that model compensates for its lack of generality and for its internal contradictions.

But we find that the older theories are manifestly inadequate to describe the internal structure of atoms and the interaction of atoms with radiation . Hence our present exploration of quantum mechanics. A study of the subject in its full generality is beyond our present scope, so we will make simplifying approximations: In particular we concentrate on the linear phenomena of atomic, molecular and optical physics that occur as two-body interactions at energies well below 20 KeV. This enables us to almost neglect relativity and to almost ignore non-linear effects (to mention the two most important approximations). So the level of quantum mechanics discussed here still represents only an intermediate stage in our drive toward understanding the physical world.

2.2 MODELS AS ABSTRACT REPRESENTATIONS

Models and Paradigms

To construct a theory, we begin by choosing a combination of visual model and symbolic representation within which experimental conditions and observations can be expressed and organized, and from which predictions can be obtained. The model may have both pictorial and abstract symbolic features. The pictorial aspects may be guided both by our visual conceptions and by the possibilties of geometric reasoning. The abstract aspects are chosen to facilitate the use of symbolic logic, algebraic reasoning, and numerical calculation so that both quanitiative and qualitative predictions can be obtained.

The obvious success of formal models and quantitative theory in the physical sciences has profoundly influenced the fundamental approach of the social sciences and the humanities of the 20th century. This is quite evident, for example in the vocabulary of economics (the "momentum" of the stock market), in the symbolic models of psychology (flow diagrams in transactional analysis), and in the metaphors of literary cricitism (a novel's "center of gravity") .

Issues in the Philosophy of Physics

In the social sciences and humanities,there are various models, paradigms and schools of thought (each with its own adherents) that change often enough so that students, as well as scholars, are quite aware of the intellectual choices they are making. As two examples among many, we can see from the histories of literary criticism and of clinical psychology how the choice of an intellectual framework and an associated terminology sets both the directions and the limits of inquiry. The half-life of these paradigms is often shorter than that of its practioners, so they are quite conversant with the notion of paradigm shift.

But physical scientists have a long and successful tradition of designing experiments and interpreting results within a single, dominant paradigm. This behavior is strongly encouraged by the long and rigorous yet conventional training of scientific apprentices in an atmosphere where philosophical differences between physicists (on the interpretation of the term *measurement* in quantum mechanics, for example) have only a minor presence in the standard physics literature. The established theoretical framework seems universally appreciated for its elegance, economy, and power. The correspondence principle and its analogs join the concepts of microscopic and macroscopic theory so smothly that workers in planetary dynamics do not perceive themselves to be in paradigm conflict with the laser physicists. Moreover, physics has no significant alternative schools of thought, at least not in the sense that alternatives exist in, say, economics or linguistics. So most physicists appear (to outsiders at least) to be dealing in natural and obvious truth. This is the track, worn deep from years of use, that Kuhn classifies as "normal science"

Clear changes in the paradigm of physics have occurred only twice in the 20th century: Relativity profoundly changed our concepts of space and time; quantum mechanics dramatically altered our concept of probability. But these last paradigm shifts occurred more than 60 years ago. So we have the superficially surprising situation in which physicists, who work on what is regarded as the forefront of knowledge, learn most of their electromagnetic theory from textbooks that were used by their parents.

Of course we can expect more paradigm shifts in the future. These may arise from new observations in terrestrial experiments (for example in Bose-Einstein condensates or in the spectroscopy of phenomena below 1 Hz). Or perhaps an interaction beyond the conventional set (strong, weak, electromagnetic, and gravitational) may be revealed from measurements on very large astrophysical systems.

There is a profound comfort in this stability of the paradigm in the physical sciences… we seem somehow to be more firmly anchored than our colleagues in the social sciences and humanities who often seem so much blown about by the winds of intellectual fashion. Each of us must take our own precautions so that the ossifying hubris arising from the comparative stability of our discipline does not block the consideration of truly new ideas.

Differences of opinion and vigorous discussion certainly do occur in the philosophy of physics and the literature on the philosophy of quantum mechanics is voluminous, but until recently most of this was in the realm of *gendankenexperimente* and thus had surprisingly little influence on the day-to-day practice of current research because physicists tend to pay more attention to predictions that are testable. This is now changing somewhat because modern apparatus and techniques (e. g. neutron interferometers, one-atom masers) make possible the measurements of which earlier theorists only dreamed.

Among the few examples where workaday physicists engage the philosophers of science in a quantitative way are in studies of Bell's theorem where experiments and their interpretation provide a deeper understanding of quantum mechanics. The results from these experiments lead to new insights but provide strong confirmation of the established paradigm.

