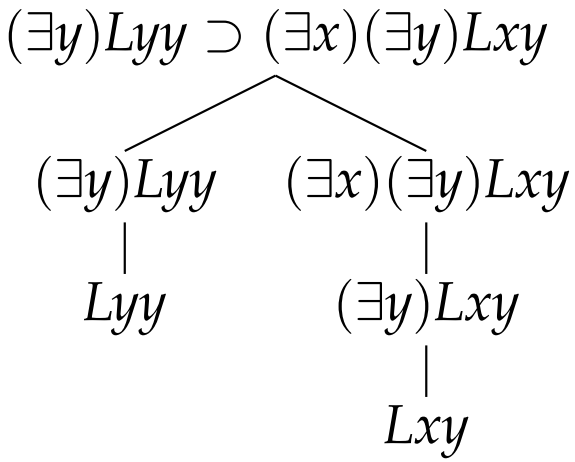
Correctness/Completeness and QL, Day 1

Philosophy 180
June 5, 2014



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An argument form is DEDUCTIVELY VALID if and only if every substitution instance of that form whose premises are all true has a true conclusion as well.

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- How well does *PL* do at its job?

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  - CORRECTNESS: If an argument is *PL*-valid, then it is deductively valid.
  - COMPLETENESS: If an argument is deductively valid, then it is *PL*-valid.
- If *PL*-validity is *correct*, then the answer to (1) is 'no'.

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  - COMPLETENESS: If an argument is deductively valid, then it is *PL*-valid.
- If *PL*-validity is *correct*, then the answer to (1) is 'no'.
- If *PL*-validity is *complete*, then the answer to (2) is 'no'.

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- First: let's consider why some think that *PL* is not even *correct*.

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  - '$\supset$' is not a perfect translation of 'if..., then...'
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  - '$\supset$' is not a perfect translation of 'if..., then...'
  - the first two arguments attempt to show that the differences between '$\supset$' and 'if..., then...' prevent the English 'if..., then...' from satisfying *modus ponens* and *modus tollens*.
- The second two arguments object not to the translation guide, but rather to *PL*'s theory about which arguments *involving the wffs of PL* are valid.

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A Counterexample to Modus Ponens?

1. If Mitt Romney doesn't win, then, if a Republican wins, then Ron Paul wins.
2. Mitt Romney doesn't win.

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3. So, if a Republican wins, then Ron Paul wins.

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- However, in the run up to the 2012 presidential election, its premises were true yet its conclusion was false.

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1	$(\sim M \bullet R) \supset P$	
2	$\sim M$	
3	$\sim M \supset (R \supset P)$	1, *Exp*
4	$R \supset P$	2, 3, *MP*

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- So, we'd have to accept the following argument as deductively valid:

  1. Shakespeare wrote Hamlet.

  ---

  2. If Shakespeare didn't write Hamlet, then Dan Brown did.

- McGee: this is unacceptable, so we must choose between *exportation* and *modus ponens* for the English 'if..., then...'.

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- Other opt for *modus ponens* and reject *exportation*.
- Still others accept both *exportation* and *modus ponens* and accept that the English 'if..., then...' is logically indistinguishable from $\supset$
  - they have stories to tell about why arguments like the ones above appear—falsely—to be invalid

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- Yalcin: in this scenario, the premises of this argument are true, yet the conclusion may easily be false.

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- So, Yalcin contends, *modus tollens* is not deductively valid for the English 'if..., then...'

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- Yalcin has responses to both of these objections, but we don't have the time to delve into them.

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  This very statement is false.

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  - false (and not true) 'F'; or
  - *both* true and false 'B'.
- updated truth-table for '$\sim$':

p	$\sim p$
T	F
F	T
B	B

A Counterexample to Disjunctive Syllogism?

