Bayesian Confirmation Theory

• Bayesian Confirmation Theory (BCT) provides a quantitative, as opposed to a qualitative, theory of confirmation. That is, it doesn’t just tell us whether a given piece of evidence confirms a hypothesis, but it additionally tells us how much, or the degree to which, a piece of evidence confirms a hypothesis.

• It does this, not by providing a formal inductive logic, as Hempel and Goodman were supposing it would, but rather by using tools from the theory of probability. So, we’re going to have to think a bit about probability theory.

The Theory of Probability

The Axioms and their Consequences

• A probability function \( P \) is a function from propositions to real numbers.

• All that it takes to count as a probability function is that it maps propositions to real numbers in accordance with the following three axioms:
  1. For any proposition \( A \), the probability of \( A \) is not negative,
     \[
     P(A) \geq 0
     \]
  2. If a proposition \( \top \) is guaranteed to be true, then the probability of \( \top \) is 1.
     \[
     P(\top) = 1
     \]
  3. If \( A \) and \( B \) are mutually exclusive (they can’t both be true), then
     \[
     P(A \lor B) = P(A) + P(B)
     \]

• All sorts of interesting properties follow from these three axioms. For instance:

Theorem 1. For any proposition \( A \),

\[
P(A) + P(\neg A) = 1
\]

Proof. \( A \lor \neg A \) is guaranteed to be true (by the truth table), so it has probability 1 (by the 2nd axiom). And, since \( A \) and \( \neg A \) are mutually exclusive, \( P(A \lor \neg A) = P(A) + P(\neg A) \) (by the 3rd axiom). So

\[
P(A) + P(\neg A) = P(A \lor \neg A) = 1
\]

\[\square\]
**Theorem 2.** If two propositions $A$ and $B$ are logically equivalent (i.e., they have the same truth table), then they have the same probability.

**Proof.** If $A$ and $B$ are logically equivalent, then $A \lor \neg B$ is guaranteed to be true (by the truth table). And since $A$ and $\neg B$ are mutually exclusive $P(A \lor \neg B) = P(A) + P(\neg B)$ (by the 3rd axiom). So

$$P(A) + P(\neg B) = P(A \lor \neg B) = 1$$

$$P(A) = 1 - P(\neg B)$$

However, $P(B) = 1 - P(\neg B)$ (by Theorem 1). So

$$P(A) = P(B)$$

\[ \square \]

**Theorem 3.** For any propositions $A$ and $B$,

$$P(A) = P(A \land B) + P(A \land \neg B)$$

**Proof.** Omitted — Problem 4 on the 2nd problem set  \[ \square \]

**Theorem 4.** For any propositions $A$ and $B$,

$$P(A \lor B) = P(A) + P(B) - P(A \land B)$$

**Proof.** By the truth table, $A \lor B$ is logically equivalent to $(A \land \neg B) \lor ((A \land B) \lor (\neg A \land B))$. So they have the same probability, by Theorem 2.

$$P(A \lor B) = P((A \land \neg B) \lor ((A \land B) \lor (\neg A \land B)))$$

And since $A \land \neg B$ and $(A \land B) \lor (\neg A \land B)$ are mutually exclusive (by the truth table),

$$P(A \lor B) = P(A \land \neg B) + P((A \land B) \lor (\neg A \land B))$$

by the 3rd axiom. And since $A \land B$ and $\neg A \land B$ are mutually exclusive (by the truth table),

$$P(A \lor B) = P(A \land \neg B) + P(A \land B) + P(\neg A \land B)$$

by the 3rd axiom. From here,

$$P(A \lor B) = P(A \land \neg B) + P(A \land B) + P(\neg A \land B)$$

$$= P(A \land \neg B) + P(A \land B) + P(\neg A \land B) + P(A \land B) - P(A \land B)$$

$$= P(A) \text{ by Theorem 3} + P(B) \text{ by Theorem 3}$$

$$P(A \lor B) = P(A) + P(B) - P(A \land B)$$

\[ \square \]
The Definitions and their Consequences

- We will define probabilistic independence and conditional probability in terms of the probability function $P$.

**Probabilistic Independence.** *Any propositions $A$ and $B$ are probabilistically independent iff*

$$P(A \land B) = P(A) \cdot P(B)$$

**Conditional Probability.** *For any propositions $A$ and $B$,*

$$P(A | B) = \frac{P(A \land B)}{P(B)}$$

- From these definitions, it follows that, if $A$ and $B$ are probabilistically independent, then $P(A | B) = P(A)$ and $P(B | A) = P(B)$.

- The following theorem also follows just from the definition of conditional probability:

**Bayes’ Theorem.** *For any propositions $A$, $B$,*

$$P(A | B) = \frac{P(A \land B)}{P(B)} \cdot P(A)$$

- We can also prove the following:

**Theorem of Total Probability.** *For any propositions $A$ and $B$,*

$$P(A) = P(A | B) \cdot P(B) + P(A | \neg B) \cdot P(\neg B)$$

**Proof.** From Theorem 3,

$$P(A) = P(A \land B) + P(A \land \neg B)$$

$$= \frac{P(A \land B)}{P(B)} \cdot P(B) + \frac{P(A \land \neg B)}{P(\neg B)} \cdot P(\neg B)$$

$$= P(A | B) \cdot P(B) + P(A | \neg B) \cdot P(\neg B)$$

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**Bayesian Confirmation Theory**

- BCT begins by supposing that an individual scientist has certain *degrees of confidence* or *credences* in some set of scientific hypotheses $h_1, h_2, ..., h_n$ and some set potential evidence $e_1, e_2, ..., e_m$.

- We can represent these degrees of confidence with a *credence function* $C$.
  - If $C(h) = 0$, then the scientist is absolutely certain that $h$ is false.
  - If $C(h) = 1$, then the scientist is absolutely certain that $h$ is true.
if $C(h) = 0.5$, then the scientist is just as confident that $h$ is false as they are that it is true.

- The first claim of BCT is Probabilism: $C$ ought to be a probability function (that is, it should obey the axioms of probability theory).

- The second claim of BCT is Conditionalization: if you start out with a prior credence function $C(\cdot)$ and you only learn that $e$, then your posterior credence function ought to be $C(\cdot | e)$.

$$C^+(\cdot) \equiv C(\cdot | e)$$

- Given Bayes' Theorem, this tells us that your credence that a hypothesis $h$ is true after acquiring evidence $e$, $C^+(h)$ ought to be

$$C^+(h) \equiv \frac{C(e | h)}{C(e)} \cdot C(h)$$

- Given the Theorem of Total Probability, it follows from this that

$$C^+(h) \equiv \frac{C(e | h)}{C(e | h) \cdot C(h) + C(e | \neg h) \cdot C(\neg h)} \cdot C(h)$$