2.3 MODELS

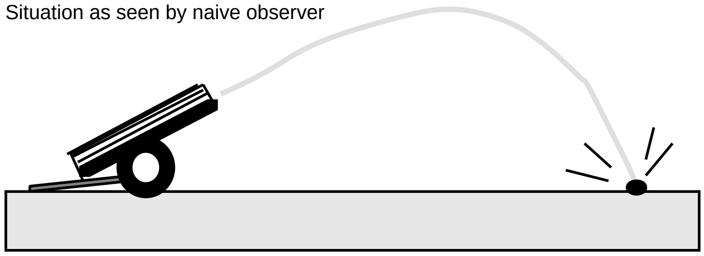
Physicists prefer to describe their models in algebraic and geometric terms so that formal calculations can yield quantitative predictions. The models range from the literal to the abstract

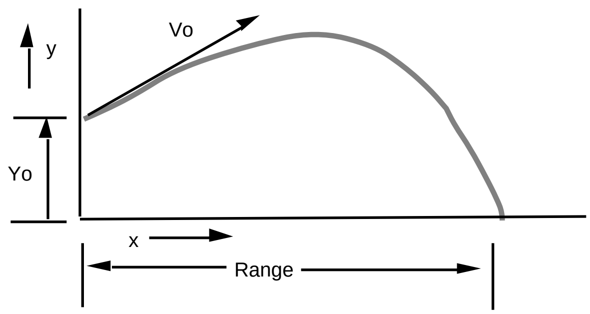
Literal Models: Concepts and Examples

By the term *literal model* we mean a construct that can be said to make a visual suggestion of the system of interest:

For example the model conventionally used to represent a projectile is that of a point mass moving along a defined trajectory (x,y,z,t) in Euclidean, three-dimensional space, and this has an obvious correspondence to the situation as it would be sketched by a representational artist.

Situation as seen by naive observer





LIteral Model as idealized by physicist

An engineer or physicist might forgo the sketch and begin at once by assuming the conventions of analytic geometry and representing the projectile as a point mass moving in a two-dimensional space. The laws of Newtonian mechanics then lead to equations that describe the trajectory:

$$x(t) = x_o + (V_o \cos\vartheta)\ t$$

$$y(t) = y_o + (V_o \sin\vartheta)\ t - \tfrac{1}{2} g\ t^2$$

where the actual situation has been simplified (for example by neglecting air resistance) in this algebraic representation of the experiment.

Models constructed for one situation are often applied to other situations that we believe (but cannot verify by direct observation) to be analogous. For example,

we cheerfully extrapolate the model originally devised to describe a cannonball to describe electrons (invisible particles that are at least seven orders of magnitude smaller) in a cathode ray tube.

Limitations and Hazards of Literal Models

Literal models are relatively easy to understand because they appeal to naive intuition and prior experience; however they have limitations and risks that become manifest when one tries to unify the description of an entire class of systems, or when one tries to use the literal model to describe systems of a vastly different scale. Literal models often have the limitation of not revealing the common aspects of apparently dissimilar systems that are actually of the same general class. For example a literal sketch of a pair of pendulums does resemble the sketch of a stretched string, but a more abstract representation allows one to use the the same normal mode analysis on each; therefore the insights gained from pendulum experiments may help better understand the vibrating string, and conversely.

Literal models derived from a system large enough to see are often used to represent systems beyond the range of conventional visibility, as already noted above in our discussion of projectiles. But this is not without risk as we know from the planetary model adapted by Bohr to describe the electrons of an atom . . . the electrons were described as though they could actually be seen by a micro-observer and the resulting atomic theory was so constrained by this preconception that it failed to describe the way that atoms interact with radiaiton. There is the hazard that our experience with macroscopic systems may lead us to attribute to the microscopic system properties that it does not have. Common sense is not always a good guide in theoretical physics: "... of course if the electron goes through either one slit or the other ...".

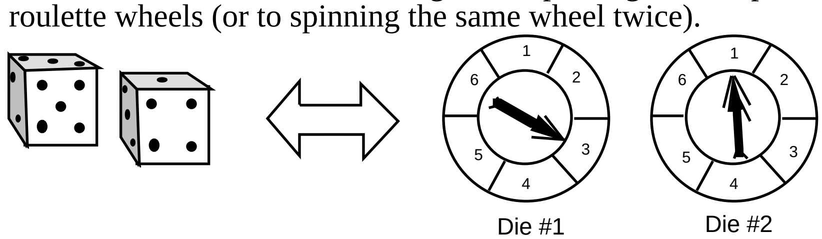
Abstract Models

Concept and Examples of Abstract Models

By the term abstract model we mean a formal construct (with rules for manipulation) which may have little obvious resemblance to the system of interest but that nevertheless exhibits responses and outcomes that resemble those of the physical system being represented. As examples we conside (1) the use of area to represent probability, and (2) the use of Fourier amplitudes to represent a vibrating string. :

