Correctness/Completeness and QL, Day 1

Philosophy 180
June 5, 2014

(∃y)Lyy ⊆ (∃x)(∃y)Lxy

(∃y)Lyy

Lyy

(∃y)Lxy

Lxy
OUTLINE

CORRECTNESS AND COMPLETENESS

ARGUMENTS THAT $PL$ IS NOT CORRECT
A COUNTEREXAMPLE TO MODUS PONENS?
A COUNTEREXAMPLE TO MODUS TOLLENS?
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?
THE SORITES PARADOX

WHY $PL$ IS NOT COMPLETE

THE LANGUAGE $QL$
PRELIMINARY ORIENTATION
THE SYNTAX OF $QL$
FREE AND BOUND VARIABLES
SEMANTICS FOR $QL$
TRUTH ON AN INTERPRETATION
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A QUICK Recap
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A QUICK Recap

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An argument is **FORMALLY DEDUCTIVELY VALID** if and only if it is a substitution instance of a deductively valid argument form.
A Quick Recap

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An argument form is **DEDUCTIVELY VALID** if and only if every substitution instance of that form whose premises are all true has a true conclusion as well.
We decided to theorize about argument forms involving the following English constructions:

- it is not the case that...
- both ... and ...
- either ... or ...
- if ..., then ...
- ... if and only if ...

We introduced a formal language PL, in which ; \_ ; and \_ are used to translate the expressions above.

We defined a notion of validity in PL—PL-validity—and used it to theorize about deductive validity.

How well does PL do at its job?
A Quick Recap

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  it is not the case that...
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- How well does $PL$ do at its job?
Two questions:

1. Are there any PL-valid arguments which are deductively invalid?
2. Are there any PL-invalid arguments which are deductively valid?

Two properties we might want PL-validity to have:

- **Correctness**: If an argument is PL-valid, then it is deductively valid.
- **Completeness**: If an argument is deductively valid, then it is PL-valid.

If PL-validity is correct, then the answer to (1) is 'no'.

If PL-validity is complete, then the answer to (2) is 'no'.

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**Correctness and Completeness**

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If *PL*-validity is *correct*, then the answer to (1) is ‘no’.
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Two properties we might want PL-validity to have:
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If PL-validity is correct, then the answer to (1) is ‘no’.
If PL-validity is complete, then the answer to (2) is ‘no’.
Many logicians believe that PL is correct.
CORRECTNESS AND COMPLETENESS

Many logicians believe that PL is correct.
However, no logician believes that PL is complete.
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Because PL is not complete, we will need to introduce additional kinds of logical forms. This will be the task of predicate logic, or, as it is also known, quantificational logic—QL.
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First: let’s consider why some think that PL is not even correct.
Correctness and Completeness

Arguments that PL is Not Correct
  A Counterexample to Modus Ponens?
  A Counterexample to Modus Tollens?
  A Counterexample to Disjunctive Syllogism?
  The Sorites Paradox

Why PL is Not Complete

The Language QL

Preliminary Orientation
The Syntax of QL
  Free and Bound Variables
Semantics for QL
  Truth on an Interpretation
Is PL Not Correct?

- A word of warning: these arguments are controversial (some more so than others); and many logicians are not moved by them to reject the correctness of PL.
Is PL NOT CORRECT?

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  1. the theory about which arguments involving the wffs of PL are valid; and
Is $PL$ Not Correct?

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- There’s two components to our theory $PL$:
  1. the theory about which arguments involving the wffs of $PL$ are valid; and
  2. the translation guide from English into $PL$. 
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  ▶ the first two arguments attempt to show that the differences between ‘⊃’ and ‘if..., then...’ prevent the English ‘if..., then...’ from satisfying modus ponens and modus tollens.
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- The second two arguments object not to the translation guide, but rather to PL’s theory about which arguments involving the wffs of PL are valid.
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A COUNTEREXAMPLE TO MODUS PONENS?

1. If Mitt Romney doesn’t win, then, if a Republican wins, then Ron Paul wins.
2. Mitt Romney doesn’t win.
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▶ McGee: this is of the form *modus ponens*

\[
\text{if } p \text{ then } q
\]

\[
\begin{array}{c}
p \\
\hline
\text{so, } q
\end{array}
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  \hline
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- However, in the run up to the 2012 presidential election, its premises were true yet its conclusion was false.
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1   \((\sim M \land R) \supset P\)
2   \(\sim M\)
3   \(\sim M \supset (R \supset P)\)  \(1, \text{Exp}\)
4   \(R \supset P\)  \(2, 3, \text{MP}\)
A COUNTEREXAMPLE TO MODUS PONENS?

- McGee shows that, if we accept both *modus ponens* and *exportation* for the English ‘if..., then...’, then the English ‘if..., then...’ will be logically indistinguishable from the material conditional ⊃.
A Counterexample to Modus Ponens?

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- So, we’d have to accept the following argument as deductively valid:
A COUNTEREXAMPLE TO MODUS PONENS?

- McGee shows that, if we accept both *modus ponens* and *exportation* for the English ‘if..., then...’, then the English ‘if..., then...’ will be logically indistinguishable from the material conditional $\supset$.

- So, we’d have to accept the following argument as deductively valid:

  1. Shakespeare wrote Hamlet.

  2. If Shakespeare didn’t write Hamlet, then Dan Brown did.

- McGee: this is unacceptable, so we must choose between *exportation* and *modus ponens* for the English ‘if..., then...’.
A COUNTEREXAMPLE TO MODUS PONENS?

- McGee opts for *exportation* and rejects *modus ponens*.
A Counterexample to Modus Ponens?

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- Other opt for *modus ponens* and reject *exportation*.
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  - they have stories to tell about why arguments like the ones above appear—falsely—to be invalid
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A COUNTEREXAMPLE TO MODUS TOLLENS?

