Chapter Five

Spooky Action at a Distance

In the last chapter we saw that observers in two different systems (Flo-Joe and the rocket ship) both measure the same speed for a beam of light. We also saw that no signal could go faster than light, as this would lead to a violation of causality: an effect could be seen as preceding its cause. But how about things that move slower than light? What do different observers measure for them? Einstein has another surprise for us here.

Suppose Flo, instead of sending a light pulse to Joe, sends a slower signal, causing Joe to paint his spot later than he did before. This means that the two painting-of-spot events will be inside the light cone and no observer could see Joe paint before Flo.

Let's say that Flo's signal had some speed (u) as measured by her and Joe. If we assume Flo paints at time zero on her clock then Joe will paint later by the amount of time it takes for the signal to reach him. Using time = distance/speed we get: Joe's time = 30 ft/u. To transform this into the rocket system we have to multiply it by $\gamma = 5/3$, which gives us:

$T_{Joe} = 50/u$ (measured in rocket system)

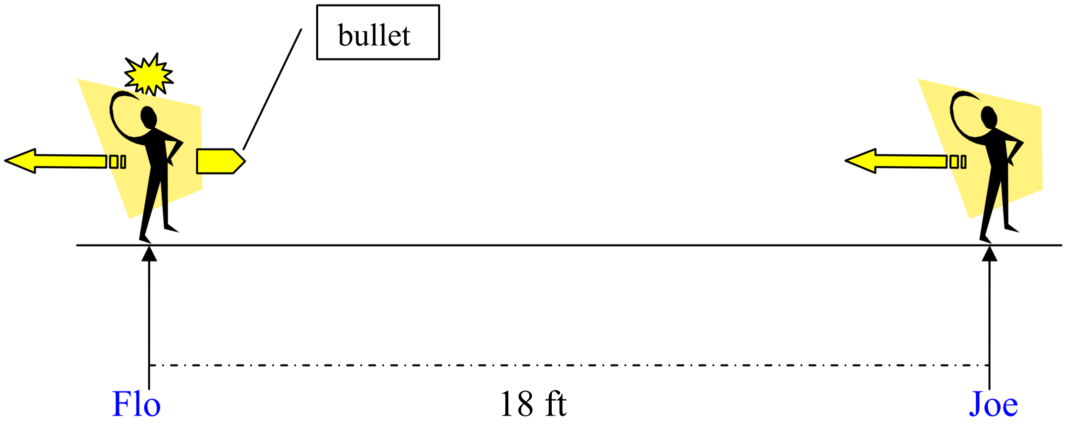
(You may recall that in Chapter Four we were using $T_{Joe} = 50/c$, but that was when the signal speed was c instead of u.)

Now, Flo's time is the same as before:

$T_{Flo} = 40/c$ (measured in rocket)

Now let's go to the rocket ship and imagine ourselves at rest. We see Flo and Joe coming towards us, 18 ft apart, traveling at $v=0.8c$. Flo paints her spot and send a signal towards Joe. Let's say the signal is a bullet.

At that moment it looks like this. (Fig. 7)



How far does Joe travel before he paints his spot? It's just his speed (v) times his time difference from Flo:

Joe's distance = $v(50/u - 40/c) = (50v/u - 40v/c) = 50v/u - 32$
(where we substituted $v/c=0.8$)

The distance the bullet travels is 18 ft – Joe's distance:

Bullet distance = $18 - (50v/u - 32) = 50 - 50v/u = 50(1 - v/u)$

To get the speed of the bullet measured by the rocket observers (let's call it u_r) we need to divide its distance by its flight time ($T_{Joe} - T_{Flo}$):

$$u_r = 50(1 - v/u)/(50/u-40/c) = (1 - v/u)/(1/u - .8/c)$$

We won't bother with the rest of the algebra, but if we substitute v/c for .8 we get, finally,

$$\boxed{u_r = (u - v)/(1-uv/c^2) \text{ = speed of bullet seen by rocket}}$$

This is Einstein's famous velocity-addition formula. It shows us how to transform the velocity (u) of something measured in one system (Flo-Joe) into the velocity (u_r) of that same thing as measured in another system (the rocket).

