PL Derivations, week 2

Philosophy UA-70

1	$\sim(A \vee \sim A)$	*AIP*
2	$\sim A \bullet \sim\sim A$	1, *DM*
3	$\sim\sim(A \vee \sim A)$	1–2, *IP*
4	$A \vee \sim A$	3, *DN*

From the Homework

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$

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4	$(E \supset\sim C) \bullet (K \supset\sim E)$	2, 3, *Conj*

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⋮		
n	$E \vee K$	???

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$n+1$	$\sim C \vee \sim E$	4, n, *CD*

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$n+1$	$\sim C \vee \sim E$	4, n, *CD*
$n+2$	$\sim (C \bullet E)$	$n+1$, *DM*

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n	$E \vee K$	???
$n+1$	$\sim C \vee \sim E$	4, n, *CD*
$n+2$	$\sim (C \bullet E)$	$n+1$, *DM*
$n+3$	$(C \bullet E) \vee (T \bullet K)$	1, *Com*

FROM THE HOMEWORK

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$(E \supset \sim C) \bullet (K \supset \sim E)$	2, 3, *Conj*
$\vdots$		
n	$E \vee K$	???
$n+1$	$\sim C \vee \sim E$	4, n, *CD*
$n+2$	$\sim (C \bullet E)$	$n+1$, *DM*
$n+3$	$(C \bullet E) \vee (T \bullet K)$	1, *Com*
$n+4$	$T \bullet K$	$n+2, n+1$, *DS*

From the Homework

$$(T \bullet K) \vee (C \bullet E)$$

FROM THE HOMEWORK

$$(T \bullet K) \vee (C \bullet E)$$

Distribution (Dist)

$$p \bullet (q \vee r) \lhd\rhd (p \bullet q) \vee (p \bullet r)$$
$$p \vee (q \bullet r) \lhd\rhd (p \vee q) \bullet (p \vee r)$$

From the Homework

$$(T \bullet K) \vee (C \bullet E)$$

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$$p \bullet (q \vee r) \dashv\vdash (p \bullet q) \vee (p \bullet r)$$

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FROM THE HOMEWORK

$$(T \bullet K) \vee (C \bullet E)$$

$$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$$

Distribution (Dist)

$$p \bullet (q \vee r) \dashv\vdash (p \bullet q) \vee (p \bullet r)$$
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2	$K \supset\sim E$	
3	$E \supset\sim C$	$/T \bullet K$
4	$(E \supset\sim C) \bullet (K \supset\sim E)$	2, 3, *Conj*
5	$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$	1, *Dist*

From the Homework

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset\sim E$	
3	$E \supset\sim C$	$/T \bullet K$
4	$(E \supset\sim C) \bullet (K \supset\sim E)$	2, 3, *Conj*
5	$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$	1, *Dist*
6	$((T \bullet K) \vee E) \bullet ((T \bullet K) \vee C)$	5, *Com*

From the Homework

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$(E \supset \sim C) \bullet (K \supset \sim E)$	2, 3, *Conj*
5	$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$	1, *Dist*
6	$((T \bullet K) \vee E) \bullet ((T \bullet K) \vee C)$	5, *Com*
7	$(T \bullet K) \vee E$	6, *Simp*

From the Homework

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$(E \supset \sim C) \bullet (K \supset \sim E)$	2, 3, *Conj*
5	$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$	1, *Dist*
6	$((T \bullet K) \vee E) \bullet ((T \bullet K) \vee C)$	5, *Com*
7	$(T \bullet K) \vee E$	6, *Simp*
8	$E \vee (T \bullet K)$	7, *Com*

From the Homework

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset\sim E$	
3	$E \supset\sim C$	$/T \bullet K$
4	$(E \supset\sim C) \bullet (K \supset\sim E)$	2, 3, *Conj*
5	$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$	1, *Dist*
6	$((T \bullet K) \vee E) \bullet ((T \bullet K) \vee C)$	5, *Com*
7	$(T \bullet K) \vee E$	6, *Simp*
8	$E \vee (T \bullet K)$	7, *Com*
9	$(E \vee T) \bullet (E \vee K)$	8, *Dist*

From the Homework

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset\sim E$	
3	$E \supset\sim C$	$/T \bullet K$
4	$(E \supset\sim C) \bullet (K \supset\sim E)$	2, 3, *Conj*
5	$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$	1, *Dist*
6	$((T \bullet K) \vee E) \bullet ((T \bullet K) \vee C)$	5, *Com*
7	$(T \bullet K) \vee E$	6, *Simp*
8	$E \vee (T \bullet K)$	7, *Com*
9	$(E \vee T) \bullet (E \vee K)$	8, *Dist*
10	$(E \vee K) \bullet (E \vee T)$	9, *Com*

From the Homework

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset\sim E$	
3	$E \supset\sim C$	$/T \bullet K$
4	$(E \supset\sim C) \bullet (K \supset\sim E)$	2, 3, *Conj*
5	$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$	1, *Dist*
6	$((T \bullet K) \vee E) \bullet ((T \bullet K) \vee C)$	5, *Com*
7	$(T \bullet K) \vee E$	6, *Simp*
8	$E \vee (T \bullet K)$	7, *Com*
9	$(E \vee T) \bullet (E \vee K)$	8, *Dist*
10	$(E \vee K) \bullet (E \vee T)$	9, *Com*
11	$E \vee K$	10, *Simp*

From the Homework

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2	$K \supset\sim E$	
3	$E \supset\sim C$	$/T \bullet K$
4	$(E \supset\sim C) \bullet (K \supset\sim E)$	2, 3, *Conj*
5	$((T \bullet K) \vee C) \bullet ((T \bullet K) \vee E)$	1, *Dist*
6	$((T \bullet K) \vee E) \bullet ((T \bullet K) \vee C)$	5, *Com*
7	$(T \bullet K) \vee E$	6, *Simp*
8	$E \vee (T \bullet K)$	7, *Com*
9	$(E \vee T) \bullet (E \vee K)$	8, *Dist*
10	$(E \vee K) \bullet (E \vee T)$	9, *Com*
11	$E \vee K$	10, *Simp*
⋮		

Outline

Four Final Inference Rules
- Subderivations
- Conditional Proof
- Indirect Proof

PL-Derivability and the Logical Notions of *PL*
- *PL*-Validity
- *PL*-Tautologies and *PL*-Self-Contradictions
- *PL*-Equivalence and *PL*-Contradiction
- *PL*-Inconsistency

OUTLINE

FOUR FINAL INFERENCE RULES
- SUBDERIVATIONS
- CONDITIONAL PROOF
- INDIRECT PROOF

PL-DERIVABILITY AND THE LOGICAL NOTIONS OF *PL*
- *PL*-VALIDITY
- *PL*-TAUTOLOGIES AND *PL*-SELF-CONTRADICTIONS
- *PL*-EQUIVALENCE AND *PL*-CONTRADICTION
- *PL*-INCONSISTENCY

OUTLINE

FOUR FINAL INFERENCE RULES
SUBDERIVATIONS
CONDITIONAL PROOF
INDIRECT PROOF

PL-DERIVABILITY AND THE LOGICAL NOTIONS OF *PL*
PL-VALIDITY
PL-TAUTOLOGIES AND *PL*-SELF-CONTRADICTIONS
PL-EQUIVALENCE AND *PL*-CONTRADICTION
PL-INCONSISTENCY

SUBDERIVATIONS

- A subderivation is a kind of *suppositional* derivation which takes place within another derivation.