- Imagine that we have an urn which contains 100 marbles.
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Yalcin: in this scenario, the premises of this argument are true, yet the conclusion may easily be false.
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▶ Suppose that we have selected a marble at random from the urn, but that we do not yet know whether it is blue or red, or whether it is big or small.
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1. If the marble is big, then it’s likely red.
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▶ But this is an instance of the argument form *modus tollens*

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- So, Yalcin contends, modus tollens is not deductively valid for the English ‘if..., then...’
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Yalcin has responses to both of these objections, but we don’t have the time to delve into them.
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A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Consider the statement:
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

Consider the statement:

This very statement is false.
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Consider the statement:
  
  \[ L := L \text{ is false.} \]
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- updated truth-table for ‘\( \sim \’:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \sim p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>( F )</td>
</tr>
<tr>
<td>( F )</td>
<td>( T )</td>
</tr>
<tr>
<td>( B )</td>
<td>( B )</td>
</tr>
</tbody>
</table>
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Updated truth-tables for $\lor$ and $\supset$:

<table>
<thead>
<tr>
<th></th>
<th>$p \lor q$</th>
<th></th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
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<tr>
<td>$F$</td>
<td>$T$</td>
<td>$F$</td>
<td>$B$</td>
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<td>$B$</td>
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<table>
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<tr>
<th></th>
<th>$p \supset q$</th>
<th></th>
<th>$q$</th>
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<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
<td>$F$</td>
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</tr>
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</table>

Updated truth-tables for $\lor$ and $\supset$:
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Let $P = \text{‘Pigs can fly’}
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Let $P = \text{‘Pigs can fly’}

\[
\begin{align*}
L \lor P \\
\sim L \\
\hline
P \\
\end{align*}
\]

\[
\begin{align*}
L \supset P \\
L \\
\hline
P \\
\end{align*}
\]
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Let $P = \text{‘Pigs can fly’}$

\[
\begin{align*}
L \lor P & \quad L \supset P \\
\sim L & \quad L \\
\hline
P & \quad P
\end{align*}
\]

- Priest: for each argument, the premises are all true (and also false), yet the conclusion is false.
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Let \( P = \text{‘Pigs can fly’} \)

- \[
\begin{align*}
L \lor P \\
\sim L \\
\hline
P \\
\end{align*}
\]

- \[
\begin{align*}
L \supset P \\
L \\
\hline
P \\
\end{align*}
\]

- Priest: for each argument, the premises are all true (and also false), yet the conclusion is false.

- Yet these arguments are of the form disjunctive syllogism and modus ponens

- \[
\begin{align*}
p \lor q \\
\sim p \\
\hline
q \\
\end{align*}
\]

- \[
\begin{align*}
p \supset q \\
p \\
\hline
q \\
\end{align*}
\]
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Let $P = ‘Pigs can fly’$

- Priest: for each argument, the premises are all true (and also false), yet the conclusion is false.

- Yet these arguments are of the form *disjunctive syllogism* and *modus ponens*

- So, Priest concludes: both *disjunctive syllogism* and *modus ponens* are deductively invalid.
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Almost everybody else is going to get off the boat by denying that $L$—or any other proposition, for that matter—is both true and false.
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Almost everybody else is going to get off the boat by denying that $L$—or any other proposition, for that matter—is both true and false.
- But then we have to say something about $L$’s truth value.
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

- Almost everybody else is going to get off the boat by denying that $L$—or any other proposition, for that matter—is both true and false.
- But then we have to say something about $L$’s truth value.
- Say that it’s neither true nor false?
A Counterexample to Disjunctive Syllogism?

- Almost everybody else is going to get off the boat by denying that $L$—or any other proposition, for that matter—is both true and false.
- But then we have to say something about $L$’s truth value.
- Say that it’s neither true nor false?

$$L' := L' \text{ is not true.}$$
OUTLINE

CORRECTNESS AND COMPLETENESS

ARGUMENTS THAT PL IS NOT CORRECT
  A COUNTEREXAMPLE TO MODUS PONENS?
  A COUNTEREXAMPLE TO MODUS TOLLENS?
  A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?

THE SORITES PARADOX

WHY PL IS NOT COMPLETE

THE LANGUAGE QL

PRELIMINARY ORIENTATION

THE SYNTAX OF QL
  FREE AND BOUND VARIABLES

SEMANTICS FOR QL
  TRUTH ON AN INTERPRETATION
THE SORITES PARADOX

- Suppose that we have 10,000 tiles lined up in a row.
The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.
  - The first tile is unmistakably red.
The Sorites Paradox

- Suppose that we have 10,000 tiles lined up in a row.
  - The first tile is unmistakably red.
  - The next tile in the sequence is perceptually indistinguishable from the first, but its color has ever-so-slightly more yellow in it than the first.
Suppose that we have 10,000 tiles lined up in a row.

- The first tile is unmistakably red.
- The next tile in the sequence is perceptually indistinguishable from the first, but its color has ever-so-slightly more yellow in it than the first.
- The third tile has ever-so-slightly more yellow in it than the second
The Sorites Paradox

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  - So on and so forth.
**The Sorites Paradox**

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  - So on and so forth.
- Any pair of sequential tiles are perceptually indistinguishable.
THE SORITES PARADOX

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  - The first tile is unmistakably red.
  - The next tile in the sequence is perceptually indistinguishable from the first, but its color has ever-so-slightly more yellow in it than the first.
  - The third tile has ever-so-slightly more yellow in it than the second.
  - So on and so forth.
- Any pair of sequential tiles are perceptually indistinguishable.
- By the end of the sequence, we have a tile that is unmistakably orange.
THE SORITES PARADOX

1) The 1\textsuperscript{st} tile is red.
**The Sorites Paradox**

1) The 1\textsuperscript{st} tile is red.
2) The 1\textsuperscript{st} tile is red \lor the 2\textsuperscript{nd} tile is red.
The Sorites Paradox

1) The 1\textsuperscript{st} tile is red.
2) The 1\textsuperscript{st} tile is red \lor the 2\textsuperscript{nd} tile is red.
3) The 2\textsuperscript{nd} tile is red \lor the 3\textsuperscript{rd} tile is red.

Something's gone wrong with this reasoning, and some people have been tempted to point the finger at modus ponens.
The Sorites Paradox

1) The 1\textsuperscript{st} tile is red.
2) The 1\textsuperscript{st} tile is red \supset the 2\textsuperscript{nd} tile is red.
3) The 2\textsuperscript{nd} tile is red \supset the 3\textsuperscript{rd} tile is red.
4) The 3\textsuperscript{rd} tile is red \supset the 4\textsuperscript{th} tile is red.
THE SORITES PARADOX

1) The 1\textsuperscript{st} tile is red.
2) The 1\textsuperscript{st} tile is red $\supset$ the 2\textsuperscript{nd} tile is red.
3) The 2\textsuperscript{nd} tile is red $\supset$ the 3\textsuperscript{rd} tile is red.
4) The 3\textsuperscript{rd} tile is red $\supset$ the 4\textsuperscript{th} tile is red.