The formula has some interesting properties. First, we note that if both u and v are much smaller than c, the denominator ($1 - uv/c^2$) is essentially equal to 1.0. In that case we get:

$$u_r \approx u - v \quad \text{(velocity-addition for low velocities)}$$

This is just we expect from our everyday experience. Think back about the car going u=100 mph past the v=80 mph convoy of trucks. Using the above formula we would compute:

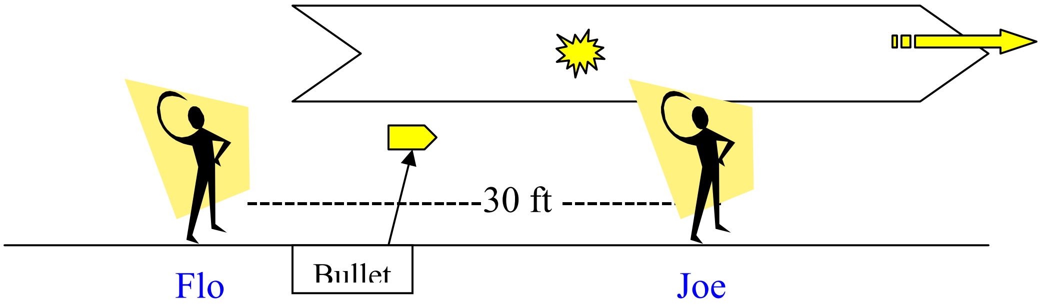
Speed of car seen by trucks = 100 mph – 80 mph = 20 mph.

Einstein's correction, the denominator of his velocity-addition formula, is only significant at very high speeds; that's why we never notice it when we're driving around.

We also see that, if u is less than v, u_r comes out negative. This is what would happen if the truck convoy were going 100 mph past a car going only 80 mph. The trucks would see the car as going backwards at –20 mph relative to them. After it passed the first truck it would continue to back up until it passed the last truck.

Similarly, what would happen if Flo sent the bullet (u) at less than the speed of the rocket (v)? Can a bullet go backwards? The answer is yes. From the Flo-Joe viewpoint, the rocket is going to the right at speed v, and the bullet is going to the right at speed u. But the rocket is going faster than the bullet. Hence the spot that Flo painted on the rocket will get to Joe before the bullet does. Joe will paint his spot behind Flo's spot! If the rocket isn't long enough Joe won't have anything to paint on.

It looks like this: (Fig. 8)



Now take the rocket observers point of view. They consider themselves at rest. Their picture looks as follows.

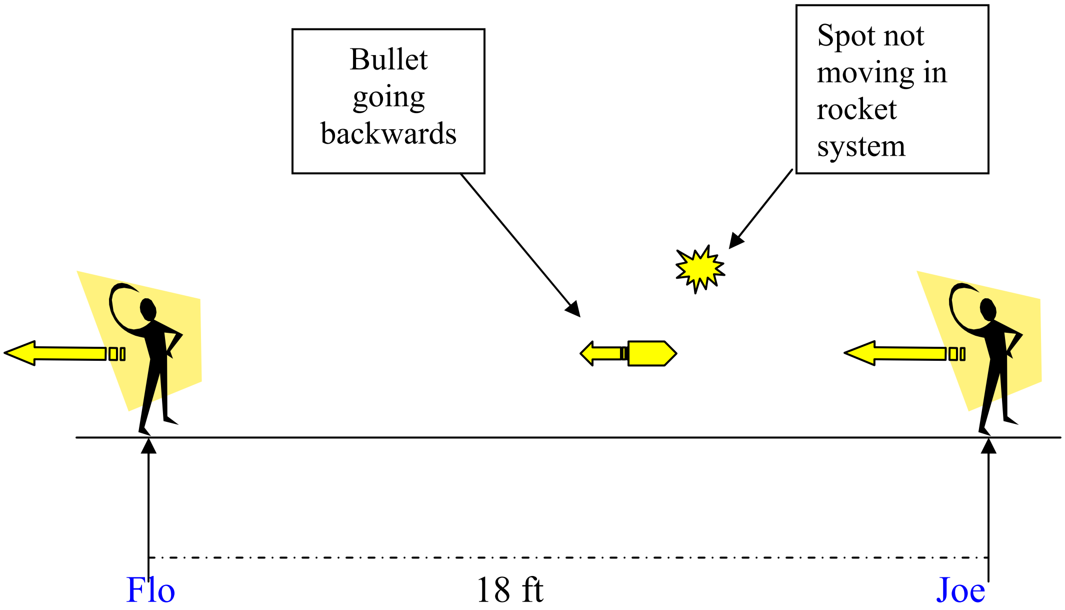


Fig. 9: Both Joe and the bullet are moving to the left, but Joe is going faster. He will catch up to the bullet and then paint his spot, behind Flo's spot.

Once Einstein had derived his velocity-addition formula he knew he had a self-consistent theory. It had several "peculiar consequences" but none of them were at odds with any experimental evidence. Moreover, these consequences would only be noticed for things moving near the speed of light.[•] At this point he must have felt that he had all the necessary ducks in line to publish his mind-boggling 1905 paper. The paper has stood without serious challenge until this day.

Actually, not many details are known about Einstein's thoughts during the years prior to 1905 when he published his famous papers on Relativity. There is evidence, from his letters, that he was thinking seriously about the subject as early as 1899. He was also aware of various experiments that were trying to measure the speed of the Earth through the so-called ether. We can imagine him struggling with the type of gedanken experiments that we've been describing: bullets going backwards, stars blowing up in reverse order, etc. Of course we have the advantage of 100 years of contemplation and verification of these ideas. He was going cold turkey. On the other hand, he had the advantage of being a lot smarter than we are.