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- *E.g.*,

1	$(A \supset B) \supset C$	
2	$B \bullet D$	$/C$
3	$\quad A$	ACP
4	$\quad B$	2, *Simp*
5	$A \supset B$	3–4, *CP*
6	C	1, 5, *MP*

ASSUMPTION FOR CONDITIONAL PROOF

Assumption for Conditional Proof (ACP)

You may, at any point in a derivation, begin a new SUBDERIVATION, and write *any wff of PL whatsoever* on the first line of that sub derivation. In the justification line, you should write '*ACP*'.

ASSUMPTION FOR INDIRECT PROOF

Assumption for Indirect Proof (AIP)

You may, at any point in a derivation, begin a new SUBDERIVATION, and write *any wff of PL whatsoever* on the first line of that sub derivation. In the justification line, you should write '*AIP*'.

ACCESSIBILITY

- You may also decide to *end* a subderivation *whenever you wish*

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- This threatens to make it too easy to derive anything at all:

1	$A \supset B$	$/ \sim A$
2	$\sim B$	*ACP*
3	$\sim A$	1, 2, *MT* $\longleftarrow$ **MISTAKE!!!**

ACCESSIBILITY

At a given line in a derivation, n, another line of the derivation, m, is ACCESSIBLE if and only if 1) line m *precedes* line n ($m < n$), and 2) either i) line m lies outside the scope of any subderivation, or ii) line m lies within a subderivation whose vertical scope line extends to line n.

ACCESSIBILITY

An *entire subderivation* is accessible at line *n* so long as 1) the subderivation *precedes* line *n*, and 2) either i) that subderivation is outside the scope of any other subderivation, or else ii) the subderivation lines within another subderivation whose vertical scope line extends to line *n*.

ACCESSIBILITY

An *entire subderivation* is accessible at line n so long as 1) the subderivation *precedes* line n, and 2) either i) that subderivation is outside the scope of any other subderivation, or else ii) the subderivation lines within another subderivation whose vertical scope line extends to line n.

- while you may end a subderivation whenever you wish, once you do so, none of the lines or subderivations appearing within the scope of that subderivation are accessible any longer.

ACCESSIBILITY

1	$C \bullet Z$	
2	$\quad A$	*ACP*
3	$\quad\quad B$	*ACP*
4	$\quad\quad C$	1, *Simp*
5	$\quad B \supset C$	3–4, *CP*
6	$A \supset (B \supset C)$	2–5, *CP*
7	$\quad B \supset A$	*ACP*
8	$\quad C$	1, *Simp*
9	$(B \supset A) \supset C$	7–8, *CP*
10	$(A \supset (B \supset C)) \bullet ((B \supset A) \supset C)$	6, 9, *Conj*

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8	$\quad C$	1, *Simp*
9	$(B \supset A) \supset C$	7–8, *CP*
10	$(A \supset (B \supset C)) \bullet ((B \supset A) \supset C)$	6, 9, *Conj*

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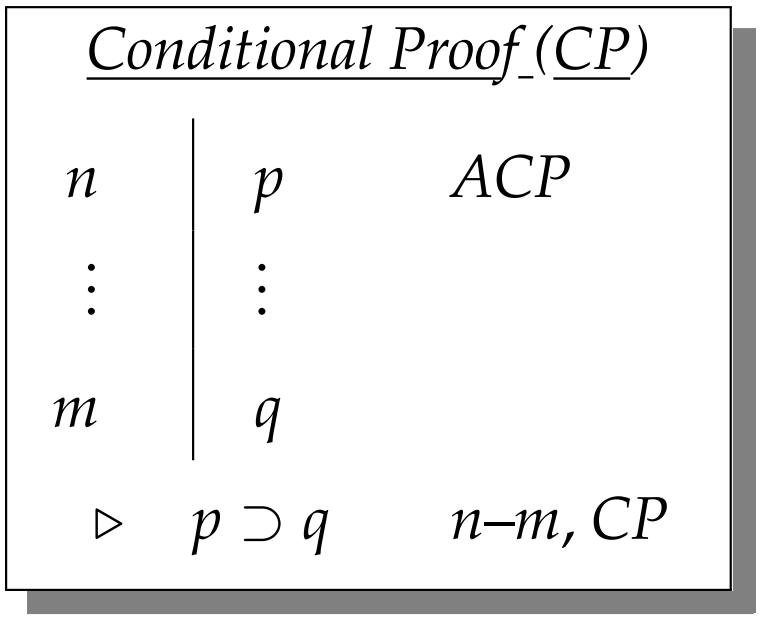
CONDITIONAL PROOF



A SAMPLE DERIVATION

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2	$B \supset C$	$/A \supset C$

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A SAMPLE DERIVATION

1	$A \supset B$	
2	$B \supset C$	$/A \supset C$
3	$\vert\ A$	ACP

A SAMPLE DERIVATION

1	$A \supset B$	
2	$B \supset C$	$/A \supset C$
3	$\quad A$	ACP
4	$\quad B$	$1, 3, MP$

A SAMPLE DERIVATION

1	$A \supset B$	
2	$B \supset C$	$/A \supset C$
3	$\quad A$	ACP
4	$\quad B$	$1, 3, MP$
5	$\quad C$	$2, 4, MP$

A SAMPLE DERIVATION

1	$A \supset B$	
2	$B \supset C$	$/A \supset C$
3	$\quad A$	ACP
4	$\quad B$	$1, 3, MP$
5	$\quad C$	$2, 4, MP$
6	$A \supset C$	$3\text{–}5, CP$

A SAMPLE DERIVATION

1	$A \vee (B \supset Q)$	
2	B	$/\sim A \supset Q$

A SAMPLE DERIVATION

1	$A \vee (B \supset Q)$		
2	B		$/\sim A \supset Q$
3	$\vert$	$\sim A$	ACP

A SAMPLE DERIVATION

1		$A \vee (B \supset Q)$	
2		B	$/\sim A \supset Q$
3	|	$\sim A$	ACP
4	|	$B \supset Q$	$1, 3, DS$

A SAMPLE DERIVATION

1		$A \vee (B \supset Q)$	
2		B	$/\sim A \supset Q$
3	|	$\sim A$	ACP
4	|	$B \supset Q$	$1, 3, DS$
5	|	Q	$2, 4, MP$

A SAMPLE DERIVATION

1	$A \vee (B \supset Q)$		
2	B		$/\sim A \supset Q$
3	$\quad \sim A$		ACP
4	$\quad B \supset Q$		1, 3, DS
5	$\quad Q$		2, 4, MP
6	$\sim A \supset Q$		3–5, CP

A SAMPLE DERIVATION

$$1 \quad A \qquad\qquad /B \supset (A \supset B)$$

A SAMPLE DERIVATION

1	A	$/B \supset (A \supset B)$
2	$\vert\ B$	ACP

A SAMPLE DERIVATION

1	A	$/B \supset (A \supset B)$
2	$\quad B$	ACP
3	$\quad\quad A$	ACP

A SAMPLE DERIVATION

1	A	$/B \supset (A \supset B)$
2	$\quad B$	ACP
3	$\quad\quad A$	ACP
4	$\quad\quad B \vee B$	2, *Taut*

A SAMPLE DERIVATION

1	A		$/B \supset (A \supset B)$
2	| B		ACP
3	| | A		ACP
4	| | $B \vee B$		2, *Taut*
5	| | B		4, *Taut*

A SAMPLE DERIVATION

1	A	$/B \supset (A \supset B)$
2	$\quad B$	*ACP*
3	$\quad\quad A$	*ACP*
4	$\quad\quad B \vee B$	2, *Taut*
5	$\quad\quad B$	4, *Taut*
6	$\quad A \supset B$	3–5, *CP*