- Updated truth-tables for $\vee$ and $\supset$:

$\boxed{p \vee q}$

		q		
		T	F	B
p	T	T	T	T
	F	T	F	B
	B	T	B	B

$\boxed{p \supset q}$

		q		
		T	F	B
p	T	T	F	B
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- Yet these arguments are of the form *disjunctive syllogism* and *modus ponens*

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$$\begin{array}{l} p \vee q \\ \sim p \\ \hline q \end{array} \qquad\qquad \begin{array}{l} p \supset q \\ p \\ \hline q \end{array}$$

- So, Priest concludes: both *disjunctive syllogism* and *modus ponens* are deductively invalid.

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$$L' := L' \textit{ is not true.}$$

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  - The third tile has ever-so-slightly more yellow in it than the second
  - So on and so forth.
- Any pair of sequential tiles are perceptually indistinguishable.
- By the end of the sequence, we have a tile that is unmistakably orange.

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The Sorites Paradox

1) The 1st tile is red.
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- Something's gone wrong with this reasoning, and some people have been tempted to point the finger at *modus ponens*.

Outline

Why *PL* is Not Complete

1. Johann knows Filipa.

---

2. So, somebody knows Filipa.

1. Everyone who owns a Ford owns a car.
2. Rohan owns a Ford.

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\begin{aligned}
Tx &= x \text{ was tall} \\
Lxy &= x \text{ loved } y \\
Kxy &= x \text{ killed } y \\
l &= \text{Abraham Lincoln} \\
b &= \text{John Wilkes Booth} \\
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- We'll also get to say things like 'everyone loves someone' and 'no one loves anyone who killed them'.

$$\begin{aligned} (x)(\exists y)Lxy &= \text{Everyone loves someone} \\ \sim(\exists x)(\exists y)(Kyx \bullet Lxy) &= \text{No one loves anyone who killed them} \end{aligned}$$

Outline

Vocabulary

The vocabulary of *QL* includes the following symbols:

1. for each $n \geq 0$, an infinite number of n-place predicates (any capital letter, along with a superscript n—perhaps with subscripts)

$$
\begin{array}{ccccccccc}
A^1 & B^1 & \dots & Z^1 & A_1^1 & \dots & Z_1^1 & A_2^1 & \dots \\
A^2 & B^2 & \dots & Z^2 & A_1^2 & \dots & Z_1^2 & A_2^2 & \dots \\
\vdots & \vdots & \dots & \vdots & \vdots & \dots & \vdots & \vdots & \dots \\
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\vdots & \vdots & \dots & \vdots & \vdots & \dots & \vdots & \vdots & \dots
\end{array}
$$

Vocabulary

2. An infinite number of *constants* (any lowercase letter between a and w—perhaps with subscripts)

$$a, b, c, \ldots, u, v, w, a_1, b_1, \ldots, v_1, w_1, a_2, b_2, \ldots$$

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Nothing else is included in the vocabulary of *QL*.

Vocabulary

- Let's call both constants and variables *terms*. That is, both '*a*' and '*x*' are *terms* of *QL*.

Grammar

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$\mathscr{F}$) If '$\mathscr{F}^n$' is an n-place predicate and '$\mathbf{a}_1$', '$\mathbf{a}_2$', . . . , '$\mathbf{a}_n$' are n terms, then '$\mathscr{F}^n\mathbf{a}_1\mathbf{a}_2 \ldots \mathbf{a}_n$' is a wff.

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– Nothing else is a wff.

Grammar

Note: none of '$\mathscr{F}$', '**a**', '**P**', and '**Q**' appear in the vocabulary of *QL*. They are not *themselves* wffs of *QL*. Rather, we are using them here as VARIABLES ranging over the formulae of *QL*.