\vdots \\
\vdots
The Sorites Paradox

1) The $1^{st}$ tile is red.
2) The $1^{st}$ tile is red $\supset$ the $2^{nd}$ tile is red.
3) The $2^{nd}$ tile is red $\supset$ the $3^{rd}$ tile is red.
4) The $3^{rd}$ tile is red $\supset$ the $4^{th}$ tile is red.
    $\vdots$
    $\vdots$
10,000) The $9,999^{th}$ tile is red $\supset$ the $10,000^{th}$ tile is red.
**The Sorites Paradox**

1) The $1^{st}$ tile is red.
2) The $1^{st}$ tile is red $\supset$ the $2^{nd}$ tile is red.
3) The $2^{nd}$ tile is red $\supset$ the $3^{rd}$ tile is red.
4) The $3^{rd}$ tile is red $\supset$ the $4^{th}$ tile is red.
   :              :
10,000) The $9,999^{th}$ tile is red $\supset$ the $10,000^{th}$ tile is red.
10,001) The $10,000^{th}$ tile is red.
**THE SORITES PARADOX**

1) The 1\textsuperscript{st} tile is red.
2) The 1\textsuperscript{st} tile is red \supset \text{the 2\textsuperscript{nd} tile is red.}
3) The 2\textsuperscript{nd} tile is red \supset \text{the 3\textsuperscript{rd} tile is red.}
4) The 3\textsuperscript{rd} tile is red \supset \text{the 4\textsuperscript{th} tile is red.}
   
   \vdots
   
   10,000) The 9,999\textsuperscript{th} tile is red \supset \text{the 10,000\textsuperscript{th} tile is red.}
10,001) The 10,000\textsuperscript{th} tile is red.

- Something’s gone wrong with this reasoning, and some people have been tempted to point the finger at *modus ponens*. 
OUTLINE

CORRECTNESS AND COMPLETENESS
ARGUMENTS THAT PL IS NOT CORRECT
A COUNTEREXAMPLE TO MODUS PONENS?
A COUNTEREXAMPLE TO MODUS TOLLENS?
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?
THE SORITES PARADOX

WHY PL IS NOT COMPLETE

THE LANGUAGE QL
PRELIMINARY ORIENTATION
THE SYNTAX OF QL
FREE AND BOUND VARIABLES
SEMANTICS FOR QL
TRUTH ON AN INTERPRETATION
**Why PL is Not Complete**

1. Johann knows Filipa.
2. So, somebody knows Filipa.

1. Everyone who owns a Ford owns a car.
2. Rohan owns a Ford.
Why PL is Not Complete

1. Johann knows Filipa.

\[\text{1. Everyone who owns a Ford owns a car.}\]

\[\text{2. So, somebody knows Filipa.}\]

1. Everyone who owns a Ford owns a car.

\[\text{2. Rohan owns a Ford.}\]

\[\text{3. So, Rohan owns a car.}\]

- Both arguments are deductively valid
**Why PL is Not Complete**

1. Johann knows Filipa.
   
   2. So, somebody knows Filipa.

1. Everyone who owns a Ford owns a car.
   
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- Both arguments are deductively valid
- Both arguments are $PL$-invalid
**Why PL is Not Complete**

1. Johann knows Filipa.
   
   2. So, somebody knows Filipa.

1. Everyone who owns a Ford owns a car.
   
   2. Rohan owns a Ford.
   

- Both arguments are deductively valid
- Both arguments are \( PL \)-invalid
  - \( J \parallel S \)
Why *PL* is Not Complete

1. Johann knows Filipa.
2. So, somebody knows Filipa.

1. Everyone who owns a Ford owns a car.
2. Rohan owns a Ford.

- Both arguments are deductively valid
- Both arguments are *PL*-invalid
  - \( J /\// S \)
  - \( E / F /\// C \)
**Correctness and Completeness**

Arguments that PL is Not Correct
- A Counterexample to Modus Ponens?
- A Counterexample to Modus Tollens?
- A Counterexample to Disjunctive Syllogism?
- The Sorites Paradox

Why PL is Not Complete

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**The Language QL**

Preliminary Orientation

The Syntax of QL
- Free and Bound Variables

Semantics for QL
- Truth on an Interpretation
**OUTLINE**

**CORRECTNESS AND COMPLETENESS**
- Arguments that PL is Not Correct
- A Counterexample to Modus Ponens?
- A Counterexample to Modus Tollens?
- A Counterexample to Disjunctive Syllogism?
- The Sorites Paradox
- Why PL is Not Complete

**THE LANGUAGE QL**
- Preliminary Orientation
  - The Syntax of QL
  - Free and Bound Variables
  - Semantics for QL
  - Truth on an Interpretation
Preliminary Orientation

- we’re going to use capital letters to denote properties and relations
PRELIMINARY ORIENTATION

- we’re going to use capital letters to denote properties and relations
- we’re going to use lowercase letters to denote things
PRELIMINARY ORIENTATION

\[ Tx = x \text{ was tall} \]
\[ Lxy = x \text{ loved } y \]
\[ Kxy = x \text{ killed } y \]

\[ l = \text{ Abraham Lincoln} \]
\[ b = \text{ John Wilkes Booth} \]
\[ c = \text{ Caesar} \]
\[ p = \text{ Pompey} \]
Preliminary Orientation

\[ Tx = x \text{ was tall} \]
\[ Lxy = x \text{ loved } y \]
\[ Kxy = x \text{ killed } y \]
\[ l = \text{ Abraham Lincoln} \]
\[ b = \text{ John Wilkes Booth} \]
\[ c = \text{ Caesar} \]
\[ p = \text{ Pompey} \]

\[ Tl = \text{ Abraham Lincoln was tall} \]
\[ Kbl = \text{ John Wilkes Booth killed Abraham Lincoln} \]
\[ Lcp = \text{ Caesar loved Pompey} \]
PRELIMINARY ORIENTATION

- we treat these statements the same way that we treated the statement letters of *PL*
Preliminary Orientation

- we treat these statements the same way that we treated the statement letters of PL
- they can be negands, disjuncts, antecedents, etc.