A SAMPLE DERIVATION

1	A	$/B \supset (A \supset B)$
2	$\quad B$	*ACP*
3	$\quad\quad A$	*ACP*
4	$\quad\quad B \vee B$	2, *Taut*
5	$\quad\quad B$	4, *Taut*
6	$\quad A \supset B$	3–5, *CP*
7	$B \supset (A \supset B)$	2–6, *CP*

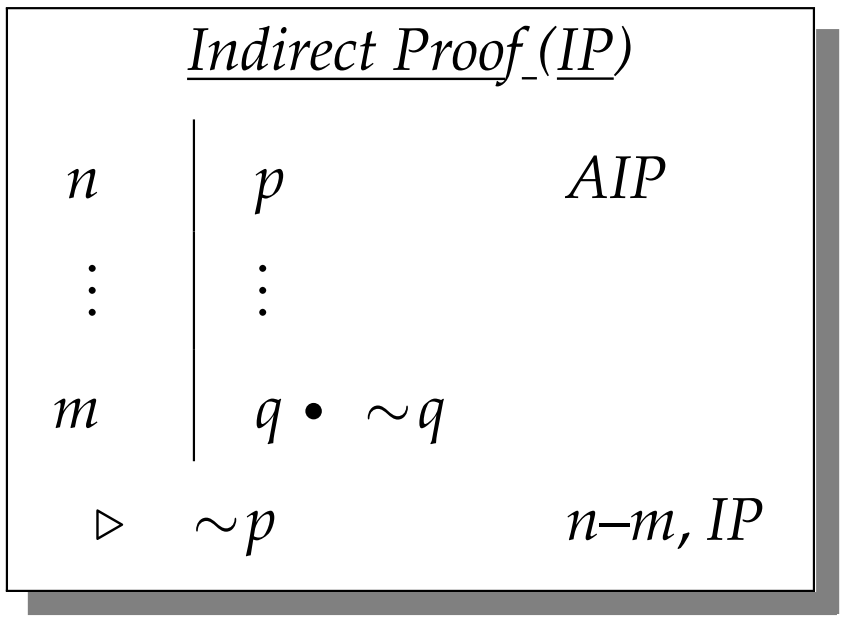
IN CLASS EXERCISE!

OUTLINE

FOUR FINAL INFERENCE RULES
SUBDERIVATIONS
CONDITIONAL PROOF
INDIRECT PROOF

PL-DERIVABILITY AND THE LOGICAL NOTIONS OF *PL*
PL-VALIDITY
PL-TAUTOLOGIES AND *PL*-SELF-CONTRADICTIONS
PL-EQUIVALENCE AND *PL*-CONTRADICTION
PL-INCONSISTENCY

INDIRECT PROOF



INDIRECT PROOF

- **NOTE**: the explicit contradiction *must be of the form* '$q \bullet \sim q$'.

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1	$A \bullet B$		
2	$\sim A$	*AIP*	
3	A	1, *Simp*	
4	$\sim A \bullet A$	2, 3, *Conj*	
5	$\sim\sim A$	2–4, *IP*	⟵ **MISTAKE!!!**

INDIRECT PROOF

- **NOTE**: the explicit contradiction *must be of the form* '$q \bullet \sim q$'.

1	$A \bullet B$		
2	$\sim A$		*AIP*
3	A		1, *Simp*
4	$A \bullet \sim A$		2, 3, *Conj*
5	$\sim\sim A$		2–4, *IP*

INDIRECT PROOF

- what you conclude outside of the subderivation *must be the negation of the thing you assumed*.

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- what you conclude outside of the subderivation *must be the negation of the thing you assumed*.

1	$\sim(A \vee \sim A)$	*AIP*	
2	$\sim A \bullet \ \sim\sim A$	1, *DM*	
3	$A \vee \sim A$	1–2, *IP*	$\longleftarrow$ **MISTAKE!!!**

INDIRECT PROOF

- what you conclude outside of the subderivation *must be the negation of the thing you assumed*.

1	$\sim(A \vee \sim A)$	*AIP*
2	$\sim A \bullet \sim\sim A$	1, *DM*
3	$\sim\sim(A \vee \sim A)$	1–2, *IP*
4	$A \vee \sim A$	3, *DN*

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$

A SAMPLE DERIVATION

1	$P \equiv Q$		
2	$P \vee Q$		$/P$
3	$\vert$	$\sim P$	AIP

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$
3	$\sim P$	*AIP*
4	Q	2, 3, *DS*

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$
3	$\sim P$	*AIP*
4	Q	2, 3, *DS*
5	$(P \supset Q) \bullet (Q \supset P)$	1, *Equiv*

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$
3	$\quad \sim P$	*AIP*
4	$\quad Q$	2, 3, *DS*
5	$\quad (P \supset Q) \bullet (Q \supset P)$	1, *Equiv*
6	$\quad (Q \supset P) \bullet (P \supset Q)$	5, *Com*

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$
3	$\sim P$	*AIP*
4	Q	2, 3, *DS*
5	$(P \supset Q) \bullet (Q \supset P)$	1, *Equiv*
6	$(Q \supset P) \bullet (P \supset Q)$	5, *Com*
7	$Q \supset P$	6, *Simp*

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$
3	$\quad \sim P$	*AIP*
4	$\quad Q$	2, 3, *DS*
5	$\quad (P \supset Q) \bullet (Q \supset P)$	1, *Equiv*
6	$\quad (Q \supset P) \bullet (P \supset Q)$	5, *Com*
7	$\quad Q \supset P$	6, *Simp*
8	$\quad P$	4, 7, *MP*

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$
3	$\sim P$	*AIP*
4	Q	2, 3, *DS*
5	$(P \supset Q) \bullet (Q \supset P)$	1, *Equiv*
6	$(Q \supset P) \bullet (P \supset Q)$	5, *Com*
7	$Q \supset P$	6, *Simp*
8	P	4, 7, *MP*
9	$P \bullet \sim P$	3, 8, *Conj*

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$
3	$\quad \sim P$	*AIP*
4	$\quad Q$	2, 3, *DS*
5	$\quad (P \supset Q) \bullet (Q \supset P)$	1, *Equiv*
6	$\quad (Q \supset P) \bullet (P \supset Q)$	5, *Com*
7	$\quad Q \supset P$	6, *Simp*
8	$\quad P$	4, 7, *MP*
9	$\quad P \bullet \sim P$	3, 8, *Conj*
10	$\sim\sim P$	3–9, *IP*

A SAMPLE DERIVATION

1	$P \equiv Q$	
2	$P \vee Q$	$/P$
3	$\quad \sim P$	*AIP*
4	$\quad Q$	2, 3, *DS*
5	$\quad (P \supset Q) \bullet (Q \supset P)$	1, *Equiv*
6	$\quad (Q \supset P) \bullet (P \supset Q)$	5, *Com*
7	$\quad Q \supset P$	6, *Simp*
8	$\quad P$	4, 7, *MP*
9	$\quad P \bullet \sim P$	3, 8, *Conj*
10	$\sim\sim P$	3–9, *IP*
11	P	10, *DN*