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f)	So, '$\sim(\exists x)(\exists z)G^2zx$' is a wff	[from (e) and ($\sim$)]
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- we can write

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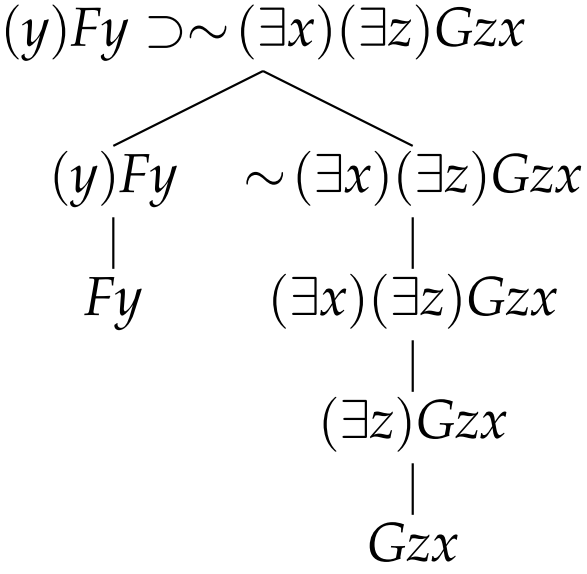
Grammar

- Syntax Trees



Outline

Free and Bound Variables

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- However, their variables are FREE.
- The variables appearing in '$(x)(y)Fxy$' are BOUND.
- In '$(x)Px \supset Qx$', the first x is bound, whereas the second one is free.

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- quantifiers are logical operators.
  - They can be the MAIN OPERATOR of a wff of *QL* or they can be the main operator of a wff's subformulae.

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- The main operator of '$(x)Fx$',

$$\begin{array}{c} (x)Fx \\ | \\ Fx \end{array}$$

is the universal quantifier '(x)'.

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  - the immediate subformula of '$(\exists y)(Fy \bullet Ga)$' is '$Fy \bullet Ga$'.

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Quantifier Scope

- The SCOPE of a quantifier is its immediate subformula.
- $$\begin{tikzcd}[column sep=small] & (\exists y)Lyy \supset (\exists x)(\exists y)Lxy \arrow[dl, dash] \arrow[dr, dash] & \\ (\exists y)Lyy \arrow[d, dash] & & (\exists x)(\exists y)Lxy \arrow[d, dash] \\ Lyy & & (\exists y)Lxy \arrow[d, dash] \\ & & Lxy \end{tikzcd}$$

FREE AND BOUND VARIABLES

A variable **x** in a wff of *PL* is BOUND if and only if it occurs within the scope of a quantifier, (**x**) or (∃**x**), whose associated variable is **x**.

A variable **x** in a wff of *PL* is FREE if and only if it does not occur within the scope of a quantifier, (**x**) or (∃**x**), whose associated variable is **x**.

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- E.g.,

$$(x)(y)Fy \supset (\exists z)Gzx$$

Free and Bound Variables

In a wff of *QL*, a quantifier $(\mathbf{x})$ or $(\exists\mathbf{x})$ BINDS a variable $\mathbf{x}$ if and only if $\mathbf{x}$ occurs *free* within that quantifier's scope.

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- E.g., in

$$(\exists x)(x)Fx$$

the variable 'x' is bound by the *universal* quantifier '(x)'. It is *not* bound by the existential quantifier '$(\exists x)$'.

Outline

Semantics for QL

- The meaning of '$\sim$, $\vee$, $\bullet$, $\supset$, and $\equiv$ are the same as before

SEMANTICS FOR *QL*

- The meaning of '$\sim$, $\vee$, $\bullet$, $\supset$, and $\equiv$ are the same as before
- In order to explain the meaning of the other expressions in *QL*, we introduce the idea of a *QL*-INTERPRETATION for a wff or set of wffs of *QL*.

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4. For every n-place predicate appearing in the wff or wffs of *QL*, a specification of the n-place property it represents.

