Chapter Six

Tidying Up

If you have come this far you now know all the basic properties of Special Relativity: time dilation, length contraction, change in simultaneity, reversal of time-ordering, preservation of causality and a new velocity-addition formula.

Armed with this information you can explain to your friends such paradoxes as, for example, the famous pole-vaulter in-the-barn paradox: An athlete, running with a 15 ft long pole, gets momentarily captured in a barn that is only 9 ft long. How is that possible? It's easy. The athlete simply runs at 80% of the speed of light so that his Lorentz contraction factor is 0.6. The door-keeper at the back of the barn agrees to close the door at the instant the front of the pole hits the front of the barn. Since the door-keeper has, effectively, marked the front and back of the pole simultaneously in the barn system the pole will be only 15ft X 0.6 = 9 ft long. It will just exactly fit in the barn at that instant. This problem is entirely equivalent to Flo and Joe simultaneously painting spots on the passing rocket ship. The spots end up 50 ft apart on the ship even thought Flo and Joe were only 30 ft apart. (I like the painting spots version better than the pole-vaulter problem because it is less destructive. Think of what happens to the poor pole-vaulter in the next instant when he slams into the front of the barn. Ugh.)

Having read about Flo and Joe in Chapter Three you also know the answer to the second part of the pole-vaulter problem. The pole-vaulter is allowed to consider himself at rest. When he does so he sees a barn coming at him moving at 80% of the speed of light. The barn door is open but the barn, as measured in his system, is only 9 ft X 0.6 = 5.4 ft long. How can those crazy barn people expect to capture his 15 ft pole in a 5.4 ft long barn? The answer is that they appear to cheat. They don't close the back door at the <u>same time</u> that the pole hits the front of the barn. They wait until the back of the pole is inside the barn before they close the door. (You're not supposed to think about the pole-vaulter crashing into the front of the barn during this interval. He gets all banged up in everybody's viewpoint.)

But, since we are privileged to know about Special Relativity, we realize that the barn people didn't in fact cheat. They did close the back door at the same instant the front of the pole hit the barn. In their system the 15 ft pole was indeed captured in the 9 ft barn. But these two simultaneous events for them were not simultaneous in the pole-vaulter's system. For him, when the front of the pole hit the barn, the back door, still open, was only 5.4 ft from the front of his 15 ft pole. The back door went another 15.0 - 5.4 = 9.6 ft before the door-keeper closed it. Now everybody is happy because they, understanding Einstein, understand exactly what happened.

Our purpose in this book has been to describe the Einsteinian view of what physicists call "kinematics": the description of motion. We have not touched on "dynamics": the description of forces, masses and acceleration This is a somewhat more difficult area which Einstein treated mainly in his second 1905 paper on Relativity. You can imagine that the drastic changes that he had made in the kinematic definitions of time, length, and velocity necessitated fundamental changes in the definitions of dynamical quantities as well. In particular, he needed to introduce the concept of the total relativistic energy of a particle:

$$E_{total} = \gamma Mc^2$$

where M is the particle's mass and $\gamma = 1/\sqrt{(1-v^2/c^2)}$ as we defined earlier.

We see that even when a particle is at rest, ($v=0$ and $\gamma = 1$), it has an energy given by:

$$E_{rest} = Mc^2.$$

This is, no doubt, the most famous equation in the world. It became so some thirty years after Einstein introduced it, marking the beginning of the age of atomic/nuclear energy.

There is one more loose end that we need to tie up before we complete our tour of Special Relativity. It's the so-called Twin Paradox. Einstein was well aware of the ambiguity that arises when he said that a moving clock runs slowly compared to a stationary clock. If two clocks are both moving with constant velocity then they can only be compared at the single instant that they pass each other. In order to compare them at a later time one of them has to turn around and come back. That would violate the rule of constant velocity so the observers in that clock's system could no longer use Special Relativity to calculate time flow. However, the observers with the clock that didn't turn around would be able to calculate that the turn-around clock would read less elapsed time when it returned. Using the same reasoning we can imagine two identical twins, born and raised on Earth, one of whom becomes an astronaut. At the age of 20 he goes off on a long rocket journey in which his speed averages 99.5 % of the speed of light. (This would give him a γ factor of 10.) Fifty years later he comes back to find his twin brother is 70 years old, but he has only aged 1/10 as much. He's only 25!

The possible paradox in this tale lies in deciding which twin will be younger. Can't the astronaut consider himself at rest and say that it was the Earth twin that went away and came back? Einstein's answer was that there is a physical difference between the two twins. The Earth twin was moving only at a slow, nearly constant, velocity during those 50 years whereas the astronaut had to undergo a large deceleration and re-acceleration in order to come back to Earth. It's that turning around that makes all the difference. Since the astronaut was not moving at constant velocity he cannot use the equations of Special Relativity to calculate how his clock was running.

Of course this particular experiment has never been done with astronauts (or eggs) but it has been done with elementary particles moving around in a circular accelerator. They are found to live longer by the appropriate amount of time compared to their "twin" identical particles that remain at rest in the laboratory.

Not being able to calculate how clocks run in an accelerated reference system left Einstein's Special Relativity in an unsatisfactory position. What if an astronaut was moving around in an orbit for an extended period of time. What would the laws of physics look like in such a system? What sort of equations could the astronaut use to calculate how clocks would run in the space ship? It was clear to Einstein and others that Special Relativity was an incomplete theory. It needed to be extended to the world of accelerated motion.

After 1905 Einstein set out to do just that. Along the way, in 1907, he had a sudden realization that there might be an intimate relationship between acceleration and the force of gravity. When you stand on the Earth the ground pushes upward on your feet with a force that equals the downward pull of the Earth's gravity on you. The force that you feel, said Einstein, is indistinguishable from the force you would feel if you were out in free space in a rocket ship that was accelerating. Likewise if you jump off the roof you are accelerate downward by the force of gravity, but you feel weightless while you are falling. This weightlessness is indistinguishable from what you would feel in a spaceship moving with constant velocity in free space or in a coasting orbit around Earth.

The indistinguishability of the effects of gravity from the effects of acceleration is known as the Principle of Equivalence. When Einstein first realized it in 1907 he referred to it as:

"the happiest thought of my life. I was sitting in a chair when all of a sudden a thought occurred to me: if a person falls freely he or she will not feel their own weight. I was startled. This simple thought made a deep impression on me."

This happy thought of Einstein was the cornerstone on which he built his new theory of gravity and acceleration: what was later called the General Theory of Relativity. It took him another eight years after 1907 to get it into the final form we still use today to understand the properties of everything from exploding supernovae to ravaging black holes.

It is interesting that the disarmingly simple Principle of Equivalence allows one to calculate how accelerated clocks, and clocks feeling the force of gravity, will run slowly compared to clocks at rest. It clears up the last shred of doubt we may have had about the twin paradox. The astronaut twin, when he turns around, will feel the force of his acceleration. He can now use General Relativity to calculate how much slower his clock is running than the clocks back home. When he returns, at the age of 25, he will not be at all surprised to see his 70-year old brother.