A SAMPLE DERIVATION

1	A	$/\sim A \supset (Z \equiv W)$

A SAMPLE DERIVATION

1	A		$/\sim A \supset (Z \equiv W)$
2	$\vert$	$\sim A$	ACP

A SAMPLE DERIVATION

1	A		$/\sim A \supset (Z \equiv W)$
2	$\vert$	$\sim A$	ACP
3	$\vert$	$\vert$ $\sim (Z \equiv W)$	AIP

A SAMPLE DERIVATION

1	A	$/\sim A \supset (Z \equiv W)$
2	$\quad \sim A$	*ACP*
3	$\quad\quad \sim(Z \equiv W)$	*AIP*
4	$\quad\quad A \bullet \sim A$	1, 2, *Conj*

A SAMPLE DERIVATION

1	A	$/\sim A \supset (Z \equiv W)$
2	$\quad \sim A$	*ACP*
3	$\quad\quad \sim (Z \equiv W)$	*AIP*
4	$\quad\quad A \bullet \sim A$	1, 2, *Conj*
5	$\quad \sim\sim (Z \equiv W)$	3–4, *IP*

A SAMPLE DERIVATION

1	A	$/\sim A \supset (Z \equiv W)$
2	$\quad \sim A$	*ACP*
3	$\quad\quad \sim(Z \equiv W)$	*AIP*
4	$\quad\quad A \bullet \sim A$	1, 2, *Conj*
5	$\quad \sim\sim(Z \equiv W)$	3–4, *IP*
6	$\quad Z \equiv W$	5, *DN*

A SAMPLE DERIVATION

1	A	$/\sim A \supset (Z \equiv W)$
2	$\quad \sim A$	*ACP*
3	$\quad\quad \sim(Z \equiv W)$	*AIP*
4	$\quad\quad A \bullet \sim A$	1, 2, *Conj*
5	$\quad \sim\sim(Z \equiv W)$	3–4, *IP*
6	$\quad Z \equiv W$	5, *DN*
7	$\sim A \supset (Z \equiv W)$	2–6, *CP*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*
5	$C \bullet E$	1, 4, *DS*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*
5	$C \bullet E$	1, 4, *DS*
6	$E \bullet C$	5, *Com*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*
5	$C \bullet E$	1, 4, *DS*
6	$E \bullet C$	5, *Com*
7	E	6, *Simp*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*
5	$C \bullet E$	1, 4, *DS*
6	$E \bullet C$	5, *Com*
7	E	6, *Simp*
8	C	5, *Simp*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*
5	$C \bullet E$	1, 4, *DS*
6	$E \bullet C$	5, *Com*
7	E	6, *Simp*
8	C	5, *Simp*
9	$\sim C$	3, 7, *MP*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*
5	$C \bullet E$	1, 4, *DS*
6	$E \bullet C$	5, *Com*
7	E	6, *Simp*
8	C	5, *Simp*
9	$\sim C$	3, 7, *MP*
10	$C \bullet \sim C$	8, 9, *Conj*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*
5	$C \bullet E$	1, 4, *DS*
6	$E \bullet C$	5, *Com*
7	E	6, *Simp*
8	C	5, *Simp*
9	$\sim C$	3, 7, *MP*
10	$C \bullet \sim C$	8, 9, *Conj*
11	$\sim\sim (T \bullet K)$	4–10, *IP*

A SAMPLE DERIVATION

1	$(T \bullet K) \vee (C \bullet E)$	
2	$K \supset \sim E$	
3	$E \supset \sim C$	$/T \bullet K$
4	$\sim (T \bullet K)$	*AIP*
5	$C \bullet E$	1, 4, *DS*
6	$E \bullet C$	5, *Com*
7	E	6, *Simp*
8	C	5, *Simp*
9	$\sim C$	3, 7, *MP*
10	$C \bullet \sim C$	8, 9, *Conj*
11	$\sim\sim (T \bullet K)$	4–10, *IP*
12	$T \bullet K$	11, *DN*

In Class Exercise!

Outline

Four Final Inference Rules
- Subderivations
- Conditional Proof
- Indirect Proof

PL-Derivability and the Logical Notions of *PL*
- *PL*-Validity
- *PL*-Tautologies and *PL*-Self-Contradictions
- *PL*-Equivalence and *PL*-Contradiction
- *PL*-Inconsistency

SOME NEW NOTATION

- If and only if the argument $p_1 \,/\, p_2 \,/\, \ldots p_N \,//\, c$ is *PL*-valid, I will write:

$$p_1, p_2, \ldots, p_N \models_{PL} c$$

SOME NEW NOTATION

- If and only if the argument $p_1 \,/\, p_2 \,/\, \ldots p_N \,//\, c$ is *PL*-valid, I will write:

$$p_1, p_2, \ldots, p_N \models_{PL} c$$

- If and only if there is a legal *PL*-derivation whose assumptions are $p_1, p_2, \ldots,$ and p_N and whose final line is c, (henceforth: c is *PL*-derivable from $p_1, p_2, \ldots,$ and p_N) I will write:

$$p_1, p_2, \ldots, p_N \vdash_{PL} c$$

Outline

Four Final Inference Rules
- Subderivations
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PL-Derivability and the Logical Notions of *PL*
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PL-VALIDITY AND *PL*-DERIVABILITY

Fact 1: $p_1 \,/\, p_2 \,/\, \ldots \,/\, p_N \,//\, c$ is *PL*-valid if and only if c is *PL*-derivable from $p_1, p_2, \ldots,$ and p_N.

$p_1, p_2 \ldots, p_N \models_{PL} c$ if and only if $p_1, p_2, \ldots, p_N \vdash_{PL} c$

Outline

Four Final Inference Rules
- Subderivations
- Conditional Proof
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PL-Derivability and the Logical Notions of *PL*
- *PL*-Validity
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PL-TAUTOLOGIES AND *PL*-DERIVABILITY

Fact 2: A wff of *PL*, p, is a *PL*-tautology if and only if

$$\vdash_{PL} p$$

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

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- What is it for a *PL*-derivation to have no assumptions?

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

Fact 2: A wff of *PL*, p, is a *PL*-tautology if and only if

$$\vdash_{PL} p$$

- What is it for a *PL*-derivation to have no assumptions?
- Consider:

1	$\sim(A \vee \sim A)$	*AIP*
2	$\sim A \bullet \ \sim\sim A$	1, *DM*
3	$\sim\sim(A \vee \sim A)$	1–2, *IP*
4	$A \vee \sim A$	3, *DN*

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	|	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2			

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \ \sim(P \supset R)$	1, *DM*
3		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \ \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \ \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \ \sim Q$	4, *DM*
6		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \ \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \ \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7	$\sim(P \supset R) \bullet \sim(\sim P \supset Q)$	2, *Com*
8		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7	$\sim(P \supset R) \bullet \sim(\sim P \supset Q)$	2, *Com*
8	$\sim(P \supset R)$	7, *Simp*
9		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7	$\sim(P \supset R) \bullet \sim(\sim P \supset Q)$	2, *Com*
8	$\sim(P \supset R)$	7, *Simp*
9	$\sim(\sim P \vee R)$	8, *Impl*
10		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7	$\sim(P \supset R) \bullet \sim(\sim P \supset Q)$	2, *Com*
8	$\sim(P \supset R)$	7, *Simp*
9	$\sim(\sim P \vee R)$	8, *Impl*
10	$\sim\sim P \bullet \sim R$	9, *DM*
11		