QL-interpretations

$$(y)Lya \vee (\exists y) \sim Lya$$

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$$\mathscr{I} = \begin{cases} \mathscr{D} & = \text{ the set of all people} \\ a & = \text{ Steve} \\ Lxy & = x \text{ loves } y \end{cases}$$

Outline

Truth on a *QL*-interpretation

1. A wff of the form '$\mathscr{F}^n \mathbf{a}_1 \ldots \mathbf{a}_n$' is true on the interpretation $\mathscr{I}$ if the things in the domain represented by $\mathbf{a}_1 \ldots \mathbf{a}_n$ have the property represented by $\mathscr{F}^n$. Otherwise, it is false on the interpretation $\mathscr{I}$.

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3. A wff of the form '$\mathbf{P} \vee \mathbf{Q}$' is true on the interpretation $\mathscr{I}$ if either $\mathbf{P}$ is true on the interpretation $\mathscr{I}$ or $\mathbf{Q}$ is true on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.

Truth on a *QL*-interpretation

4. A wff of the form '**P** • **Q**' is true on the interpretation $\mathscr{I}$ if both **P** is true on the interpretation $\mathscr{I}$ and **Q** is true on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.

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5. A wff of the form '**P** ⊃ **Q**' is true on the interpretation $\mathscr{I}$ if either **P** is false on the interpretation $\mathscr{I}$ or **Q** is true on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.

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4. A wff of the form '$\mathbf{P} \bullet \mathbf{Q}$' is true on the interpretation $\mathscr{I}$ if both **P** is true on the interpretation $\mathscr{I}$ and **Q** is true on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.
5. A wff of the form '$\mathbf{P} \supset \mathbf{Q}$' is true on the interpretation $\mathscr{I}$ if either **P** is false on the interpretation $\mathscr{I}$ or **Q** is true on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.
6. A wff of the form '$\mathbf{P} \equiv \mathbf{Q}$' is true on the interpretation $\mathscr{I}$ if both **P** and **Q** have the same truth value on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.

Substitution Instances

- A substitution instance of a quantified wff of the form '$(\mathbf{x})\mathbf{P}$' or '$(\exists\mathbf{x})\mathbf{P}$' is the wff that you get by removing the quantifier, leaving behind just its immediate subformula, and *uniformly replacing every instance of the variable* **x** *which is bound by the quantifier with some one constant* **a**

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  - **it must be the same constant throughout**
- Substitution instances of '$(\exists y)((Ay \bullet \sim Lay) \supset (x) \sim Lax)$':

$$(Ab \bullet \sim Lab) \supset (x) \sim Lax$$
$$(Ac \bullet \sim Lac) \supset (x) \sim Lax$$
$$(Ak \bullet \sim Lak) \supset (x) \sim Lax$$
$$(Aa \bullet \sim Laa) \supset (x) \sim Lax$$

Substitution Instances

- *Not* substitution instances of '$(\exists y)(\sim Lay \supset (x) \sim Lax)$':

$$(Ax \bullet \sim Lax) \supset (x) \sim Lax$$
$$(Aa \bullet \sim Lab) \supset (x) \sim Lax$$
$$(Ab \bullet \sim Lbb) \supset (x) \sim Lcx$$
$$(Ar \bullet \sim Lra) \supset (x) \sim Lax$$

Truth on a *QL*-Interpretation

7. A wff of the form '$(\mathbf{x})\mathbf{P}$' is true on the interpretation $\mathscr{I}$ if every substitution instance of '$(\mathbf{x})\mathbf{P}$' is true on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.

Truth on a *QL*-Interpretation

7. A wff of the form '$(\mathbf{x})\mathbf{P}$' is true on the interpretation $\mathscr{I}$ if every substitution instance of '$(\mathbf{x})\mathbf{P}$' is true on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.
8. A wff of the form '$(\exists\mathbf{x})\mathbf{P}$' is true on the interpretation $\mathscr{I}$ if there is *some* substitution instance of '$(\mathbf{x})\mathbf{P}$' which is true on the interpretation $\mathscr{I}$. Otherwise, it is false on the interpretation $\mathscr{I}$.