\[ \sim Lbl = \text{John Wilkes Booth didn’t love Abraham Lincoln} \]
\[ Kcp \supset \sim Lcp = \text{If Caesar killed Pompey, then he didn’t love him} \]
\[ Tc \lor Tb = \text{Either Caesar or John Wilkes Booth is tall} \]
Preliminary Orientation

- we treat these statements the same way that we treated the statement letters of PL
- they can be negands, disjuncts, antecedents, etc.

\[ \sim Lbl = \text{John Wilkes Booth didn’t love Abraham Lincoln} \]
\[ Kcp \supset \sim Lcp = \text{If Caesar killed Pompey, then he didn’t love him} \]
\[ Tc \lor Tb = \text{Either Caesar or John Wilkes Booth is tall} \]

- We’ll also get to say things like ‘everyone loves someone’ and ‘no one loves anyone who killed them’.

\[ (x)(\exists y)L_{xy} = \text{Everyone loves someone} \]
\[ \sim (\exists x)(\exists y)(Ky\, x \land L_{xy}) = \text{No one loves anyone who killed them} \]
OUTLINE

CORRECTNESS AND COMPLETENESS

ARGUMENTS THAT PL IS NOT CORRECT
A Counterexample to Modus Ponens?
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A Counterexample to Disjunctive Syllogism?
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WHY PL IS NOT COMPLETE

THE LANGUAGE QL

PRELIMINARY ORIENTATION

THE SYNTAX OF QL

FREE AND BOUND VARIABLES

SEMANTICS FOR QL

TRUTH ON AN INTERPRETATION
The vocabulary of QL includes the following symbols:

1. for each $n \geq 0$, an infinite number of $n$-place predicates (any capital letter, along with a superscript $n$—perhaps with subscripts)

$$A^1 \ B^1 \ \ldots \ \ Z^1 \ A^1_1 \ \ldots \ \ Z^1_1 \ A^1_2 \ \ldots$$

$$A^2 \ B^2 \ \ldots \ \ Z^2 \ A^2_1 \ \ldots \ \ Z^2_1 \ A^2_2 \ \ldots$$

$$\vdots \ \vdots \ \ldots \ \vdots \ \vdots \ \ldots \ \vdots \ \ldots$$

$$A^n \ B^n \ \ldots \ \ Z^n \ A^n_1 \ \ldots \ \ Z^n_1 \ A^n_2 \ \ldots$$

$$\vdots \ \vdots \ \ldots \ \vdots \ \vdots \ \ldots \ \vdots \ \ldots$$
2. An infinite number of constants (any lowercase letter between $a$ and $w$—perhaps with subscripts)

$$a, b, c, \ldots, u, v, w, a_1, b_1, \ldots, v_1, w_1, a_2, b_2, \ldots$$
VOCABULARY

2. An infinite number of constants (any lowercase letter between $a$ and $w$—perhaps with subscripts)

   $a, b, c, \ldots, u, v, w, a_1, b_1, \ldots, v_1, w_1, a_2, b_2, \ldots$

3. An infinite number of variables (lowercase $x, y, z$—perhaps with subscripts)

   $x, y, z, x_1, y_2, z_2, x_3 \ldots$
Vocabulary

2. An infinite number of constants (any lowercase letter between $a$ and $w$—perhaps with subscripts)

   
   $a, b, c, \ldots, u, v, w, a_1, b_1, \ldots, v_1, w_1, a_2, b_2, \ldots$

3. An infinite number of variables (lowercase $x, y$, or $z$—perhaps with subscripts)

   $x, y, z, x_1, y_2, z_2, x_3 \ldots$

4. Logical operators

   $\neg, \lor, \bullet, \exists, \equiv$
Vocabulary

2. An infinite number of *constants* (any lowercase letter between \(a\) and \(w\)—perhaps with subscripts)

\[ a, b, c, \ldots, u, v, w, a_1, b_1, \ldots, v_1, w_1, a_2, b_2, \ldots \]

3. An infinite number of *variables* (lowercase \(x, y,\) or \(z\)—perhaps with subscripts)

\[ x, y, z, x_1, y_2, z_2, x_3 \ldots \]

4. Logical operators

\[ \sim, \lor, \cdot, \exists, \equiv, \forall \]

5. Parentheses

\[ (, ) \]
VOCABULARY

2. An infinite number of constants (any lowercase letter between a and w—perhaps with subscripts)

\[a, b, c, \ldots, u, v, w, a_1, b_1, \ldots, v_1, w_1, a_2, b_2, \ldots\]

3. An infinite number of variables (lowercase x, y, or z—perhaps with subscripts)

\[x, y, z, x_1, y_2, z_2, x_3 \ldots\]

4. Logical operators

\[\sim, \lor, \cdot, \forall, \equiv, \exists\]

5. Parentheses

\[(, )\]

Nothing else is included in the vocabulary of QL.
VOCABULARY

- Let’s call both constants and variables terms. That is, both ‘a’ and ‘x’ are terms of QL.
GRAMMAR

- Any sequence of the symbols in the vocabulary of QL is a formula of QL.
GRAMMAR

Any sequence of the symbols in the vocabulary of QL is a formula of QL.

All of the following are formulae of QL:

\[ V^{2800}x \sim (\varnothing \varnothing \text{ anv} \]
\[ P^1Q^2R^3S^4T^5 \sim \sim \]
\[ ((x)F^3xab \supset \sim (\exists y)P^4ynst) \]
\[ N^{54}xy\lor \sim (\exists x)B^2x \]
Any sequence of the symbols in the vocabulary of QL is a formula of QL.

All of the following are formulae of QL:

\[ V^{2800}x \sim (\lor \land an \lor \land an \lor \land an) \]
\[ P^1 Q^2 R^3 S^4 T^5 \sim \sim \]
\[ ((x)F^3xab \lor \sim (\exists y)P^4ynst) \]
\[ N^{54}xy \lor \sim (\exists x)B^2x \]
If \( \mathcal{F}^n \) is an \( n \)-place predicate and \( a_1, a_2, \ldots, a_n \) are \( n \) terms, then \( \mathcal{F}^n a_1 a_2 \ldots a_n \) is a wff.
**Grammar**

1) If ‘$\mathcal{F}^n$’ is an $n$-place predicate and ‘$a_1$’, ‘$a_2$’, …, ‘$a_n$’ are $n$ terms, then ‘$\mathcal{F}^n a_1 a_2 \ldots a_n$’ is a wff.