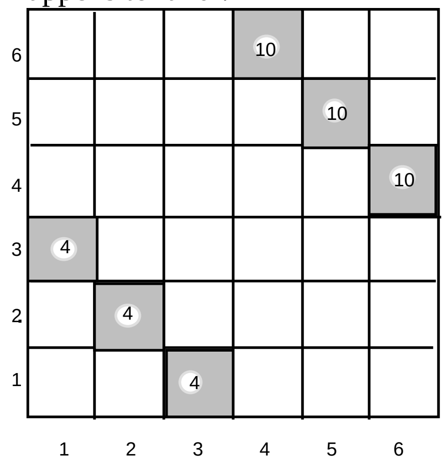
Example: Area representation of Probability

The throw of two dice is is analogous to spinning two simple roulette wheels (or to spinning the same wheel twice).



Dice are equivalent to Wheels of Chance

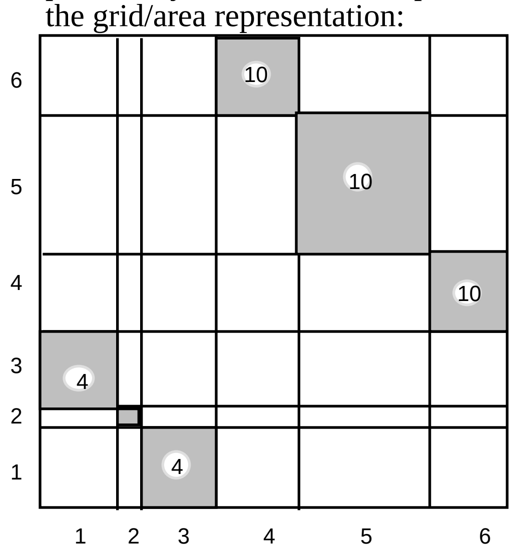
We see that games of different physical form may be essentially equivalent, and a useful model would not only show this equivalence but would also facilitate calculations. In this situation, we are dealing with pairs of random numbers as is common in many games of chance, and another equivalent game is the choice of a random point within a large square, where the square is divided into a 6x6 cellular grid. The score of a single trial is determined by the cell in which the randomly chosen point happens to land :



Fair dice in area representation

The probability of a given score depends on the how much area of the square is associated with that score. For example, in the figure above the grid is shaded for those combinations that yield a scores of 4 and scores of 10. Since relative areas represent probability, we easily see that the probability of getting 4 is the same as that of getting 10, each of these having the same probability: 3/36. These conclusions flow easily from the grid/area representation; they are somewhat harder to obtain from the dice or roulette wheel visualizations. Moreover, the grid/area representation is more easily extended to complicated situations. For example the throw of 3 dice is equivalent to the random choice of a point within a cube divided into 6x6x6 elements.

The area representation is adaptable to the situation in which the dice are loaded, for example if both dice have an enhanced probability for "5" at the expense of "2". This is easily shown on the grid/area representation:

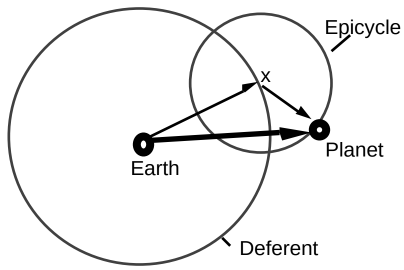


Loaded dice in the area representation

We see (not surprisingly) that 10 is now far more likely than 4. But we also have a qualitative measure of how other scores may be affected. This shows that the ease of understanding and the rapiditty of calculations can be enhanced if one makes a clever choice of representation.

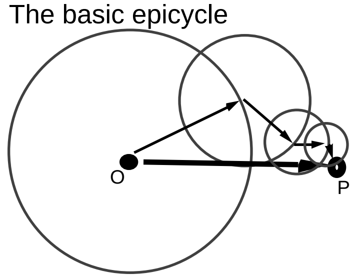
Example: Fourier representation of periodic motions

An obvious precursor of Fourier analysis is the method of epicycles devised in the times of the ancient Greeks (200 BC) that represents the observed motion of a planet as a resultant of two or more uniform circular motions:.