PL-TAUTOLOGIES AND PL-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7	$\sim(P \supset R) \bullet \sim(\sim P \supset Q)$	2, *Com*
8	$\sim(P \supset R)$	7, *Simp*
9	$\sim(\sim P \vee R)$	8, *Impl*
10	$\sim\sim P \bullet \sim R$	9, *DM*
11	$\sim\sim P$	10, *Simp*
12		

PL-TAUTOLOGIES AND PL-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7	$\sim(P \supset R) \bullet \sim(\sim P \supset Q)$	2, *Com*
8	$\sim(P \supset R)$	7, *Simp*
9	$\sim(\sim P \vee R)$	8, *Impl*
10	$\sim\sim P \bullet \sim R$	9, *DM*
11	$\sim\sim P$	10, *Simp*
12	$\sim\sim P \bullet \sim\sim\sim P$	6, 11, *Conj*
13		

PL-TAUTOLOGIES AND PL-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7	$\sim(P \supset R) \bullet \sim(\sim P \supset Q)$	2, *Com*
8	$\sim(P \supset R)$	7, *Simp*
9	$\sim(\sim P \vee R)$	8, *Impl*
10	$\sim\sim P \bullet \sim R$	9, *DM*
11	$\sim\sim P$	10, *Simp*
12	$\sim\sim P \bullet \sim\sim\sim P$	6, 11, *Conj*
13	$\sim\sim((\sim P \supset Q) \vee (P \supset R))$	1–12, *IP*
14		

PL-TAUTOLOGIES AND PL-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
6	$\sim\sim\sim P$	5, *Simp*
7	$\sim(P \supset R) \bullet \sim(\sim P \supset Q)$	2, *Com*
8	$\sim(P \supset R)$	7, *Simp*
9	$\sim(\sim P \vee R)$	8, *Impl*
10	$\sim\sim P \bullet \sim R$	9, *DM*
11	$\sim\sim P$	10, *Simp*
12	$\sim\sim P \bullet \sim\sim\sim P$	6, 11, *Conj*
13	$\sim\sim((\sim P \supset Q) \vee (P \supset R))$	1–12, *IP*
14	$(\sim P \supset Q) \vee (P \supset R)$	13, *DN*

PL-TAUTOLOGIES AND PL-DERIVABILITY

1	$\sim((\sim P \supset Q) \vee (P \supset R))$	*AIP*
2	$\sim(\sim P \supset Q) \bullet \sim(P \supset R)$	1, *DM*
3	$\sim(\sim P \supset Q)$	2, *Simp*
4	$\sim(\sim\sim P \vee Q)$	3, *Impl*
5	$\sim\sim\sim P \bullet \sim Q$	4, *DM*
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10	$\sim\sim P \bullet \sim R$	9, *DM*
11	$\sim\sim P$	10, *Simp*
12	$\sim\sim P \bullet \sim\sim\sim P$	6, 11, *Conj*
13	$\sim\sim((\sim P \supset Q) \vee (P \supset R))$	1–12, *IP*
14	$(\sim P \supset Q) \vee (P \supset R)$	13, *DN*

PL-TAUTOLOGIES AND PL-DERIVABILITY

1	|	$\sim(\sim P \supset Q)$	*ACP*
2			

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4	$\sim\sim\sim P$	3, *Simp*
5		

PL-TAUTOLOGIES AND PL-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4	$\sim\sim\sim P$	3, *Simp*
5	$\sim P$	4, *DN*
6		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4	$\sim\sim\sim P$	3, *Simp*
5	$\sim P$	4, *DN*
6	$\sim P \vee R$	5, *Add*
7		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4	$\sim\sim\sim P$	3, *Simp*
5	$\sim P$	4, *DN*
6	$\sim P \vee R$	5, *Add*
7	$P \supset R$	6, *Impl*
8		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4	$\sim\sim\sim P$	3, *Simp*
5	$\sim P$	4, *DN*
6	$\sim P \vee R$	5, *Add*
7	$P \supset R$	6, *Impl*
8	$\sim(\sim P \supset Q) \supset (P \supset R)$	1–7, *CP*
9		

PL-TAUTOLOGIES AND PL-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4	$\sim\sim\sim P$	3, *Simp*
5	$\sim P$	4, *DN*
6	$\sim P \vee R$	5, *Add*
7	$P \supset R$	6, *Impl*
8	$\sim(\sim P \supset Q) \supset (P \supset R)$	1–7, *CP*
9	$\sim\sim(\sim P \supset Q) \vee (P \supset R)$	8, *Impl*
10		

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4	$\sim\sim\sim P$	3, *Simp*
5	$\sim P$	4, *DN*
6	$\sim P \vee R$	5, *Add*
7	$P \supset R$	6, *Impl*
8	$\sim(\sim P \supset Q) \supset (P \supset R)$	1–7, *CP*
9	$\sim\sim(\sim P \supset Q) \vee (P \supset R)$	8, *Impl*
10	$(\sim P \supset Q) \vee (P \supset R)$	9, *DN*

PL-TAUTOLOGIES AND *PL*-DERIVABILITY

1	$\sim(\sim P \supset Q)$	*ACP*
2	$\sim(\sim\sim P \vee Q)$	1, *Impl*
3	$\sim\sim\sim P \bullet \ \sim Q$	2, *DM*
4	$\sim\sim\sim P$	3, *Simp*
5	$\sim P$	4, *DN*
6	$\sim P \vee R$	5, *Add*
7	$P \supset R$	6, *Impl*
8	$\sim(\sim P \supset Q) \supset (P \supset R)$	1–7, *CP*
9	$\sim\sim(\sim P \supset Q) \vee (P \supset R)$	8, *Impl*
10	$(\sim P \supset Q) \vee (P \supset R)$	9, *DN*

IN CLASS EXERCISE!

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

Fact 3: A wff of *PL*, p, is a *PL*-self-contradiction if and only if

$$p \vdash_{PL} A \bullet \sim A$$

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1 $\quad \sim (P \supset Q) \bullet \sim (Q \supset P)$

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim (P \supset Q) \bullet \sim (Q \supset P)$	
2	$\sim (Q \supset P) \bullet \sim (P \supset Q)$	1, *Com*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*

PL-SELF-CONTRADICTION AND *PL*-DERIVABILITY

1	$\sim (P \supset Q) \bullet \sim (Q \supset P)$	
2	$\sim (Q \supset P) \bullet \sim (P \supset Q)$	1, *Com*
3	$\quad \sim (A \bullet \sim A)$	*AIP*
4	$\quad \sim (P \supset Q)$	1, *Simp*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim (P \supset Q) \bullet \sim (Q \supset P)$	
2	$\sim (Q \supset P) \bullet \sim (P \supset Q)$	1, *Com*
3	$\sim (A \bullet \sim A)$	*AIP*
4	$\sim (P \supset Q)$	1, *Simp*
5	$\sim (Q \supset P)$	2, *Simp*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim (P \supset Q) \bullet \sim (Q \supset P)$	
2	$\sim (Q \supset P) \bullet \sim (P \supset Q)$	1, *Com*
3	$\sim (A \bullet \sim A)$	*AIP*
4	$\sim (P \supset Q)$	1, *Simp*
5	$\sim (Q \supset P)$	2, *Simp*
6	$\sim (\sim P \vee Q)$	4, *Impl*