2) If ‘$P$’ is a wff, then ‘$\sim P$’ is a wff.
**Grammar**

1) If ‘\( F^n \)’ is an \( n \)-place predicate and ‘\( a_1 \)’, ‘\( a_2 \)’, \ldots , ‘\( a_n \)’ are \( n \) terms, then ‘\( F^n a_1 a_2 \ldots a_n \)’ is a wff.

2) If ‘\( P \)’ is a wff, then ‘\( \sim P \)’ is a wff.

3) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \cdot Q) \)’ is a wff.
CORRECTNESS AND COMPLETENESS

THE LANGUAGE QL

GRAMMAR

If \( \mathcal{F} \) is an \( n \)-place predicate and \( \mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n \) are \( n \) terms, then \( \mathcal{F}^{n} \mathbf{a}_1 \mathbf{a}_2 \ldots \mathbf{a}_n \) is a wff.

\( \sim \) If \( \mathbf{P} \) is a wff, then \( \sim \mathbf{P} \) is a wff.

\( \bullet \) If \( \mathbf{P} \) and \( \mathbf{Q} \) are wffs, then \((\mathbf{P} \cdot \mathbf{Q})\) is a wff.

\( \vee \) If \( \mathbf{P} \) and \( \mathbf{Q} \) are wffs, then \((\mathbf{P} \lor \mathbf{Q})\) is a wff.

Nothing else is a wff.
**Grammar**

\( F \) If \( F^n \) is an \( n \)-place predicate and \( a_1, a_2, \ldots, a_n \) are \( n \) terms, then \( F^n a_1 a_2 \ldots a_n \) is a wff.

\( \sim \) If \( P \) is a wff, then \( \sim P \) is a wff.

\( \cdot \) If \( P \) and \( Q \) are wffs, then \( (P \cdot Q) \) is a wff.

\( \lor \) If \( P \) and \( Q \) are wffs, then \( (P \lor Q) \) is a wff.

\( \supset \) If \( P \) and \( Q \) are wffs, then \( (P \supset Q) \) is a wff.

\( \exists \) If \( P \) is a wff and \( x \) is a variable, then \( (x) P \) is a wff.

\( \forall \) If \( P \) is a wff and \( x \) is a variable, then \( (\forall x) P \) is a wff.

Nothing else is a wff.
**Grammar**

1. If \( \mathcal{F}^n \) is an \( n \)-place predicate and \('a_1', 'a_2', \ldots, 'a_n'\) are \( n \) terms, then \('\mathcal{F}^n a_1 a_2 \ldots a_n'\) is a wff.

2. If \('P'\) is a wff, then \('\sim P'\) is a wff.

3. If \('P'\) and \('Q'\) are wffs, then \('(P \cdot Q)'\) is a wff.

4. If \('P'\) and \('Q'\) are wffs, then \('(P \lor Q)'\) is a wff.

5. If \('P'\) and \('Q'\) are wffs, then \('(P \supset Q)'\) is a wff.

6. If \('P'\) and \('Q'\) are wffs, then \('(P \equiv Q)'\) is a wff.

7. If \('P'\) is a wff and \(x\) is a variable, then \('(x) P'\) is a wff.

8. If \('P'\) is a wff and \(x\) is a variable, then \('(\exists x) P'\) is a wff.

Nothing else is a wff.
GRAMMAR

\[ F \] If ‘\( F^n \)’ is an \( n \)-place predicate and ‘\( a_1 \)’, ‘\( a_2 \)’, \ldots, ‘\( a_n \)’ are \( n \) terms, then ‘\( F^n a_1 a_2 \ldots a_n \)’ is a wff.

\( \sim \) If ‘\( P \)’ is a wff, then ‘\( \sim P \)’ is a wff.

\( \cdot \) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \cdot Q) \)’ is a wff.

\( \lor \) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \lor Q) \)’ is a wff.

\( \supset \) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \supset Q) \)’ is a wff.

\( \equiv \) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \equiv Q) \)’ is a wff.

\( x \) If ‘\( P \)’ is a wff and \( x \) is a variable, then ‘\( (x)P \)’ is a wff.
**GRAMMAR**

\( \mathcal{F} \)  If ‘\( \mathcal{F}^n \)’ is an \( n \)-place predicate and ‘\( a_1 \), ‘\( a_2 \), . . . , ‘\( a_n \)’ are \( n \) terms, then ‘\( \mathcal{F}^n a_1 a_2 \ldots a_n \)’ is a wff.

\( \sim \) If ‘\( P \)’ is a wff, then ‘\( \sim P \)’ is a wff.

\( \bullet \) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \bullet Q) \)’ is a wff.

\( \forall \) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \lor Q) \)’ is a wff.

\( \exists \) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \supset Q) \)’ is a wff.

\( \equiv \) If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \equiv Q) \)’ is a wff.

\( x \) If ‘\( P \)’ is a wff and \( x \) is a variable, then ‘\( (x)P \)’ is a wff.

\( \exists \) If ‘\( P \)’ is a wff and \( x \) is a variable, then ‘\( (\exists x)P \)’ is a wff.
**Grammar**

\[ F \]  If ‘\( F^n \)’ is an \( n \)-place predicate and ‘\( a_1 \)’, ‘\( a_2 \)’, \ldots , ‘\( a_n \)’ are \( n \) terms, then ‘\( F^n a_1 a_2 \ldots a_n \)’ is a wff.

\[ \neg \] If ‘\( P \)’ is a wff, then ‘\( \neg P \)’ is a wff.

\[ \land \] If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \land Q) \)’ is a wff.

\[ \lor \] If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \lor Q) \)’ is a wff.

\[ \supset \] If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \supset Q) \)’ is a wff.

\[ \equiv \] If ‘\( P \)’ and ‘\( Q \)’ are wffs, then ‘\( (P \equiv Q) \)’ is a wff.

\[ x \] If ‘\( P \)’ is a wff and \( x \) is a variable, then ‘\( (x)P \)’ is a wff.

\[ \exists \] If ‘\( P \)’ is a wff and \( x \) is a variable, then ‘\( (\exists x)P \)’ is a wff.

