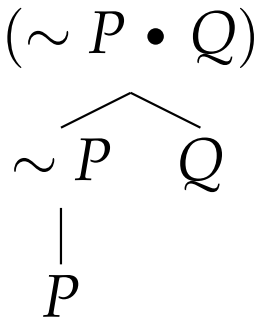
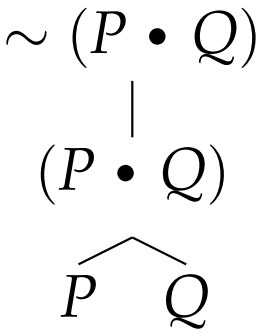
Propositional Logic, day 1

Philosophy 180
May 20, 2014





OUTLINE

Outline

THE PLAN

- Construct an artificial language, '*PL*', within which we can be incredibly precise about which arguments are deductively valid and which are deductively invalid.

THE PLAN

- Construct an artificial language, '*PL*', within which we can be incredibly precise about which arguments are deductively valid and which are deductively invalid.
- Provide a method for translating statements of English into *PL* and statements of *PL* into English

THE PLAN

- Construct an artificial language, '*PL*', within which we can be incredibly precise about which arguments are deductively valid and which are deductively invalid.
- Provide a method for translating statements of English into *PL* and statements of *PL* into English
- One advantage: we can theorize about relations of deductive validity without having to worry about the ambiguity of English (*e.g.*, equivocation and amphiboly).

LANGUAGES IN GENERAL

- To specify a language, we need to provide:

LANGUAGES IN GENERAL

- To specify a language, we need to provide:
  - a vocabulary for the language

LANGUAGES IN GENERAL

- To specify a language, we need to provide:
  - a vocabulary for the language
    - *e.g.*, a list of the words of English and their parts of speech

LANGUAGES IN GENERAL

- To specify a language, we need to provide:
  - a vocabulary for the language
    - *e.g.*, a list of the words of English and their parts of speech
  - a grammar for the language

LANGUAGES IN GENERAL

- To specify a language, we need to provide:
  - a vocabulary for the language
    - *e.g.*, a list of the words of English and their parts of speech
  - a grammar for the language
    - *e.g.*, rules for saying which strings of words are grammatical and which aren't

LANGUAGES IN GENERAL

- To specify a language, we need to provide:
  - a vocabulary for the language
    - *e.g.*, a list of the words of English and their parts of speech
  - a grammar for the language
    - *e.g.*, rules for saying which strings of words are grammatical and which aren't
      'Bubbie makes pickles' ✓

LANGUAGES IN GENERAL

- To specify a language, we need to provide:
  - a vocabulary for the language
    - *e.g.*, a list of the words of English and their parts of speech
  - a grammar for the language
    - *e.g.*, rules for saying which strings of words are grammatical and which aren't
      'Bubbie makes pickles' ✓
      'Up bouncy ball door John variously catapult' ×

LANGUAGES IN GENERAL

- To specify a language, we need to provide:
  - a vocabulary for the language
    - *e.g.*, a list of the words of English and their parts of speech
  - a grammar for the language
    - *e.g.*, rules for saying which strings of words are grammatical and which aren't
      'Bubbie makes pickles' ✓
      'Up bouncy ball door John variously catapult' ×
  - a way to interpret the *meaning* of every grammatical expression of the language

LANGUAGES IN GENERAL

- To specify a language, we need to provide:
  - a vocabulary for the language
    - *e.g.*, a list of the words of English and their parts of speech
  - a grammar for the language
    - *e.g.*, rules for saying which strings of words are grammatical and which aren't
      'Bubbie makes pickles' ✓
      'Up bouncy ball door John variously catapult' ×
  - a way to interpret the *meaning* of every grammatical expression of the language
    - *e.g.*, a dictionary entry for every word of English and rules for constructing the meaning of sentences out of the meanings of words

LANGUAGES IN GENERAL

SYNTAX —{ 1. Vocabulary
2. Grammar

SEMANTICS —3. Meaning

OUTLINE

THE VOCABULARY OF *PL*

- The vocabulary of *PL* includes the following symbols:

THE VOCABULARY OF *PL*

- The vocabulary of *PL* includes the following symbols:
  1. An infinite number of statement letters:

$$A, B, C, ..., Y, Z, A_1, B_1, C_1, ..., Y_1, Z_1, A_2, B_2, C_2, ...$$

THE VOCABULARY OF *PL*

- The vocabulary of *PL* includes the following symbols:
  1. An infinite number of statement letters:

$$A, B, C, ..., Y, Z, A_1, B_1, C_1, ..., Y_1, Z_1, A_2, B_2, C_2, ...$$

  2. logical operators:

$$\sim, \bullet, \vee, \supset, \equiv$$

THE VOCABULARY OF *PL*

- The vocabulary of *PL* includes the following symbols:
  1. An infinite number of statement letters:

     $$A, B, C, ..., Y, Z, A_1, B_1, C_1, ..., Y_1, Z_1, A_2, B_2, C_2, ...$$

  2. logical operators:

     $$\sim, \bullet, \vee, \supset, \equiv$$

  3. parenthases

     $$(\ ,\ )$$

THE VOCABULARY OF *PL*

- The vocabulary of *PL* includes the following symbols:
  1. An infinite number of statement letters:

     $$A, B, C, ..., Y, Z, A_1, B_1, C_1, ..., Y_1, Z_1, A_2, B_2, C_2, ...$$

  2. logical operators:

     $$\sim, \bullet, \vee, \supset, \equiv$$

  3. parenthases

     $$(\ ,\ )$$

- Nothing else is included in the vocabulary of *PL*.

THE GRAMMAR OF *PL*

- Any sequence of the symbols in the vocabulary of *PL* is a *formula* of *PL*.

THE GRAMMAR OF *PL*

- Any sequence of the symbols in the vocabulary of *PL* is a *formula* of *PL*.

$$(((\,)A_{23} \bullet \ \bullet \ \supset\supset Z$$
$$P \supset (Q) \supset \ \bullet \ (\,))$$
$$(P \supset (Q \supset (R \supset (S \supset T))))$$
$$A \bullet B \bullet (C \sim D)))$$

THE GRAMMAR OF *PL*

- Any sequence of the symbols in the vocabulary of *PL* is a *formula* of *PL*.

$$(((\,)A_{23} \bullet \ \bullet \ \supset\supset Z$$

$$P \supset (Q) \supset \ \bullet \ (\,))$$

$$(P \supset (Q \supset (R \supset (S \supset T))))$$

$$A \bullet B \bullet (C \sim D)))$$

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.

  $\sim$) If 'p' is a wff, then '$\sim p$' is a wff.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.

  $\sim$) If 'p' is a wff, then '$\sim p$' is a wff.