Planet moves uniformly on a circular path (the Epicycle) the center of which (x) moves uniformly on a circular path (the Deferent) about the earth

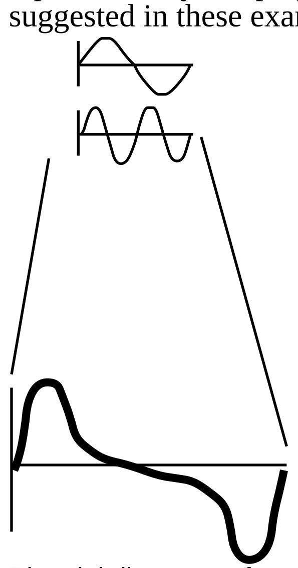
The basic epicycle

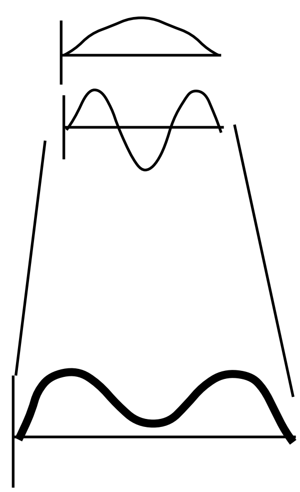


Arbitrary periodic motion of point P about center O synthesized by superposing several uniform circular motions of frequecies that are an integer multiple of the basic frequency of the motion.

More complicated epicycles and the essence of Fourier's method

A vibrating string with a complicated waveform can be represented by a superposition of fundamental plus harmonics, as suggested in these examples:





Pictorial diagrams of two-state superpositions

One can also represent these superpositions in a more abstract way by plotting the amplitude of Fourier component in each; this is called a *spectrum*. [note that the spectrum, as sketched here, does not convey the relative phases of the various contributions.]

Spectral diagrams of two-state superpositions

The algebraic expression of the Fourier series is quite abstract but is very useful for the representation of periodic phenomena:

$$F(t) = \sum_k A_k \cos\omega_k t + B_k \sin\omega_k t \quad or \quad F(t) = \sum_k C_k e^{i(\omega_k t + \phi_k)}$$

To see a pair of engineers at work, one with construction blueprints and the other with Fourier methods, is to appreciate the contrast between literal and abstract models. Blueprints give an unambiguous image of the object at hand, whereas it would be hard to tell whether the series of equations on paper applied to a violin, a telephone, or an automobile.

Abstract Models Provide Insights

Abstract models often provide unexpected insights and useful methods for calculations. We can see this from the examples cited above. In the dice example, the geometric representation quickly and intuitively shows the probability for getting a particular total from a throw of the dice, whether fair or loaded. One can easily generalize to throws of 3 or more dice.

Abstract Models can Unify

Abstract models are sometimes found to apply to two apparently disparate phenomena and this suggests that a deeper physical principle underlies them both. For example (as we will show later) the abstract operator representation of the mass subject to a linear restoring force (harmonic oscillator) also applies to the electromagnetic field. And for another example (also shown later), the operator representation of orbital angular momentum turns out to be valid for the representation of spins and (as *isotopic* spin) for representation of nucleon charge states.

Abstract Models may seduce their users

Abstract models are often hard to understand. But a model, once mastered, may captivate the user by its elegance and broad span. *Se no e vero, e ben trovato*… one may be lured by that elegance into discarding data that does not fit the model. Or the aesthetic pleasures offered by the model may, by themselves, be a justification for its study… it may not matter whether the model applies to observable phenomena.

Models and Reality

The distinction between literal and abstract is a subjective one, since what seems abstract and remote to one person may seem almost pictorial to another. A very important point that we can now understand is:

The models we use are not identical with the systems themselves. No one would claim that a set of Fourier amplitudes *is* a vibrating string, or that the point on the 6x6 grid *is* a pair of dice.

Why belabor this obvious point?

Because later in our discussions of electrons and photons we will use periodic functions (waves) to represent predictions for the future; we will use point-like functions (particles) to represent observations that have been made to test those predictions. And we will not claim that electrons and photons "are" either waves or particles.

What sort of problems are appropriate for this chapter?

Historical Assignments on

discovery of helium

Brahe, Copernicus, Galileo
Hubble and red shift
Black body radiation

given a series of numbers, find the pattern,

predict the next few values
pick missing interpolated values

are there pattern discovery games?

Examples of models:

Modify the Hatfied McCoy game;

make up a CIA vs KGB game with matrix representation for annihilation, for conversion to one of our own,

Make up a drill field representation with Left face, right face, about face, and stay as you are. What is the group multiplication table for this?

Asides

Should a phenomenological "theory" actually be called a phenomenological "model" for the situation at hand? Also, we should recognize that a theory may be crassly phenomnological or it might be on its way to being a fundamental theory The glossing of this statement might be a large subject in its own right, more than the scope of this text would encompass

We cannot, for example, talk about the color of an electron in the context of ordinary atomic theory. Yet, with suitable definitions, we can talk about the flavor of a quark.

Among the few examples where workaday physicists engage the philosophers of science in a quantitative way are in studies of Bell's theorem where experiments and their interpretation provide a deeper understanding of quantum mechanics even as they confirm the established paradigm.