– Nothing else is a wff.
Note: none of ‘F’, ‘a’, ‘P’, and ‘Q’ appear in the vocabulary of QL. They are not themselves wffs of QL. Rather, we are using them here as VARIABLES ranging over the formulae of QL.
To show that ‘((y)F₁y ⊃ (∃x)(∃z)G₂zx)’ is a wff of QL:
To show that ‘$((y)F^1y \sim (\exists x)(\exists z)G^2zx)$’ is a wff of QL:

a) ‘$F^1y$’ is a wff [from \(\mathcal{F}\)]
To show that ‘((y)F^1y ∨ (y)(∃x)(∃z)G^2zx)’ is a wff of QL:

a) ‘F^1y’ is a wff  
   [from (F)]

b) So, ‘(y)F^1y’ is a wff  
   [from (a) and (x)]
To show that ‘((y)F^1y ⊨ (x)(z)G^2zx)’ is a wff of QL:

a) ‘F^1y’ is a wff [from (F)]

b) So, ‘(y)F^1y’ is a wff [from (a) and (x)]

c) ‘G^2zx’ is a wff [from (F)]
To show that ‘\(((y)F^1y \supset \exists x)(\exists z)G^2zx\)’ is a wff of QL:

a) ‘\(F^1y\)’ is a wff [from (\(\exists\))]  
b) So, ‘\((y)F^1y\)’ is a wff [from (a) and (\(\exists\))]  
c) ‘\(G^2zx\)’ is a wff [from (\(\exists\))]  
d) So, ‘\((\exists z)G^2zx\)’ is a wff [from (c) and (\(\exists\))]
Grammar

To show that ‘((y)F₁y ⊃ (exists x)(exists z)G₂zx)’ is a wff of QL:

a) ‘F₁y’ is a wff [from (∃)]
b) So, ‘(y)F₁y’ is a wff [from (a) and (x)]
c) ‘G₂zx’ is a wff [from (∃)]
d) So, ‘(exists z)G₂zx’ is a wff [from (c) and (∃)]
e) So, ‘(exists x)(exists z)G₂zx’ is a wff [from (d) and (∃)]
To show that ‘((y)F^1y ⊃ (∃x)(∃z)G^2zx’) is a wff of QL:

a) ‘F^1y’ is a wff [from (I)]
b) So, ‘(y)F^1y’ is a wff [from (a) and (x)]
c) ‘G^2zx’ is a wff [from (I)]
d) So, ‘(∃z)G^2zx’ is a wff [from (c) and (∃)]
e) So, ‘(∃x)(∃z)G^2zx’ is a wff [from (d) and (∃)]
f) So, ‘~(∃x)(∃z)G^2zx’ is a wff [from (e) and (~)]
To show that ‘\((y)F^1y \supset (\exists x)(\exists z)G^2xz\)’ is a wff of QL:

a) ‘\(F^1y\)’ is a wff [from (\(\mathcal{F}\))]
b) So, ‘\((y)F^1y\)’ is a wff [from (a) and (\(\exists\))]
c) ‘\(G^2xz\)’ is a wff [from (\(\mathcal{F}\))]
d) So, ‘\((\exists z)G^2xz\)’ is a wff [from (c) and (\(\exists\))]
e) So, ‘\((\exists x)(\exists z)G^2xz\)’ is a wff [from (d) and (\(\exists\))]
f) So, ‘\(\sim (\exists x)(\exists z)G^2xz\)’ is a wff [from (e) and (\(\sim\))]
g) So, ‘\((y)F^1y \supset (\exists x)(\exists z)G^2xz\)’ is a wff [from (b), (f), and (\(\supset\))]

GRAMMAR
GRAMMAR

- Conventions:
Conventions:

- Omit the outermost parentheses in a wff of QL.
GRAMMAR

- Conventions:
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  - Omit the superscripts on the predicates of QL.
Grammar

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  - Omit the outermost parentheses in a wff of QL.
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- So, rather than

\[ ((y)F^1 y \supset (\exists x)(\exists z)G^2 zx) \]
**Grammar**

- Conventions:
  - Omit the outermost parentheses in a wff of $QL$.
  - Omit the superscripts on the predicates of $QL$.

- So, rather than

$$((y)F^1y \supset (\exists x)(\exists z)G^2zx)$$

- we can write

$$(y)Fy \supset (\exists x)(\exists z)Gzx$$
GRAMMAR

- Syntax Trees

\[(y)Fy \supset \sim (\exists x)(\exists z)Gzx\]

\[(y)Fy \sim (\exists x)(\exists z)Gzx\]

\[Fy \sim (\exists x)(\exists z)Gzx\]

\[Fy \sim (\exists x)(\exists z)Gzx\]

\[(\exists z)Gzx\]

\[Gzx\]
OUTLINE

CORRECTNESS AND COMPLETENESS

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A COUNTEREXAMPLE TO MODUS TOLLENS?
A COUNTEREXAMPLE TO DISJUNCTIVE SYLLOGISM?
THE SORITES PARADOX

WHY PL IS NOT COMPLETE

THE LANGUAGE QL

PRELIMINARY ORIENTATION

THE SYNTAX OF QL
FREE AND BOUND VARIABLES

SEMANTICS FOR QL
TRUTH ON AN INTERPRETATION
FREE AND BOUND VARIABLES

- ‘Fx’ and ‘Ay<sub>c</sub>’ as wffs.
Free and Bound Variables

- ‘Fx’ and ‘AyCy’ as wffs.
- However, their variables are **FREE**.
Free and Bound Variables

- ‘Fx’ and ‘Ayc’ as wffs.
- However, their variables are FREE.
- The variables appearing in ‘(x)(y)Fxy’ are BOUND.
FREE AND BOUND VARIABLES

- ‘Fx’ and ‘Ay c’ as wffs.
- However, their variables are FREE.
- The variables appearing in ‘(x)(y)Fxy’ are BOUND.
- In ‘(x)Px ⊃ Qx’, the first x is bound, whereas the second one is free.
QUANTIFIERS

- For any variable \( x \), both ‘(x)’ and ‘(\exists x)’ are quantifiers.
QUANTIFIERS

- For any variable $x$, both ‘$(x)$’ and ‘$(\exists x)$’ are quantifiers.
  - ‘$(x)$’ is the UNIVERSAL QUANTIFIER
Quantifiers

- For any variable \( x \), both \( (x) \) and \( (\exists x) \) are quantifiers.
  - \( (x) \) is the UNIVERSAL QUANTIFIER
  - \( (\exists x) \) is the EXISTENTIAL QUANTIFIER.
For any variable $x$, both $'(x)'$ and $'\exists x'\) are quantifiers.