  $\bullet$) If 'p' and 'q' are wffs, then '$(p \bullet q)$' is a wff.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.
  $\sim$) If 'p' is a wff, then '$\sim p$' is a wff.
  $\bullet$) If 'p' and 'q' are wffs, then '$(p \bullet q)$' is a wff.
  $\vee$) If 'p' and 'q' are wffs, then '$(p \vee q)$' is a wff.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.
  $\sim$) If 'p' is a wff, then '$\sim p$' is a wff.
  $\bullet$) If 'p' and 'q' are wffs, then '$(p \bullet q)$' is a wff.
  $\vee$) If 'p' and 'q' are wffs, then '$(p \vee q)$' is a wff.
  $\supset$) 'p' and 'q' are wffs, then '$(p \supset q)$' is a wff.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.
  $\sim$) If 'p' is a wff, then '$\sim p$' is a wff.
  $\bullet$) If 'p' and 'q' are wffs, then '$(p \bullet q)$' is a wff.
  $\vee$) If 'p' and 'q' are wffs, then '$(p \vee q)$' is a wff.
  $\supset$) 'p' and 'q' are wffs, then '$(p \supset q)$' is a wff.
  $\equiv$) 'p' and 'q' are wffs, then '$(p \equiv q)$' is a wff.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.
  $\sim$) If 'p' is a wff, then '$\sim p$' is a wff.
  $\bullet$) If 'p' and 'q' are wffs, then '$(p \bullet q)$' is a wff.
  $\vee$) If 'p' and 'q' are wffs, then '$(p \vee q)$' is a wff.
  $\supset$) 'p' and 'q' are wffs, then '$(p \supset q)$' is a wff.
  $\equiv$) 'p' and 'q' are wffs, then '$(p \equiv q)$' is a wff.
  $-$) Nothing else is a wff.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.
  $\sim$) If 'p' is a wff, then '$\sim p$' is a wff.
  $\bullet$) If 'p' and 'q' are wffs, then '$(p \bullet q)$' is a wff.
  $\vee$) If 'p' and 'q' are wffs, then '$(p \vee q)$' is a wff.
  $\supset$) 'p' and 'q' are wffs, then '$(p \supset q)$' is a wff.
  $\equiv$) 'p' and 'q' are wffs, then '$(p \equiv q)$' is a wff.
  $-$) Nothing else is a wff.

- 'p' and 'q' do not appear in the vocabulary of *PL*.

THE GRAMMAR OF *PL*

- We define a *well-formed formula* ('wff') of *PL* with the following rules.

  SL) Any statement letter, by itself, is a wff.
  $\sim$) If 'p' is a wff, then '$\sim p$' is a wff.
  $\bullet$) If 'p' and 'q' are wffs, then '$(p \bullet q)$' is a wff.
  $\vee$) If 'p' and 'q' are wffs, then '$(p \vee q)$' is a wff.
  $\supset$) 'p' and 'q' are wffs, then '$(p \supset q)$' is a wff.
  $\equiv$) 'p' and 'q' are wffs, then '$(p \equiv q)$' is a wff.
  $-$) Nothing else is a wff.

- 'p' and 'q' do not appear in the vocabulary of *PL*.
  - They are *formulae variables*—variables whose potential values are formulae of *PL*.

THE GRAMMAR OF *PL*

- To show that '$(\sim(P \vee Q) \supset R)$' is a wff:

THE GRAMMAR OF *PL*

- To show that '$(\sim(P \vee Q) \supset R)$' is a wff:
  a) 'P' is a wff [from (SL)]

THE GRAMMAR OF *PL*

- To show that '$(\sim(P \vee Q) \supset R)$' is a wff:
  a) 'P' is a wff [from (SL)]
  b) 'Q' is a wff [from (SL)]

THE GRAMMAR OF *PL*

- To show that '$(\sim(P \vee Q) \supset R)$' is a wff:
  a) 'P' is a wff [from (SL)]
  b) 'Q' is a wff [from (SL)]
  c) So, '$(P \vee Q)$' is a wff [from (a) and (b) and ($\vee$)]

THE GRAMMAR OF *PL*

- To show that '$(\sim(P \vee Q) \supset R)$' is a wff:

a)	'P' is a wff	[from (SL)]
b)	'Q' is a wff	[from (SL)]
c)	So, '$(P \vee Q)$' is a wff	[from (a) and (b) and ($\vee$)]
d)	So, '$\sim(P \vee Q)$' is a wff	[from (c) and ($\sim$)]

THE GRAMMAR OF *PL*

- To show that '$(\sim(P \vee Q) \supset R)$' is a wff:

a)	'P' is a wff	[from (SL)]
b)	'Q' is a wff	[from (SL)]
c)	So, '$(P \vee Q)$' is a wff	[from (a) and (b) and ($\vee$)]
d)	So, '$\sim(P \vee Q)$' is a wff	[from (c) and ($\sim$)]
e)	'R' is a wff	[from (SL)]

THE GRAMMAR OF *PL*

- To show that '$(\sim(P \vee Q) \supset R)$' is a wff:

a)	'P' is a wff	[from (SL)]
b)	'Q' is a wff	[from (SL)]
c)	So, '$(P \vee Q)$' is a wff	[from (a) and (b) and ($\vee$)]
d)	So, '$\sim(P \vee Q)$' is a wff	[from (c) and ($\sim$)]
e)	'R' is a wff	[from (SL)]
f)	So, '$(\sim(P \vee Q) \supset R)$' is a wff	[from (d), (e), and ($\supset$)]

THE GRAMMAR OF *PL*

- To show that '$(\sim(P \vee Q) \supset R)$' is a wff:

a)	'P' is a wff	[from (SL)]
b)	'Q' is a wff	[from (SL)]
c)	So, '$(P \vee Q)$' is a wff	[from (a) and (b) and ($\vee$)]
d)	So, '$\sim(P \vee Q)$' is a wff	[from (c) and ($\sim$)]
e)	'R' is a wff	[from (SL)]
f)	So, '$(\sim(P \vee Q) \supset R)$' is a wff	[from (d), (e), and ($\supset$)]

- A convention: we will allow ourselves to omit the outermost parenthases, writing '$\sim(P \vee Q) \supset R$' instead of '$(\sim(P \vee Q) \supset R)$'

MAIN OPERATORS

- A wff's *main operator* is just the operator associated with the last rule which would have to be applied if we were building the formula up by applying the rules for wffs.

MAIN OPERATORS

- A wff's *main operator* is just the operator associated with the last rule which would have to be applied if we were building the formula up by applying the rules for wffs.
- E.g., what is the main operator of '$\sim P \bullet Q$'?