PL-SELF-CONTRADICTION AND *PL*-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*
9	$\sim\sim Q \bullet \sim P$	8, *DM*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*
9	$\sim\sim Q \bullet \sim P$	8, *DM*
10	$\sim P \bullet \sim\sim Q$	9, *Com*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*
9	$\sim\sim Q \bullet \sim P$	8, *DM*
10	$\sim P \bullet \sim\sim Q$	9, *Com*
11	$\sim P$	10, *Simp*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*
9	$\sim\sim Q \bullet \sim P$	8, *DM*
10	$\sim P \bullet \sim\sim Q$	9, *Com*
11	$\sim P$	10, *Simp*
12	$\sim\sim P$	7, *Simp*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*
9	$\sim\sim Q \bullet \sim P$	8, *DM*
10	$\sim P \bullet \sim\sim Q$	9, *Com*
11	$\sim P$	10, *Simp*
12	$\sim\sim P$	7, *Simp*
13	$\sim P \bullet \sim\sim P$	11, 12, *Conj*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*
9	$\sim\sim Q \bullet \sim P$	8, *DM*
10	$\sim P \bullet \sim\sim Q$	9, *Com*
11	$\sim P$	10, *Simp*
12	$\sim\sim P$	7, *Simp*
13	$\sim P \bullet \sim\sim P$	11, 12, *Conj*
14	$\sim\sim(A \bullet \sim A)$	3–13, *IP*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*
9	$\sim\sim Q \bullet \sim P$	8, *DM*
10	$\sim P \bullet \sim\sim Q$	9, *Com*
11	$\sim P$	10, *Simp*
12	$\sim\sim P$	7, *Simp*
13	$\sim P \bullet \sim\sim P$	11, 12, *Conj*
14	$\sim\sim(A \bullet \sim A)$	3–13, *IP*
15	$A \bullet \sim A$	14, *DN*

PL-SELF-CONTRADICTION AND PL-DERIVABILITY

1	$\sim(P \supset Q) \bullet \sim(Q \supset P)$	
2	$\sim(Q \supset P) \bullet \sim(P \supset Q)$	1, *Com*
3	$\sim(A \bullet \sim A)$	*AIP*
4	$\sim(P \supset Q)$	1, *Simp*
5	$\sim(Q \supset P)$	2, *Simp*
6	$\sim(\sim P \vee Q)$	4, *Impl*
7	$\sim\sim P \bullet \sim Q$	6, *DM*
8	$\sim(\sim Q \vee P)$	5, *Impl*
9	$\sim\sim Q \bullet \sim P$	8, *DM*
10	$\sim P \bullet \sim\sim Q$	9, *Com*
11	$\sim P$	10, *Simp*
12	$\sim\sim P$	7, *Simp*
13	$\sim P \bullet \sim\sim P$	11, 12, *Conj*
14	$\sim\sim(A \bullet \sim A)$	3–13, *IP*
15	$A \bullet \sim A$	14, *DN*

Outline

Four Final Inference Rules
- Subderivations
- Conditional Proof
- Indirect Proof

PL-Derivability and the Logical Notions of *PL*
- *PL*-Validity
- *PL*-Tautologies and *PL*-Self-Contradictions
- *PL*-Equivalence and *PL*-Contradiction
- *PL*-Inconsistency

PL-EQUIVALENCE AND *PL*-DERIVABILITY

Fact 4: Two wffs of *PL*, p and q are *PL*-equivalent if and only if

$$\vdash_{PL} p \equiv q$$

PL-EQUIVALENCE AND *PL*-DERIVABILITY

Fact 4: Two wffs of *PL*, p and q are *PL*-equivalent if and only if

$$\vdash_{PL} p \equiv q$$

- Suppose we wish to show that $\sim(P \equiv Q)$ and $P \equiv \sim Q$ are *PL*-equivalent.

PL-EQUIVALENCE AND *PL*-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \ \sim Q))$	1, *Equiv*

PL-EQUIVALENCE AND PL-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*

PL-EQUIVALENCE AND PL-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*
4	$(\sim P \vee \sim Q) \bullet \sim (\sim P \bullet \sim Q)$	3, *DM*

PL-EQUIVALENCE AND PL-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*
4	$(\sim P \vee \sim Q) \bullet \sim (\sim P \bullet \sim Q)$	3, *DM*
5	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	4, *DM*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*
4	$(\sim P \vee \sim Q) \bullet \sim (\sim P \bullet \sim Q)$	3, *DM*
5	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	4, *DM*
6	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	5, *Impl*

PL-EQUIVALENCE AND PL-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*
4	$(\sim P \vee \sim Q) \bullet \sim (\sim P \bullet \sim Q)$	3, *DM*
5	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	4, *DM*
6	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	5, *Impl*
7	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	6, *Com*

PL-EQUIVALENCE AND PL-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*
4	$(\sim P \vee \sim Q) \bullet \sim (\sim P \bullet \sim Q)$	3, *DM*
5	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	4, *DM*
6	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	5, *Impl*
7	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	6, *Com*
8	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	7, *DN*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*
4	$(\sim P \vee \sim Q) \bullet \sim (\sim P \bullet \sim Q)$	3, *DM*
5	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	4, *DM*
6	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	5, *Impl*
7	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	6, *Com*
8	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	7, *DN*
9	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	8, *Impl*

PL-EQUIVALENCE AND PL-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*
4	$(\sim P \vee \sim Q) \bullet \sim (\sim P \bullet \sim Q)$	3, *DM*
5	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	4, *DM*
6	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	5, *Impl*
7	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	6, *Com*
8	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	7, *DN*
9	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	8, *Impl*
10	$P \equiv \sim Q$	9, *Equiv*

PL-EQUIVALENCE AND PL-DERIVABILITY

1	$\sim (P \equiv Q)$	*ACP*
2	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	1, *Equiv*
3	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	2, *DM*
4	$(\sim P \vee \sim Q) \bullet \sim (\sim P \bullet \sim Q)$	3, *DM*
5	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	4, *DM*
6	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	5, *Impl*
7	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	6, *Com*
8	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	7, *DN*
9	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	8, *Impl*
10	$P \equiv \sim Q$	9, *Equiv*
11	$\sim (P \equiv Q) \supset (P \equiv \sim Q)$	1–10, *CP*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv\sim Q$	*ACP*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*

PL-EQUIVALENCE AND PL-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*
17	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	16, *Impl*

PL-EQUIVALENCE AND PL-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*
17	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	16, *Impl*
18	$\sim (P \bullet Q) \bullet (\sim\sim P \vee \sim\sim Q)$	17, *DM*

PL-EQUIVALENCE AND PL-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*
17	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	16, *Impl*
18	$\sim (P \bullet Q) \bullet (\sim\sim P \vee \sim\sim Q)$	17, *DM*
19	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	18, *DM*