- $'(x)'$ is the universal quantifier
- $'\exists x'\) is the existential quantifier.

Quantifiers are logical operators.
QUANTIFIERS

- For any variable $x$, both $(x)$ and $(\exists x)$ are quantifiers.
  - $(x)$ is the UNIVERSAL QUANTIFIER
  - $(\exists x)$ is the EXISTENTIAL QUANTIFIER.

- quantifiers are logical operators.
  - They can be the MAIN OPERATOR of a wff of $QL$ or they can be the main operator of a wff's subformulae.
**Main Operators**

- Define the **main operator** of a wff of QL to be the logical operator whose associated rule is *last* appealed to when building the wff up according to the rules for well-formed formulae.
Define the **main operator** of a wff of QL to be the logical operator whose associated rule is *last* appealed to when building the wff up according to the rules for well-formed formulae.

- E.g., the main operator of ‘(y)Fy ⊃ (∃x)(∃z)Gzx’ is the horseshoe ‘⊃’.
**Main Operators**

- Define the **main operator** of a wff of QL to be the logical operator whose associated rule is *last* appealed to when building the wff up according to the rules for well-formed formulae.
  - E.g., the main operator of ‘(y)Fy ⊃ (∃x)(∃z)Gzx’ is the horseshoe ‘⊃’.

- The main operator of ‘(x)Fx’, 
  
  \[
  \begin{array}{c}
  \text{(x)Fx} \\
  \hline
  Fx
  \end{array}
  \]
  
  is the universal quantifier ‘(x)’.
SUBFORMULAE

- $P$ is a subformula of $Q$ if and only if $P$ lies somewhere on $Q$’s syntax tree.
SUBFORMULAE

- P is a subformula of Q if and only if P lies somewhere on Q’s syntax tree.
- P is an immediate subformula of Q iff P lies immediately below Q on the syntax tree.
**SUBFORMULAE**

- $P$ is a subformula of $Q$ if and only if $P$ lies somewhere on $Q$’s syntax tree.
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  - the immediate subformula of ‘$(x)Fx$’ is ‘$Fx$’
**SUBFORMULAE**

- P is a subformula of Q if and only if P lies somewhere on Q’s syntax tree.
- P is an immediate subformula of Q iff P lies immediately below Q on the syntax tree.
  - the immediate subformula of ‘(x)Fx’ is ‘Fx’
  - the immediate subformula of ‘(\exists y)(Fy \land Ga)’ is ‘Fy \land Ga’.
Quantifier Scope

- The scope of a quantifier is its immediate subformula.
The scope of a quantifier is its immediate subformula.

\[(\exists y)Lyy \supset (\exists x)(\exists y)Lxy\]

\[\begin{array}{c}
(\exists y)Lyy \\
Lyy
\end{array}
\quad
\begin{array}{c}
(\exists x)(\exists y)Lxy \\
(\exists y)Lxy \\
Lxy
\end{array}\]
A variable $x$ in a wff of $PL$ is **bound** if and only if it occurs within the scope of a quantifier, $(x)$ or $(\exists x)$, whose associated variable is $x$.

A variable $x$ in a wff of $PL$ is **free** if and only if it does not occur within the scope of a quantifier, $(x)$ or $(\exists x)$, whose associated variable is $x$. 
Free and Bound Variables

A variable $x$ in a wff of $PL$ is bound if and only if it occurs within the scope of a quantifier, $(x)$ or $(\exists x)$, whose associated variable is $x$.

A variable $x$ in a wff of $PL$ is free if and only if it does not occur within the scope of a quantifier, $(x)$ or $(\exists x)$, whose associated variable is $x$.

- E.g.,

$$(x)(y)Fy \supset (\exists z) Gzx$$
Free and Bound Variables

In a wff of QL, a quantifier (\(x\)) or (\(\exists x\)) binds a variable \(x\) if and only if \(x\) occurs free within that quantifier’s scope.
**Free and Bound Variables**

In a wff of QL, a quantifier $(x)$ or $(\exists x)$ binds a variable $x$ if and only if $x$ occurs free within that quantifier’s scope.

- E.g., in $(\exists x)(x)Fx$
  
  the variable ‘$x$’ is bound by the *universal* quantifier ‘$(x)$’. It is *not* bound by the existential quantifier ‘$(\exists x)$’.
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The meaning of ‘∼, ∨, •, ⊃, and ≡ are the same as before
SEMANTICS FOR QL

- The meaning of ‘∼, ∨, •, ⊃, and ≡ are the same as before
- In order to explain the meaning of the other expressions in QL, we introduce the idea of a QL-INTERPRETATION for a wff or set of wffs of QL.
QL-INTERPRETATIONS

A QL-INTERPRETATION, $\mathcal{I}$, of a wff or set of wffs of QL provides:
QL-INTERPRETATIONS

A QL-INTERPRETATION, $\mathcal{I}$, of a wff or set of wffs of QL provides:

1. A specification of which things fall in the domain, $\mathcal{D}$, of the interpretation.
QL-INTERPRETATIONS

A QL-INTERPRETATION, $\mathcal{I}$, of a wff or set of wffs of QL provides:

1. A specification of which things fall in the domain, $\mathcal{D}$, of the interpretation.
2. A specification of which things in the domain the constants appearing in the wff or wffs of QL represent.
QL-INTERPRETATIONS

A QL-INTERPRETATION, $\mathcal{I}$, of a wff or set of wffs of QL provides:

1. A specification of which things fall in the domain, $\mathcal{D}$, of the interpretation.
2. A specification of which things in the domain the constants appearing in the wff or wffs of QL represent.
3. A specification of which things in the domain the free variables appearing in the wff or wffs of QL represent.
QL-INTERPRETATIONS

A QL-INTERPRETATION, $\mathcal{I}$, of a wff or set of wffs of QL provides:

1. A specification of which things fall in the domain, $\mathcal{D}$, of the interpretation.
2. A specification of which things in the domain the constants appearing in the wff or wffs of QL represent.
3. A specification of which things in the domain the free variables appearing in the wff or wffs of QL represent.
4. For every $n$-place predicate appearing in the wff or wffs of QL, a specification of the $n$-place property it represents.
QL-INTERPRETATIONS

(y)\text{Lya} \lor (\exists y) \sim \text{Lya}
QL-INTERPRETATIONS

\((y)\text{Ly}a \lor (\exists y) \sim \text{Ly}a\)