MAIN OPERATORS

- A wff's *main operator* is just the operator associated with the last rule which would have to be applied if we were building the formula up by applying the rules for wffs.
- E.g., what is the main operator of '$\sim P \bullet Q$'?
  a) 'P' is a wff [from (*SL*)]

MAIN OPERATORS

- A wff's *main operator* is just the operator associated with the last rule which would have to be applied if we were building the formula up by applying the rules for wffs.
- E.g., what is the main operator of '$\sim P \bullet Q$'?

a)	'P' is a wff	[from (SL)]
b)	So, '$\sim P$' is a wff	[from (a) and ($\sim$)]

MAIN OPERATORS

- A wff's *main operator* is just the operator associated with the last rule which would have to be applied if we were building the formula up by applying the rules for wffs.
- E.g., what is the main operator of '$\sim P \bullet Q$'?

a)	'P' is a wff	[from (SL)]
b)	So, '$\sim P$' is a wff	[from (a) and ($\sim$)]
c)	'Q' is a wff	[from (SL)]

MAIN OPERATORS

- A wff's *main operator* is just the operator associated with the last rule which would have to be applied if we were building the formula up by applying the rules for wffs.
- E.g., what is the main operator of '$\sim P \bullet Q$'?

a)	'P' is a wff	[from (SL)]
b)	So, '$\sim P$' is a wff	[from (a) and ($\sim$)]
c)	'Q' is a wff	[from (SL)]
d)	So, '$(\sim P \bullet Q)$' is a wff	[from (b), (c), and ($\bullet$)]

MAIN OPERATORS

- A wff's *main operator* is just the operator associated with the last rule which would have to be applied if we were building the formula up by applying the rules for wffs.
- E.g., what is the main operator of '$\sim P \bullet Q$'?

a)	'P' is a wff	[from (SL)]
b)	So, '$\sim P$' is a wff	[from (a) and ($\sim$)]
c)	'Q' is a wff	[from (SL)]
d)	So, '$(\sim P \bullet Q)$' is a wff	[from (b), (c), and ($\bullet$)]

- So, the main operator is ' $\bullet$ '

MAIN OPERATORS

- What if we had applied the rule ($\bullet$) first, and then applied the rule ($\sim$)?

MAIN OPERATORS

- What if we had applied the rule (•) first, and then applied the rule ($\sim$)?
  a) 'P' is a wff [from (SL)]

MAIN OPERATORS

- What if we had applied the rule (•) first, and then applied the rule ($\sim$)?
  a) 'P' is a wff [from (SL)]
  b) 'Q' is a wff [from (SL)]

MAIN OPERATORS

- What if we had applied the rule (•) first, and then applied the rule ($\sim$)?

a)	'P' is a wff	[from (SL)]
b)	'Q' is a wff	[from (SL)]
c)	So, '$(P \bullet Q)$' is a wff	[from (a), (b), and (•)]

MAIN OPERATORS

- What if we had applied the rule (•) first, and then applied the rule ($\sim$)?

a)	'P' is a wff	[from (SL)]
b)	'Q' is a wff	[from (SL)]
c)	So, '$(P \bullet Q)$' is a wff	[from (a), (b), and (•)]
d)	So, '$\sim(P \bullet Q)$' is a wff	[from (c) and ($\sim$)]

MAIN OPERATORS

- What if we had applied the rule (•) first, and then applied the rule (∼)?

a)	'P' is a wff	[from (SL)]
b)	'Q' is a wff	[from (SL)]
c)	So, '$(P \bullet Q)$' is a wff	[from (a), (b), and (•)]
d)	So, '$\sim(P \bullet Q)$' is a wff	[from (c) and (∼)]

- '$\sim(P \bullet Q)$' is not the same as '$(\sim P \bullet Q)$'

SUBFORMULAE

- p is a *subformula* of q if and only if, in the course of building up q by applying the rules for wffs, p appears on a line before q.

SUBFORMULAE

- p is a *subformula* of q if and only if, in the course of building up q by applying the rules for wffs, p appears on a line before q.
  - '$\sim P$' is a subformula of '$\sim P \bullet Q$'

SUBFORMULAE

- p is a *subformula* of q if and only if, in the course of building up q by applying the rules for wffs, p appears on a line before q.
  - '$\sim P$' is a subformula of '$\sim P \bullet Q$'
  - '$\sim P$' is *not* a subformula of '$\sim (P \bullet Q)$'

SUBFORMULAE

- p is a *subformula* of q if and only if, in the course of building up q by applying the rules for wffs, p appears on a line before q.
  - '$\sim P$' is a subformula of '$\sim P \bullet Q$'
  - '$\sim P$' is *not* a subformula of '$\sim (P \bullet Q)$'
- A formula's *immediate subformulae* are those wffs whose lines were appealed to in the final step of building to formula up.

SUBFORMULAE

- p is a *subformula* of q if and only if, in the course of building up q by applying the rules for wffs, p appears on a line before q.
  - '$\sim P$' is a subformula of '$\sim P \bullet Q$'
  - '$\sim P$' is *not* a subformula of '$\sim (P \bullet Q)$'
- A formula's *immediate subformulae* are those wffs whose lines were appealed to in the final step of building to formula up.
  - the immediate subformulae of '$\sim P \bullet Q$' are '$\sim P$' and 'Q'

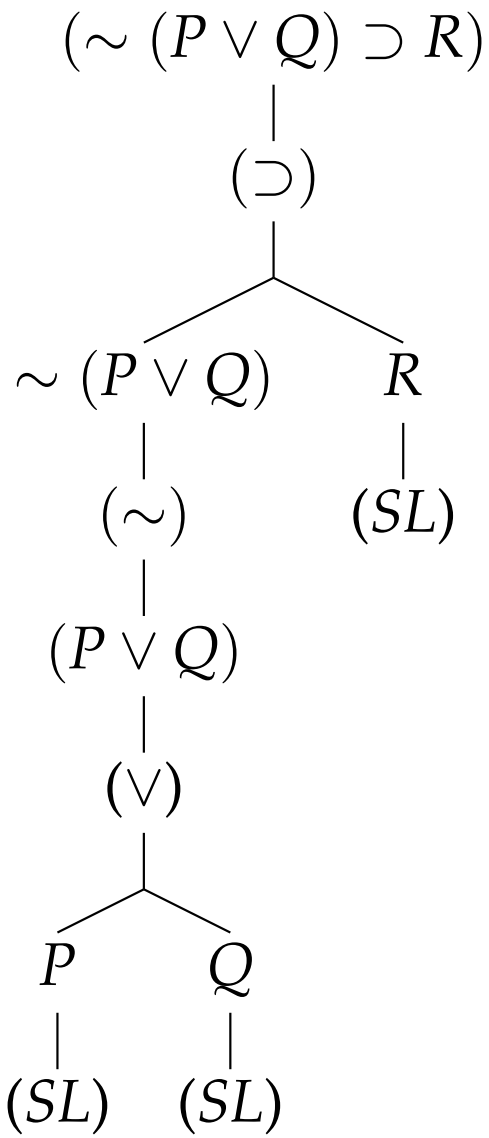
SUBFORMULAE

- p is a *subformula* of q if and only if, in the course of building up q by applying the rules for wffs, p appears on a line before q.
  - '$\sim P$' is a subformula of '$\sim P \bullet Q$'
  - '$\sim P$' is *not* a subformula of '$\sim (P \bullet Q)$'
- A formula's *immediate subformulae* are those wffs whose lines were appealed to in the final step of building to formula up.
  - the immediate subformulae of '$\sim P \bullet Q$' are '$\sim P$' and 'Q'
  - the immediate subformula of '$\sim (P \bullet Q)$' is '$(P \bullet Q)$'