As we were saying, it took him several years prior to 1905 to get all his ducks in line. When he did, he published. There was a duck born later, however, which caused a little trouble. In fact, it still is an ugly duckling for a lot of people. We need to say something about it.

The duck of which I speak first reared its head in the late 1920's when the implications of the new theory of Quantum Mechanics were being mulled about. The predictions of Quantum Mechanics are strange --even stranger than those of Special Relativity-- but they are also very successful in describing how Nature behaves. You are asked to believe that waves of light sometimes behave like single particles and that single particles, like electrons, can behave like waves. The sphere of influence of a particle can be spread out fuzzily over a huge volume --- possibly miles in diameter. You have no idea where it is until it interacts with another particle or wave.

Einstein never liked the fuzzy nature of Quantum Mechanics. He felt that particles must have some underlying hidden properties, that were yet to be discovered, which would make them behave more like the tiny little localized bits of matter that we normally think of. Here is one of his remarks about the subject:

[•] Note that if we choose the bullet to be a pulse of light we would put $u = c$ into the velocity-addition formula. If you do this you can show, with a little algebra, that u_r in the rocket system also equals c. This again agrees with the fact that all observers measure the same speed for light, but not for material objects like bullets.

> *“Quantum Mechanics is very impressive. But an inner voice tells me that it is not yet the real thing. The theory yields a lot, but it hardly brings us any closer to the secret or the Old One.*[•]*”*

Einstein had several heated exchanges with Niels Bohr on the subject, trying to trap him into some logical corner because of Bohr’s steadfast belief in the validity of Quantum Mechanics. It’s a long and interesting story, but suffice it to say that Bohr always came up with a suitable answer, sending Einstein back to the drawing board.

For our purposes we want to focus on one of Quantum Mechanic’s weirdest predictions… one that Einstein referred to as “spooky action at a distance”. It involves the behavior of two particles, or atoms, that are put into an “entangled” Quantum Mechanical state next to each other. They are then transported far away from each other without destroying their entanglement. (Don’t try this at home.) The rules of Quantum Mechanics predict that such atoms can influence each other in strange ways. If one of them is measured to be spinning clockwise, for example, the other one will always be found spinning counter-clockwise. The “spooky” part is that the information that the first atom has been measured seems to be conveyed to the second atom instantly, no matter how far away it is. It’s like having two people at opposite ends of the Earth flipping coins. When one of them gets “heads” the other one is pre-determined to get “tails” on his next flip, no matter how soon he does it, and despite the fact that there was no apparent means of communication between them. This is definitely weird. Don’t let anybody tell you it’s not.

But the thing is, Quantum Mechanics seems to describe perfectly what goes on in the real world. It’s how Nature behaves, so until something better comes along we’re stuck with it.

If you haven’t already guessed, the reason I bring all this up is that the correlated behavior of two entangled atoms would seem, on the face of it, to violate one of the basic tenets of Special Relativity: no signal (or information) can travel faster than light. Indeed, there have been several recent experiments with entangled pairs of atoms in which the times between the correlated measurements have been outside the light-cone. That is, the difference in time between the measurements was less than the amount of

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[•] Einstein’s reference to his belief in an intelligent creator of the universe.

time it would have taken for a light pulse to go from one atom to the other.[•] Does this mean that the weird behavior of Quantum Mechanics is at odds with Special Relativity?

Let’s think back to our two people flipping coins at opposite ends of the Earth. At the instant person A sees “tails” he knows that person B will get “heads”. Doesn’t this amount to sending a signal faster than light? The answer is no; the reason being that person A cannot choose whether he will get heads or tails. Quantum Mechanics is completely random in that respect. If person A could determine heads or tails before he flipped then he could also pre-determine how B’s flip would come out. That would mean he could send information, or a signal, to B faster than light, which would violate special Relativity. But the coin flippers have to obey the rules of Quantum Mechanics which state that the first flip must be completely random. Thus B’s results, although completely correlated to A’s, will also be completely random and contain no information.[••]

The message you should take away from this long-winded discussion is that Special Relativity and Quantum Mechanics are not in conflict with one another. Both are needed to describe the world in which we live.

[•] The current world record for the distance between two entangled quantum states is something like 100 Km.

[••] There is much talk nowadays about using entangled quantum states to create identical random bit-strings at widely separated locations and using them as a means of decoding encrypted messages. Person A would keep track of his coin flip results to create a key with which to code his message. The key would be securely, and instantly, transmitted to person B using entangled quantum states. Then A would send his coded message to B, who, in turn, would decode it with the key.