\(\mathcal{I} = \begin{cases} 
\mathcal{D} & = \text{the set of all people} \\
a & = \text{Steve} \\
Lxy & = x \text{ loves } y 
\end{cases}\)
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SEMANTICS FOR QL
TRUTH ON AN INTERPRETATION
**Truth on a QL-interpretation**

1. A wff of the form ‘$\forall^n a_1 \ldots a_n$’ is true on the interpretation $\mathcal{I}$ if the things in the domain represented by $a_1 \ldots a_n$ have the property represented by $\forall^n$. Otherwise, it is false on the interpretation $\mathcal{I}$. 
Truth on a QL-interpretation

1. A wff of the form \( \mathcal{F}^n a_1 \ldots a_n \) is true on the interpretation \( \mathcal{I} \) if the things in the domain represented by \( a_1 \ldots a_n \) have the property represented by \( \mathcal{F}^n \). Otherwise, it is false on the interpretation \( \mathcal{I} \).

2. A wff of the form \( \sim P \) is true on the interpretation \( \mathcal{I} \) if \( P \) is false on the interpretation \( \mathcal{I} \). Otherwise, it is false on the interpretation \( \mathcal{I} \).
TRUTH ON A QL-INTERPRETATION

1. A wff of the form ‘$F^na_1 \ldots a_n$’ is true on the interpretation $\mathcal{I}$ if the things in the domain represented by $a_1 \ldots a_n$ have the property represented by $F^n$. Otherwise, it is false on the interpretation $\mathcal{I}$.

2. A wff of the form ‘$\neg P$’ is true on the interpretation $\mathcal{I}$ if $P$ is false on the interpretation $\mathcal{I}$. Otherwise, it is false on the interpretation $\mathcal{I}$.

3. A wff of the form ‘$P \lor Q$’ is true on the interpretation $\mathcal{I}$ if either $P$ is true on the interpretation $\mathcal{I}$ or $Q$ is true on the interpretation $\mathcal{I}$. Otherwise, it is false on the interpretation $\mathcal{I}$.
**Truth on a QL-Interpretation**

4. A wff of the form ‘\( P \land Q \)’ is true on the interpretation \( \mathcal{I} \) if both \( P \) is true on the interpretation \( \mathcal{I} \) and \( Q \) is true on the interpretation \( \mathcal{I} \). Otherwise, it is false on the interpretation \( \mathcal{I} \).
4. A wff of the form ‘P • Q’ is true on the interpretation $\mathcal{I}$ if both P is true on the interpretation $\mathcal{I}$ and Q is true on the interpretation $\mathcal{I}$. Otherwise, it is false on the interpretation $\mathcal{I}$.

5. A wff of the form ‘P ⊃ Q’ is true on the interpretation $\mathcal{I}$ if either P is false on the interpretation $\mathcal{I}$ or Q is true on the interpretation $\mathcal{I}$. Otherwise, it is false on the interpretation $\mathcal{I}$. 

Truth on a QL-interpretation
4. A wff of the form ‘$P \cdot Q$’ is true on the interpretation $I$ if both $P$ is true on the interpretation $I$ and $Q$ is true on the interpretation $I$. Otherwise, it is false on the interpretation $I$.

5. A wff of the form ‘$P \supset Q$’ is true on the interpretation $I$ if either $P$ is false on the interpretation $I$ or $Q$ is true on the interpretation $I$. Otherwise, it is false on the interpretation $I$.

6. A wff of the form ‘$P \equiv Q$’ is true on the interpretation $I$ if both $P$ and $Q$ have the same truth value on the interpretation $I$. Otherwise, it is false on the interpretation $I$. 
SUBSTITUTION INSTANCES

- A substitution instance of a quantified wff of the form ‘(x)P’ or ‘(∃x)P’ is the wff that you get by removing the quantifier, leaving behind just its immediate subformula, and uniformly replacing every instance of the variable x which is bound by the quantifier with some one constant a.
SUBSTITUTION INSTANCES

▶ A substitution instance of a quantified wff of the form ‘(x)P’ or ‘(∃x)P’ is the wff that you get by removing the quantifier, leaving behind just its immediate subformula, and uniformly replacing every instance of the variable x which is bound by the quantifier with some one constant a
  ▶ it must be the same constant throughout
**Substitution Instances**

- A substitution instance of a quantified wff of the form ‘(x)P’ or ‘(∃x)P’ is the wff that you get by removing the quantifier, leaving behind just its immediate subformula, and uniformly replacing every instance of the variable x which is bound by the quantifier with some one constant a
  - it must be the same constant throughout

- Substitution instances of ‘(∃y)((Ay • ∼Lay) ⊃ (x) ∼Lax)’:
  
  \[
  \begin{align*}
  (Ab \cdot \sim Lab) & \supset (x) \sim Lax \\
  (Ac \cdot \sim Lac) & \supset (x) \sim Lax \\
  (Ak \cdot \sim Lak) & \supset (x) \sim Lax \\
  (Aa \cdot \sim Laa) & \supset (x) \sim Lax 
  \end{align*}
  \]
Substitution Instances

- *Not* substitution instances of ‘$(\exists y)(\sim Lay \supset (x) \sim Lax)$’:

$(Ax \cdot \sim Lax) \supset (x) \sim Lax$

$(Aa \cdot \sim Lab) \supset (x) \sim Lax$

$(Ab \cdot \sim Lbb) \supset (x) \sim Lcx$

$(Ar \cdot \sim Lra) \supset (x) \sim Lax$
TRUTH ON A QL-INTERPRETATION

7. A wff of the form ‘(x)P’ is true on the interpretation $\mathcal{I}$ if every substitution instance of ‘(x)P’ is true on the interpretation $\mathcal{I}$. Otherwise, it is false on the interpretation $\mathcal{I}$. 
Truth on a QL-Interpretation

7. A wff of the form ‘(x)P’ is true on the interpretation $\mathcal{I}$ if every substitution instance of ‘(x)P’ is true on the interpretation $\mathcal{I}$. Otherwise, it is false on the interpretation $\mathcal{I}$.

8. A wff of the form ‘(\exists x)P’ is true on the interpretation $\mathcal{I}$ if there is some substitution instance of ‘(x)P’ which is true on the interpretation $\mathcal{I}$. Otherwise, it is false on the interpretation $\mathcal{I}$. 