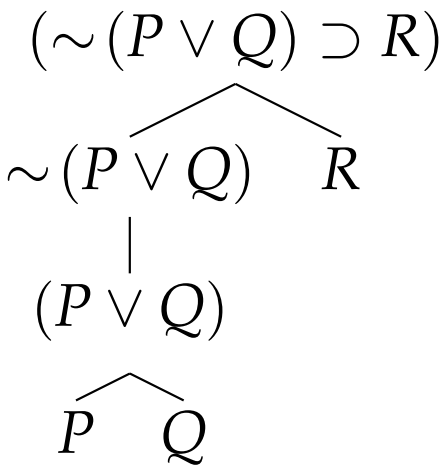
SUBFORMULAE

- p is a *subformula* of q if and only if, in the course of building up q by applying the rules for wffs, p appears on a line before q.
  - '$\sim P$' is a subformula of '$\sim P \bullet Q$'
  - '$\sim P$' is *not* a subformula of '$\sim (P \bullet Q)$'
- A formula's *immediate subformulae* are those wffs whose lines were appealed to in the final step of building to formula up.
  - the immediate subformulae of '$\sim P \bullet Q$' are '$\sim P$' and 'Q'
  - the immediate subformula of '$\sim (P \bullet Q)$' is '$(P \bullet Q)$'
- Another way of notating proofs that a formulae of *PL* is well-formed: *syntax trees*

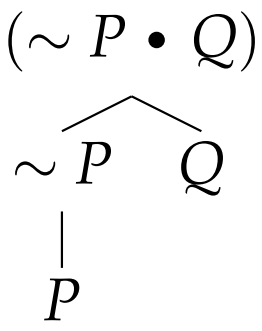
SYNTACTIC STRUCTURE

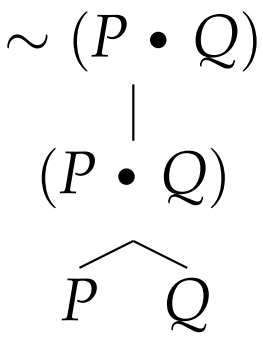


SYNTACTIC STRUCTURE



SYNTACTIC STRUCTURE





OUTLINE

MEANING

- Our guiding assumption: what it is to understand the meaning of an expression is just to understand the circumstances in which it is true

MEANING

- Our guiding assumption: what it is to understand the meaning of an expression is just to understand the circumstances in which it is true
- Parenthases just make syntactic structure explicit. They do not make any contribution to meaning beyond that.

MEANING

- Our guiding assumption: what it is to understand the meaning of an expression is just to understand the circumstances in which it is true
- Parenthases just make syntactic structure explicit. They do not make any contribution to meaning beyond that.
- So: we must say what the meanings of the statements letters are and what the meanings of the logical operators are

THE MEANING OF THE STATEMENT LETTERS

- Each statement letter represents a statement in English.

THE MEANING OF THE STATEMENT LETTERS

- Each statement letter represents a statement in English.
- The statement letter is true if and only if the statement in English is true.

THE MEANING OF '$\sim$'

- '$\sim$' is called the *tilde*.

THE MEANING OF '$\sim$'

- '$\sim$' is called the *tilde*.
- A wff whose main operator is the tilde is called a *negation*.

THE MEANING OF '$\sim$'

- '$\sim$' is called the *tilde*.
- A wff whose main operator is the tilde is called a *negation*.
- Its immediate subformula is called the *negand*.

THE MEANING OF '$\sim$'

- '$\sim$' is called the *tilde*.
- A wff whose main operator is the tilde is called a *negation*.
- Its immediate subformula is called the *negand*.

p	$\sim p$
T	F
F	T

THE MEANING OF '$\sim$'

- '$\sim$' is called the *tilde*.
- A wff whose main operator is the tilde is called a *negation*.
- Its immediate subformula is called the *negand*.

p	$\sim p$
T	F
F	T

- Note: 'p' is not a wff of *PL*

THE MEANING OF '$\sim$'

- '$\sim$' is called the *tilde*.
- A wff whose main operator is the tilde is called a *negation*.
- Its immediate subformula is called the *negand*.

p	$\sim p$
T	F
F	T

- Note: 'p' is not a wff of *PL*
  - we are using 'p' and 'q' as variables ranging over the wffs of *PL*

THE MEANING OF '•'

- '•' is known as the *dot*.

THE MEANING OF '•'

- '•' is known as the *dot*.
- A wff whose main operator is the dot is known as a *conjunction*.

THE MEANING OF '•'

- '•' is known as the *dot*.
- A wff whose main operator is the dot is known as a *conjunction*.
- Its immediate subformulae are called *conjuncts*.

THE MEANING OF '•'

- '•' is known as the *dot*.
- A wff whose main operator is the dot is known as a *conjunction*.
- Its immediate subformulae are called *conjuncts*.

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

THE MEANING OF '$\vee$'

- The operator '$\vee$' is known as the *wedge*.

THE MEANING OF '$\vee$'

- The operator '$\vee$' is known as the *wedge*.
- A wff whose main operator is the wedge is known as a *disjunction*.

THE MEANING OF '$\vee$'

- The operator '$\vee$' is known as the *wedge*.
- A wff whose main operator is the wedge is known as a *disjunction*.
- Its immediate subformulae are called *disjuncts*.

THE MEANING OF '$\vee$'

- The operator '$\vee$' is known as the *wedge*.
- A wff whose main operator is the wedge is known as a *disjunction*.
- Its immediate subformulae are called *disjuncts*.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

THE MEANING OF '⊃'

- The operator '⊃' is known as the *horseshoe*.

THE MEANING OF '⊃'

- The operator '⊃' is known as the *horseshoe*.
- A wff whose main operator is the horseshoe is known as a *material conditional*.