PL-EQUIVALENCE AND PL-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*
17	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	16, *Impl*
18	$\sim (P \bullet Q) \bullet (\sim\sim P \vee \sim\sim Q)$	17, *DM*
19	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	18, *DM*
20	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	19, *DM*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*
17	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	16, *Impl*
18	$\sim (P \bullet Q) \bullet (\sim\sim P \vee \sim\sim Q)$	17, *DM*
19	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	18, *DM*
20	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	19, *DM*
21	$\sim (P \equiv Q)$	20, *Equiv*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*
17	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	16, *Impl*
18	$\sim (P \bullet Q) \bullet (\sim\sim P \vee \sim\sim Q)$	17, *DM*
19	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	18, *DM*
20	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	19, *DM*
21	$\sim (P \equiv Q)$	20, *Equiv*
22	$(P \equiv \sim Q) \supset \sim (P \equiv Q)$	12–21, *CP*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv \sim Q$	*ACP*
13	$(P \supset \sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset \sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset \sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*
17	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	16, *Impl*
18	$\sim (P \bullet Q) \bullet (\sim\sim P \vee \sim\sim Q)$	17, *DM*
19	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	18, *DM*
20	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	19, *DM*
21	$\sim (P \equiv Q)$	20, *Equiv*
22	$(P \equiv \sim Q) \supset \sim (P \equiv Q)$	12–21, *CP*
23	$(\sim (P \equiv Q) \supset (P \equiv \sim Q)) \bullet ((P \equiv \sim Q) \supset \sim (P \equiv Q))$	11, 22, *Conj*

PL-EQUIVALENCE AND *PL*-DERIVABILITY

12	$P \equiv\sim Q$	*ACP*
13	$(P \supset\sim Q) \bullet (\sim Q \supset P)$	12, *Equiv*
14	$(P \supset\sim Q) \bullet (\sim\sim Q \vee P)$	13, *Impl*
15	$(P \supset\sim Q) \bullet (\sim\sim Q \vee \sim\sim P)$	14, *DN*
16	$(P \supset\sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	15, *Com*
17	$(\sim P \vee \sim Q) \bullet (\sim\sim P \vee \sim\sim Q)$	16, *Impl*
18	$\sim (P \bullet Q) \bullet (\sim\sim P \vee \sim\sim Q)$	17, *DM*
19	$\sim (P \bullet Q) \bullet \sim (\sim P \bullet \sim Q)$	18, *DM*
20	$\sim ((P \bullet Q) \vee (\sim P \bullet \sim Q))$	19, *DM*
21	$\sim (P \equiv Q)$	20, *Equiv*
22	$(P \equiv\sim Q) \supset\sim (P \equiv Q)$	12–21, *CP*
23	$(\sim (P \equiv Q) \supset (P \equiv\sim Q)) \bullet ((P \equiv\sim Q) \supset\sim (P \equiv Q))$	11, 22, *Conj*
24	$\sim (P \equiv Q) \equiv (P \equiv\sim Q)$	23, *Equiv*

PL-CONTRADICTORIES AND PL-DERIVABILITY

Fact 5: Two wffs of *PL*, p and q are *PL*-contradictories if and only if

$$\vdash_{PL} p \equiv \sim q$$

- Suppose we wish to show that $J \supset K$ and $J \bullet \sim K$, are *PL*-contradictories.

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*
6	$\sim (J \bullet \sim K)$	*ACP*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*
6	$\sim (J \bullet \sim K)$	*ACP*
7	$\sim J \vee \sim\sim K$	6, *DM*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*
6	$\sim (J \bullet \sim K)$	*ACP*
7	$\sim J \vee \sim\sim K$	6, *DM*
8	$J \supset \sim\sim K$	7, *Impl*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*
6	$\sim (J \bullet \sim K)$	*ACP*
7	$\sim J \vee \sim\sim K$	6, *DM*
8	$J \supset \sim\sim K$	7, *Impl*
9	$J \supset K$	8, *DN*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*
6	$\sim (J \bullet \sim K)$	*ACP*
7	$\sim J \vee \sim\sim K$	6, *DM*
8	$J \supset \sim\sim K$	7, *Impl*
9	$J \supset K$	8, *DN*
10	$\sim (J \bullet \sim K) \supset (J \supset K)$	6–9, *CP*

PL-CONTRADICTORIES AND PL-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*
6	$\sim (J \bullet \sim K)$	*ACP*
7	$\sim J \vee \sim\sim K$	6, *DM*
8	$J \supset \sim\sim K$	7, *Impl*
9	$J \supset K$	8, *DN*
10	$\sim (J \bullet \sim K) \supset (J \supset K)$	6–9, *CP*
11	$((J \supset K) \supset \sim (J \bullet \sim K)) \bullet (\sim (J \bullet \sim K) \supset (J \supset K))$	5, 10, *Conj*

PL-CONTRADICTORIES AND PL-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*
6	$\sim (J \bullet \sim K)$	*ACP*
7	$\sim J \vee \sim\sim K$	6, *DM*
8	$J \supset \sim\sim K$	7, *Impl*
9	$J \supset K$	8, *DN*
10	$\sim (J \bullet \sim K) \supset (J \supset K)$	6–9, *CP*
11	$((J \supset K) \supset \sim (J \bullet \sim K)) \bullet (\sim (J \bullet \sim K) \supset (J \supset K))$	5, 10, *Conj*
12	$(J \supset K) \equiv \sim (J \bullet \sim K)$	11, *Equiv*

PL-CONTRADICTORIES AND *PL*-DERIVABILITY

1	$J \supset K$	*ACP*
2	$\sim J \vee K$	1, *Impl*
3	$\sim J \vee \sim\sim K$	2, *DN*
4	$\sim (J \bullet \sim K)$	3, *DM*
5	$(J \supset K) \supset \sim (J \bullet \sim K)$	1–4, *CP*
6	$\sim (J \bullet \sim K)$	*ACP*
7	$\sim J \vee \sim\sim K$	6, *DM*
8	$J \supset \sim\sim K$	7, *Impl*
9	$J \supset K$	8, *DN*
10	$\sim (J \bullet \sim K) \supset (J \supset K)$	6–9, *CP*
11	$((J \supset K) \supset \sim (J \bullet \sim K)) \bullet (\sim (J \bullet \sim K) \supset (J \supset K))$	5, 10, *Conj*
12	$(J \supset K) \equiv \sim (J \bullet \sim K)$	11, *Equiv*

IN CLASS EXERCISE!

OUTLINE

FOUR FINAL INFERENCE RULES
SUBDERIVATIONS
CONDITIONAL PROOF
INDIRECT PROOF

PL-DERIVABILITY AND THE LOGICAL NOTIONS OF *PL*
PL-VALIDITY
PL-TAUTOLOGIES AND *PL*-SELF-CONTRADICTIONS
PL-EQUIVALENCE AND *PL*-CONTRADICTION
PL-INCONSISTENCY

PL-DERIVABILITY AND *PL*-INCONSISTENCY

Fact 6: A set of wffs of *PL*, $\{q_1, q_2, ..., q_N\}$ is *PL*-inconsistent if and only if

$$q_2, ..., q_N \vdash_{PL} \sim q_1$$

PL-DERIVABILITY AND *PL*-INCONSISTENCY

Fact 6: A set of wffs of *PL*, $\{q_1, q_2, ..., q_N\}$ is *PL*-inconsistent if and only if

$$q_2, ..., q_N \vdash_{PL} \sim q_1$$

- That is: if you can derive the negation of one of the members of the set from the other members of the set, then the set is *PL*-inconsistent.

PL-DERIVABILITY AND *PL*-INCONSISTENCY

Fact 6: A set of wffs of *PL*, $\{q_1, q_2, ..., q_N\}$ is *PL*-inconsistent if and only if

$$q_2, ..., q_N \vdash_{PL} \sim q_1$$

- That is: if you can derive the negation of one of the members of the set from the other members of the set, then the set is *PL*-inconsistent.
- Suppose that we wish to show that the set

$$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$$

is *PL*-inconsistent.