THE MEANING OF '⊃'

- The operator '⊃' is known as the *horseshoe*.
- A wff whose main operator is the horseshoe is known as a *material conditional*.
- The immediate subformulae which precedes the horseshoe is known as the *antecedent*.

THE MEANING OF '⊃'

- The operator '⊃' is known as the *horseshoe*.
- A wff whose main operator is the horseshoe is known as a *material conditional*.
- The immediate subformulae which precedes the horseshoe is known as the *antecedent*.
- The immediate subformulae which follows the horseshoe is known as the *consequent*.

THE MEANING OF '$\supset$'

- The operator '$\supset$' is known as the *horseshoe*.
- A wff whose main operator is the horseshoe is known as a *material conditional*.
- The immediate subformulae which precedes the horseshoe is known as the *antecedent*.
- The immediate subformulae which follows the horseshoe is known as the *consequent*.

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

THE MEANING OF '$\supset$'

- The operator '$\supset$' is known as the *horseshoe*.
- A wff whose main operator is the horseshoe is known as a *material conditional*.
- The immediate subformulae which precedes the horseshoe is known as the *antecedent*.
- The immediate subformulae which follows the horseshoe is known as the *consequent*.

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

- Note: this is the only binary operator which is not symmetric.

THE MEANING OF '$\supset$'

- The operator '$\supset$' is known as the *horseshoe*.
- A wff whose main operator is the horseshoe is known as a *material conditional*.
- The immediate subformulae which precedes the horseshoe is known as the *antecedent*.
- The immediate subformulae which follows the horseshoe is known as the *consequent*.

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

- Note: this is the only binary operator which is not symmetric.
  - '$p \supset q$' *does not* have the same meaning as '$q \supset p$'

THE MEANING OF '≡'

- The operator '≡' is known as the *triple bar*.

THE MEANING OF '≡'

- The operator '≡' is known as the *triple bar*.
- A wff whose main operator is the triple bar is known as a *material biconditional*.

THE MEANING OF '≡'

- The operator '≡' is known as the *triple bar*.
- A wff whose main operator is the triple bar is known as a *material biconditional*.
  - The immediate subformula which appears before the triple bar is known as the biconditional's *left hand side*

THE MEANING OF '≡'

- The operator '≡' is known as the *triple bar*.
- A wff whose main operator is the triple bar is known as a *material biconditional*.
  - The immediate subformula which appears before the triple bar is known as the biconditional's *left hand side*
  - The immediate subformula which appears after the triple bar is known as the biconditional's *right hand side*.

THE MEANING OF '≡'

- The operator '≡' is known as the *triple bar*.
- A wff whose main operator is the triple bar is known as a *material biconditional*.
  - The immediate subformula which appears before the triple bar is known as the biconditional's *left hand side*
  - The immediate subformula which appears after the triple bar is known as the biconditional's *right hand side*.

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

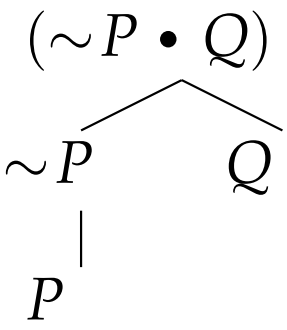
- Suppose 'P' is true and 'Q' is false

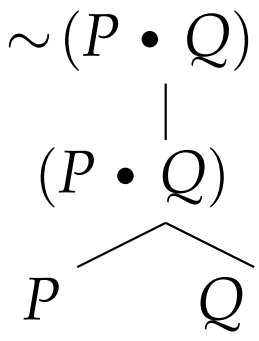
DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

- Suppose 'P' is true and 'Q' is false
- What are the truth-values of '$\sim P \bullet Q$' and '$\sim(P \bullet Q)$'?

DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

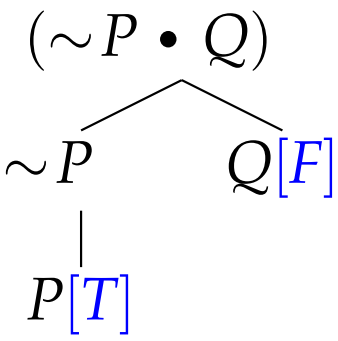
- Suppose 'P' is true and 'Q' is false
- What are the truth-values of '$\sim P \bullet Q$' and '$\sim(P \bullet Q)$'?

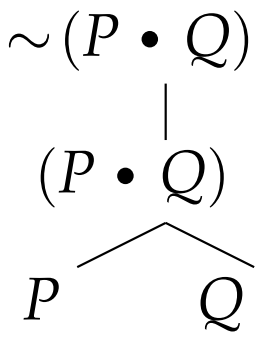




DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

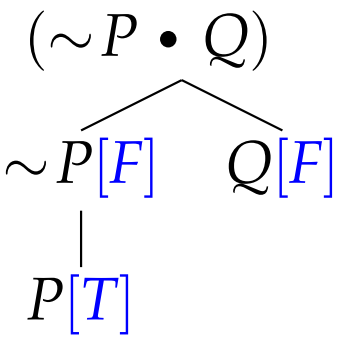
- Suppose 'P' is true and 'Q' is false
- What are the truth-values of '$\sim P \bullet Q$' and '$\sim(P \bullet Q)$'?

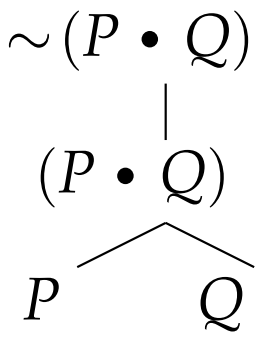




DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

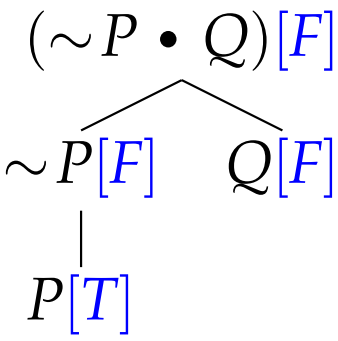
- Suppose 'P' is true and 'Q' is false
- What are the truth-values of '$\sim P \bullet Q$' and '$\sim(P \bullet Q)$'?

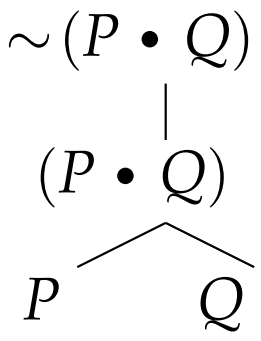




DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

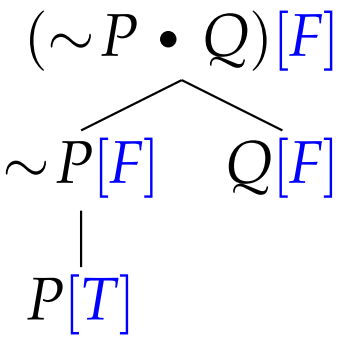
- Suppose 'P' is true and 'Q' is false
- What are the truth-values of '$\sim P \bullet Q$' and '$\sim(P \bullet Q)$'?

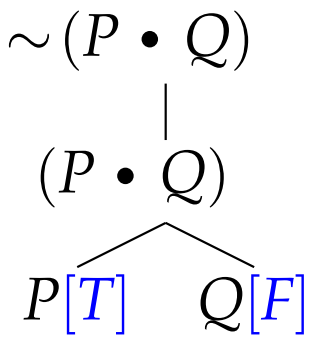




DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

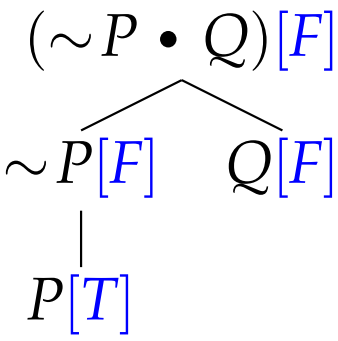
- Suppose 'P' is true and 'Q' is false
- What are the truth-values of '$\sim P \bullet Q$' and '$\sim(P \bullet Q)$'?

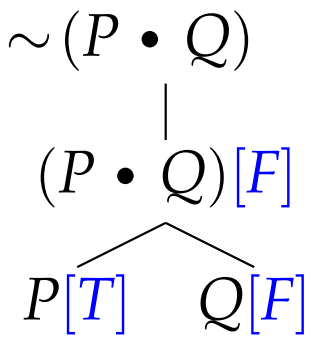




DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

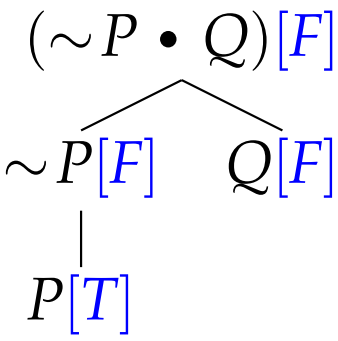
- Suppose 'P' is true and 'Q' is false
- What are the truth-values of '$\sim P \bullet Q$' and '$\sim(P \bullet Q)$'?

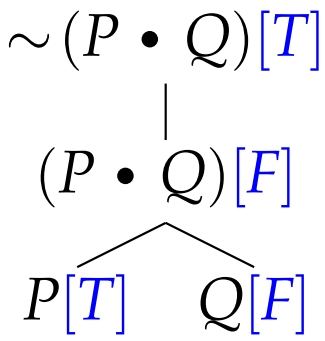




DETERMINING THE TRUTH-VALUE OF A WFF OF *PL*

- Suppose 'P' is true and 'Q' is false
- What are the truth-values of '$\sim P \bullet Q$' and '$\sim(P \bullet Q)$'?





TRUTH TABLES

- Truth-table for '$\sim P \bullet Q$':

TRUTH TABLES

- Truth-table for '$\sim P \bullet Q$':

P	Q	$\sim$	P	$\bullet$	Q

TRUTH TABLES

- Truth-table for '$\sim P \bullet Q$':

P	Q	$\sim$	P	$\bullet$	Q
T	T				
T	F				
F	T				
F	F				

TRUTH TABLES

- Truth-table for '$\sim P \bullet Q$':

P	Q	$\sim$	P	$\bullet$	Q
T	T		T		T
T	F		T		F
F	T		F		T
F	F		F		F

TRUTH TABLES

- Truth-table for '$\sim P \bullet Q$':

P	Q	$\sim$	P	$\bullet$	Q
T	T	F	T		T
T	F	F	T		F
F	T	T	F		T
F	F	T	F		F

TRUTH TABLES

- Truth-table for '$\sim P \bullet Q$':

P	Q	$\sim$	P	$\bullet$	Q
T	T	F	T	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	F	F	F

TRUTH TABLES

P	Q	$\sim P$	$\bullet$	Q
T	T	F	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	F	F

TRUTH TABLES

- Truth-table for '$\sim(P \bullet Q)$':

TRUTH TABLES

- Truth-table for '$\sim(P \bullet Q)$':

P	Q	$\sim$	$(P$	$\bullet$	$Q)$

TRUTH TABLES

- Truth-table for '$\sim(P \bullet Q)$':

P	Q	$\sim$	$(P$	$\bullet$	$Q)$
T	T				
T	F				
F	T				
F	F				

TRUTH TABLES

- Truth-table for '$\sim(P \bullet Q)$':

P	Q	$\sim$	$(P$	$\bullet$	$Q)$
T	T		T		T
T	F		T		F
F	T		F		T
F	F		F		F

TRUTH TABLES

- Truth-table for '$\sim(P \bullet Q)$':

P	Q	$\sim$	$(P$	$\bullet$	$Q)$
T	T		T	T	T
T	F		T	F	F
F	T		F	F	T
F	F		F	F	F

TRUTH TABLES

- Truth-table for '$\sim(P \bullet Q)$':

P	Q	$\sim$	$(P$	$\bullet$	$Q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	T	F	F	T
F	F	T	F	F	F

Outline

Outline

A TRANSLATION GUIDE FROM *PL* TO ENGLISH

- Submitted for your approval:

$\sim p$	$\longrightarrow$	It is not the case that p
$p \bullet q$	$\longrightarrow$	Both p and q
$p \vee q$	$\longrightarrow$	Either p or q
$p \supset q$	$\longrightarrow$	If p, then q
$p \equiv q$	$\longrightarrow$	p if and only if q

'$p \supset q$' AND 'IF p, THEN q'

- If 'p' is false, then '$p \supset q$' is automatically true, *no matter what statement q represents*

'$p \supset q$' AND 'IF p, THEN q'

- If 'p' is false, then '$p \supset q$' is automatically true, *no matter what statement q represents*
- But 'if John Adams was America's first president, then eating soap cures cancer' doesn't seem true.