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$
2	$A \bullet B$

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \bullet B$	
3	$A \supset \sim C$	*AIP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1		$A \supset (B \supset C)$	
2		$A \bullet B$	
3	|	$A \supset \sim C$	*AIP*
4	|	$(A \bullet B) \supset C$	1, *Exp*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \bullet B$	
3	$A \supset \sim C$	*AIP*
4	$(A \bullet B) \supset C$	1, *Exp*
5	C	2, 4, *MP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \bullet B$	
3	$A \supset \sim C$	*AIP*
4	$(A \bullet B) \supset C$	1, *Exp*
5	C	2, 4, *MP*
6	A	2, *Simp*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \bullet B$	
3	$A \supset \sim C$	*AIP*
4	$(A \bullet B) \supset C$	1, *Exp*
5	C	2, 4, *MP*
6	A	2, *Simp*
7	$\sim C$	3, 6, *MP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \bullet B$	
3	$A \supset \sim C$	*AIP*
4	$(A \bullet B) \supset C$	1, *Exp*
5	C	2, 4, *MP*
6	A	2, *Simp*
7	$\sim C$	3, 6, *MP*
8	$C \bullet \sim C$	5, 7, *Conj*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \bullet B$	
3	$A \supset \sim C$	*AIP*
4	$(A \bullet B) \supset C$	1, *Exp*
5	C	2, 4, *MP*
6	A	2, *Simp*
7	$\sim C$	3, 6, *MP*
8	$C \bullet \sim C$	5, 7, *Conj*
9	$\sim (A \supset \sim C)$	3–8, *IP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \bullet B$	
3	$A \supset \sim C$	*AIP*
4	$(A \bullet B) \supset C$	1, *Exp*
5	C	2, 4, *MP*
6	A	2, *Simp*
7	$\sim C$	3, 6, *MP*
8	$C \bullet \sim C$	5, 7, *Conj*
9	$\sim (A \supset \sim C)$	3–8, *IP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$
2	$A \supset \sim C$

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \supset \sim C$	
3	$A \bullet B$	*AIP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset {\sim}C\}$

1		$A \supset (B \supset C)$	
2		$A \supset {\sim}C$	
3	|	$A \bullet B$	*AIP*
4	|	$B \bullet A$	3, *Com*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset\!\sim C\}$

1	$A \supset (B \supset C)$	
2	$A \supset\!\sim C$	
3	$A \bullet B$	*AIP*
4	$B \bullet A$	3, *Com*
5	A	3, *Simp*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1		$A \supset (B \supset C)$	
2		$A \supset \sim C$	
3	|	$A \bullet B$	*AIP*
4	|	$B \bullet A$	3, *Com*
5	|	A	3, *Simp*
6	|	$B \supset C$	1, 5, *MP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \supset \sim C$	
3	$A \bullet B$	*AIP*
4	$B \bullet A$	3, *Com*
5	A	3, *Simp*
6	$B \supset C$	1, 5, *MP*
7	B	4, *Simp*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \supset \sim C$	
3	$A \bullet B$	*AIP*
4	$B \bullet A$	3, *Com*
5	A	3, *Simp*
6	$B \supset C$	1, 5, *MP*
7	B	4, *Simp*
8	C	6, 7, *MP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \supset \sim C$	
3	$A \bullet B$	*AIP*
4	$B \bullet A$	3, *Com*
5	A	3, *Simp*
6	$B \supset C$	1, 5, *MP*
7	B	4, *Simp*
8	C	6, 7, *MP*
9	$\sim C$	2, 5, *MP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \supset (B \supset C)$	
2	$A \supset \sim C$	
3	$A \bullet B$	*AIP*
4	$B \bullet A$	3, *Com*
5	A	3, *Simp*
6	$B \supset C$	1, 5, *MP*
7	B	4, *Simp*
8	C	6, 7, *MP*
9	$\sim C$	2, 5, *MP*
10	$C \bullet \sim C$	8, 9, *Conj*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset {\sim}C\}$

1	$A \supset (B \supset C)$	
2	$A \supset {\sim}C$	
3	$A \bullet B$	*AIP*
4	$B \bullet A$	3, *Com*
5	A	3, *Simp*
6	$B \supset C$	1, 5, *MP*
7	B	4, *Simp*
8	C	6, 7, *MP*
9	${\sim}C$	2, 5, *MP*
10	$C \bullet {\sim}C$	8, 9, *Conj*
11	${\sim}(A \bullet B)$	3–10, *IP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset {\sim}C\}$

1	$A \supset (B \supset C)$	
2	$A \supset {\sim}C$	
3	$A \bullet B$	*AIP*
4	$B \bullet A$	3, *Com*
5	A	3, *Simp*
6	$B \supset C$	1, 5, *MP*
7	B	4, *Simp*
8	C	6, 7, *MP*
9	${\sim}C$	2, 5, *MP*
10	$C \bullet {\sim}C$	8, 9, *Conj*
11	${\sim}(A \bullet B)$	3–10, *IP*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$
2	$A \supset \sim C$

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*

PL-DERIVABILITY AND PL-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*

PL-DERIVABILITY AND *PL*-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*
5	$B \supset C$	3, 4, *MP*

PL-DERIVABILITY AND PL-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*
5	$B \supset C$	3, 4, *MP*
6	$B \bullet A$	1, *Com*

PL-DERIVABILITY AND PL-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*
5	$B \supset C$	3, 4, *MP*
6	$B \bullet A$	1, *Com*
7	B	6, *Simp*

PL-DERIVABILITY AND PL-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*
5	$B \supset C$	3, 4, *MP*
6	$B \bullet A$	1, *Com*
7	B	6, *Simp*
8	C	5, 7, *MP*

PL-DERIVABILITY AND PL-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*
5	$B \supset C$	3, 4, *MP*
6	$B \bullet A$	1, *Com*
7	B	6, *Simp*
8	C	5, 7, *MP*
9	$\sim C$	2, 4, *MP*

PL-DERIVABILITY AND PL-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset {\sim}C\}$

1	$A \bullet B$	
2	$A \supset {\sim}C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*
5	$B \supset C$	3, 4, *MP*
6	$B \bullet A$	1, *Com*
7	B	6, *Simp*
8	C	5, 7, *MP*
9	${\sim}C$	2, 4, *MP*
10	$C \bullet {\sim}C$	8, 9, *Conj*

PL-DERIVABILITY AND PL-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*
5	$B \supset C$	3, 4, *MP*
6	$B \bullet A$	1, *Com*
7	B	6, *Simp*
8	C	5, 7, *MP*
9	$\sim C$	2, 4, *MP*
10	$C \bullet \sim C$	8, 9, *Conj*
11	$\sim(A \supset (B \supset C))$	3–10, *IP*

PL-DERIVABILITY AND PL-INCONSISTENCY

$\{A \supset (B \supset C), A \bullet B, A \supset \sim C\}$

1	$A \bullet B$	
2	$A \supset \sim C$	
3	$A \supset (B \supset C)$	*AIP*
4	A	1, *Simp*
5	$B \supset C$	3, 4, *MP*
6	$B \bullet A$	1, *Com*
7	B	6, *Simp*
8	C	5, 7, *MP*
9	$\sim C$	2, 4, *MP*
10	$C \bullet \sim C$	8, 9, *Conj*
11	$\sim(A \supset (B \supset C))$	3–10, *IP*

IN CLASS EXERCISE!