'$p \supset q$' AND 'IF p, THEN q'

- If 'p' is false, then '$p \supset q$' is automatically true, *no matter what statement q represents*
- But 'if John Adams was America's first president, then eating soap cures cancer' doesn't seem true.
- I plead guilty

'$p \supset q$' AND 'IF p, THEN q'

- If 'p' is false, then '$p \supset q$' is automatically true, *no matter what statement q represents*
- But 'if John Adams was America's first president, then eating soap cures cancer' doesn't seem true.
- I plead guilty
  - '$p \supset q$' isn't a perfect translation of 'if p, then q'

'$p \supset q$' AND 'IF p, THEN q'

- If 'p' is false, then '$p \supset q$' is automatically true, *no matter what statement q represents*
- But 'if John Adams was America's first president, then eating soap cures cancer' doesn't seem true.
- I plead guilty
  - '$p \supset q$' isn't a perfect translation of 'if p, then q'
  - *But*

'$p \supset q$' AND 'IF p, THEN q'

- If 'p' is false, then '$p \supset q$' is automatically true, *no matter what statement q represents*
- But 'if John Adams was America's first president, then eating soap cures cancer' doesn't seem true.
- I plead guilty
  - '$p \supset q$' isn't a perfect translation of 'if p, then q'
  - *But* it's not as bad as you might think.

'$p \supset q$' AND 'IF p, THEN q'

- Dmitri: 'If it's a weekday, then I'm on campus.'

'$p \supset q$' AND 'IF p, THEN q'

- Dmitri: 'If it's a weekday, then I'm on campus.'
- Steve: 'If I'm on campus, then it's a weekday.'

'$p \supset q$' AND 'IF p, THEN q'

- Dmitri: 'If it's a weekday, then I'm on campus.'
- Steve: 'If I'm on campus, then it's a weekday.'
  - $D :=$ 'Dmitri is on campus'

'$p \supset q$' AND 'IF p, THEN q'

- Dmitri: 'If it's a weekday, then I'm on campus.'
- Steve: 'If I'm on campus, then it's a weekday.'
  - $D :=$ 'Dmitri is on campus'
  - $S :=$ 'Steve is on campus'

'$p \supset q$' AND 'IF p, THEN q'

- Dmitri: 'If it's a weekday, then I'm on campus.'
- Steve: 'If I'm on campus, then it's a weekday.'
  - $D :=$ 'Dmitri is on campus'
  - $S :=$ 'Steve is on campus'
  - $W :=$ 'It is a weekday'

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T		
T	F		
F	T		
F	F		

S	W	if S, then W	$S \supset W$
T	T		
T	F		
F	T		
F	F		

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	
T	F		
F	T		
F	F		

S	W	if S, then W	$S \supset W$
T	T		
T	F		
F	T		
F	F		

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	
T	F	didn't lie	
F	T		
F	F		

S	W	if S, then W	$S \supset W$
T	T		
T	F		
F	T		
F	F		

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	
T	F	didn't lie	
F	T	lied	
F	F		

S	W	if S, then W	$S \supset W$
T	T		
T	F		
F	T		
F	F		

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	
T	F	didn't lie	
F	T	lied	
F	F	didn't lie	

S	W	if S, then W	$S \supset W$
T	T		
T	F		
F	T		
F	F		

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	
T	F	didn't lie	
F	T	lied	
F	F	didn't lie	

S	W	if S, then W	$S \supset W$
T	T	didn't lie	
T	F		
F	T		
F	F		

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	
T	F	didn't lie	
F	T	lied	
F	F	didn't lie	

S	W	if S, then W	$S \supset W$
T	T	didn't lie	
T	F	lied	
F	T		
F	F		

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	
T	F	didn't lie	
F	T	lied	
F	F	didn't lie	

S	W	if S, then W	$S \supset W$
T	T	didn't lie	
T	F	lied	
F	T	didn't lie	
F	F		

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	
T	F	didn't lie	
F	T	lied	
F	F	didn't lie	

S	W	if S, then W	$S \supset W$
T	T	didn't lie	
T	F	lied	
F	T	didn't lie	
F	F	didn't lie	

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	T
T	F	didn't lie	
F	T	lied	
F	F	didn't lie	

S	W	if S, then W	$S \supset W$
T	T	didn't lie	
T	F	lied	
F	T	didn't lie	
F	F	didn't lie	

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	T
T	F	didn't lie	T
F	T	lied	
F	F	didn't lie	

S	W	if S, then W	$S \supset W$
T	T	didn't lie	
T	F	lied	
F	T	didn't lie	
F	F	didn't lie	

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	T
T	F	didn't lie	T
F	T	lied	F
F	F	didn't lie	

S	W	if S, then W	$S \supset W$
T	T	didn't lie	
T	F	lied	
F	T	didn't lie	
F	F	didn't lie	

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	T
T	F	didn't lie	T
F	T	lied	F
F	F	didn't lie	T

S	W	if S, then W	$S \supset W$
T	T	didn't lie	
T	F	lied	
F	T	didn't lie	
F	F	didn't lie	

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	T
T	F	didn't lie	T
F	T	lied	F
F	F	didn't lie	T

S	W	if S, then W	$S \supset W$
T	T	didn't lie	T
T	F	lied	
F	T	didn't lie	
F	F	didn't lie	

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	T
T	F	didn't lie	T
F	T	lied	F
F	F	didn't lie	T

S	W	if S, then W	$S \supset W$
T	T	didn't lie	T
T	F	lied	F
F	T	didn't lie	
F	F	didn't lie	

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	T
T	F	didn't lie	T
F	T	lied	F
F	F	didn't lie	T

S	W	if S, then W	$S \supset W$
T	T	didn't lie	T
T	F	lied	F
F	T	didn't lie	T
F	F	didn't lie	

'$p \supset q$' AND 'IF p, THEN q'

D	W	if W, then D	$W \supset D$
T	T	didn't lie	T
T	F	didn't lie	T
F	T	lied	F
F	F	didn't lie	T

S	W	if S, then W	$S \supset W$
T	T	didn't lie	T
T	F	lied	F
F	T	didn't lie	T
F	F	didn't lie	T

Inclusive and Exclusive 'or'

The Inclusive 'or'

In the inclusive sense, 'p or q' means 'Either p or q or both.'

INCLUSIVE AND EXCLUSIVE 'OR'

The Inclusive 'or'

In the inclusive sense, 'p or q' means 'Either p or q or both.'

- 'Either the elevator or the escalator is working'

INCLUSIVE AND EXCLUSIVE 'OR'

The Inclusive 'or'

In the inclusive sense, 'p or q' means 'Either p or q or both.'

- 'Either the elevator or the escalator is working'

The Exclusive 'or'

In the exclusive sense, 'p or q' means 'Either p or q, but not both.'

INCLUSIVE AND EXCLUSIVE 'OR'

The Inclusive 'or'

In the inclusive sense, 'p or q' means 'Either p or q or both.'

- 'Either the elevator or the escalator is working'

The Exclusive 'or'

In the exclusive sense, 'p or q' means 'Either p or q, but not both.'

- 'Either you clean your room, or you're grounded'

INCLUSIVE AND EXCLUSIVE 'OR'

The Inclusive 'or'

In the inclusive sense, 'p or q' means 'Either p or q or both.'

- 'Either the elevator or the escalator is working'

The Exclusive 'or'

In the exclusive sense, 'p or q' means 'Either p or q, but not both.'

- 'Either you clean your room, or you're grounded'

'$\vee$' translates to the *inclusive* 'or'

Outline

CANONICAL LOGICAL FORM

- Call the phrases on the right-hand-side of the translation guide the *canonical logical expressions* of English.

Canonical Logical Form

- Call the phrases on the right-hand-side of the translation guide the *canonical logical expressions* of English.
- If the logical structure of an English statement is written in this form, then that statement is in *canonical logical form*.

Canonical Logical Form

- Call the phrases on the right-hand-side of the translation guide the *canonical logical expressions* of English.
- If the logical structure of an English statement is written in this form, then that statement is in *canonical logical form*.
- E.g.,

CANONICAL LOGICAL FORM

- Call the phrases on the right-hand-side of the translation guide the *canonical logical expressions* of English.
- If the logical structure of an English statement is written in this form, then that statement is in *canonical logical form*.
- E.g.,

  If both John loves Andrew and it is not the case that Andrew loves John, then it is not the case that John and Andrew will be friends.

CANONICAL LOGICAL FORM

- Call the phrases on the right-hand-side of the translation guide the *canonical logical expressions* of English.
- If the logical structure of an English statement is written in this form, then that statement is in *canonical logical form*.
- E.g.,

  If both John loves Andrew and it is not the case that Andrew loves John, then it is not the case that John and Andrew will be friends.

- Expressions in this form are easily translated into *PL*. Let $J =$ 'John loves Andrew', $A =$ 'Andrew loves John', and $F =$ 'John and Andrew will be friends'. Then,

$$(J \bullet \sim A) \supset \sim F$$

English Sentences Not in Canonical Logical Form

ENGLISH SENTENCES NOT IN CANONICAL LOGICAL FORM

- *John and Andrew won't be friends if John loves Andrew but Andrew doesn't love him back.*

ENGLISH SENTENCES NOT IN CANONICAL LOGICAL FORM

- *John and Andrew won't be friends if John loves Andrew but Andrew doesn't love him back.*
- We'll have to say a bit more about how to translate sentences like this into *PL*.

Outline

Outline

Translating into Canonical Logical Form

- Consider:

Translating into Canonical Logical Form

- Consider:

  Harry doesn't like chestnuts

Translating into Canonical Logical Form

- Consider:

  Harry doesn't like chestnuts

- This has the same meaning as

Translating into Canonical Logical Form

- Consider:

  Harry doesn't like chestnuts

- This has the same meaning as

  It is not the case that Harry likes chestnuts.

TRANSLATING INTO CANONICAL LOGICAL FORM

- Consider:

  Harry doesn't like chestnuts

- This has the same meaning as

  It is not the case that Harry likes chestnuts.

- So, if we let $H =$ 'Harry likes chestnuts', then we can translate it into *PL* as follows:

$$\sim H$$

Translating into Canonical Logical Form

- A general strategy:

Translating into Canonical Logical Form

- A general strategy:
  - for any sentence of English, find another sentence of English which has the same meaning as the first, and which is in canonical logical form.

Translating into Canonical Logical Form

- A general strategy:
  - for any sentence of English, find another sentence of English which has the same meaning as the first, and which is in canonical logical form.

TRANSLATING INTO CANONICAL LOGICAL FORM

- A general strategy:
  - for any sentence of English, find another sentence of English which has the same meaning as the first, and which is in canonical logical form.
  - Then, translate the sentence in canonical logical form into *PL* following the translation guide:

It is not the case that p	$\longrightarrow$	$\sim p$
Both p and q	$\longrightarrow$	$p \bullet q$
Either p or q	$\longrightarrow$	$p \vee q$
If p, then q	$\longrightarrow$	$p \supset q$
p if and only if q	$\longrightarrow$	$p \equiv q$

Some Issues: Negation

SOME ISSUES: NEGATION

- *I hate not getting what I want and I hate getting what I want.*

SOME ISSUES: NEGATION

- *I hate not getting what I want and I hate getting what I want.*

Some Issues: Negation

- *I hate not getting what I want and I hate getting what I want.*
- *It is not the case that I hate getting what I want, and I hate getting what I want.*

Some Issues: Negation

- *I hate not getting what I want and I hate getting what I want.*
- *It is not the case that I hate getting what I want, and I hate getting what I want.*

SOME ISSUES: NEGATION

- *I hate not getting what I want and I hate getting what I want.*
- *It is not the case that I hate getting what I want, and I hate getting what I want.*
- $\sim$ *I hate getting what I want* $\bullet$ *I hate getting what I want.*

SOME ISSUES: NEGATION

- *I hate not getting what I want and I hate getting what I want.*
- *It is not the case that I hate getting what I want, and I hate getting what I want.*
- $\sim$ *I hate getting what I want* $\bullet$ *I hate getting what I want.*
- $$\sim H \bullet H$$

Some Issues: Negation

H	$\sim$	H	$\boxed{\bullet}$	H
T	F	T	F	T
F	T	F	F	F

SOME ISSUES: CONJUNCTION

SOME ISSUES: CONJUNCTION

- *Hannes loves peaches and he loves apples.*
  Hannes loves peaches but he loves apples.

SOME ISSUES: CONJUNCTION

- *Hannes loves peaches and he loves apples.*
  Hannes loves peaches but he loves apples.
- $P =$ 'Hannes loves peaches' and $A =$ 'Hannes loves apples'

SOME ISSUES: CONJUNCTION

- *Hannes loves peaches and he loves apples.*
  Hannes loves peaches but he loves apples.
- $P =$ 'Hannes loves peaches' and $A =$ 'Hannes loves apples'
- $$P \bullet A$$

SOME ISSUES: CONJUNCTION

$$\left.\begin{array}{c} p \text{ and } q \\ p\text{, but } q \\ p\text{; however, } q \\ p\text{, though } q \\ p \text{ as well as } q \end{array}\right\} \longrightarrow p \bullet q$$

Some Issues: Disjunction

$$\left.\begin{array}{c} p \text{ or } q \\ p \text{ unless } q \end{array}\right\} \longrightarrow p \vee q$$

SOME ISSUES: THE MATERIAL CONDITIONAL

$$\left.\begin{array}{c}\text{If } p\text{, then } q\\ p \text{ only if } q\\ q \text{ if } p\\ p \text{ is sufficient for } q\\ q \text{ is necessary for } p\end{array}\right\} \longrightarrow p \supset q$$

SOME ISSUES: THE MATERIAL BICONDITIONAL

$$\left.\begin{array}{c} p \text{ if and only if } q \\ p \text{ is necessary and sufficient for } q \end{array}\right\} \longrightarrow p \equiv q$$