Chapter Three

"What I tell you three times is true"

Ok here's the plan: The two painters (let's call them Flo and Joe) agree to do what they did before. They will use their synchronized clocks to paint spots on the rocket ship simultaneously at some pre-arranged time. They are 30 ft apart and so the spots on the ship will end up 50 ft apart as before. The two photographers, Sue and Tom, are 18 ft apart in the rocket ship. They also have clocks synchronized in their system. They consider themselves at rest and agree to take pictures of Flo's and Joe's clocks as the painters go past them.

This is what happens: Joe paints his spot at the instant he passes Tom. Tom notes the time on his own clock and also takes a picture of Joe's clock and records that.

Now remember our rule about the Lorentz Contraction. Since Sue and Tom are taking pictures of Flo and Joe at the <u>same time</u> in the rocket system they will find the rest length between Flo and Joe (30 ft) will be contracted to 18 ft. That is why the photographers positioned themselves 18 ft apart in the first place. Now precisely when Joe took Tom's picture Sue took Flo's picture. Figure 4 shows the situation at that instant as seen by Sue and Tom.

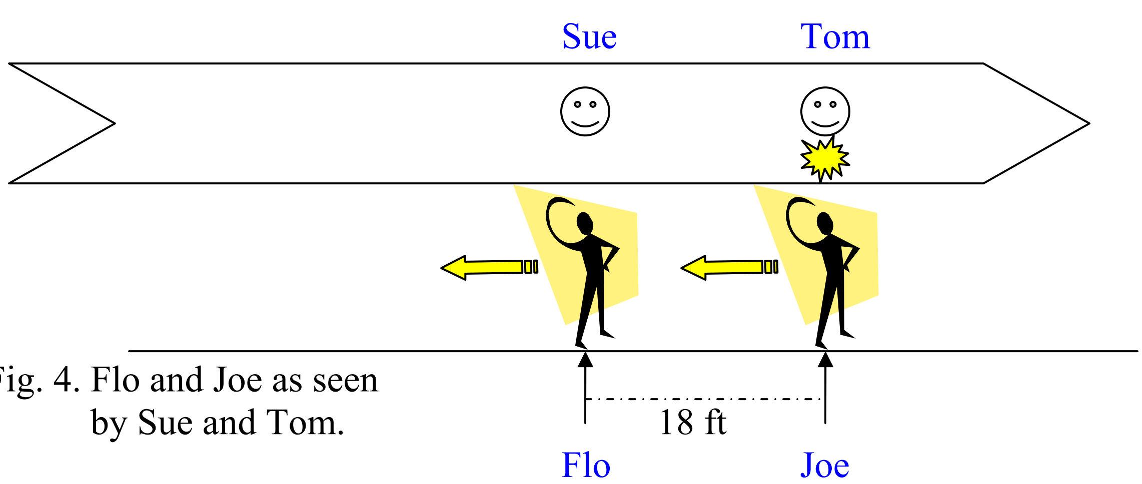

Fig. 4. Flo and Joe as seen by Sue and Tom.

What the rocket people see is that Joe has painted his spot but Flo has not! What happened? Did Flo forget her instructions?

Actually, a short time later, Flo does paint her spot. It ends up just where it did in the first experiment, 50 ft away from Joe's spot on the rocket.

All of this gets sorted out in a debriefing session after the rocket comes back to the barn. Flo and Joe, as measured by their clocks, did indeed paint their spots at the same time. But, as measured on Sue's and Tom's clocks, first Joe painted and then Flo waited until she got 50 ft away from Joe's spot before she painted. The amount of time she waited was just the distance she had to go (50 ft – 18 ft = 32 ft) divided by the relative speed (v) between the rocket and the Earth.*

So we have:

Flo's delay (clocked by Sue and Tom) = 32 ft/v

Flo's delay (clocked by Flo and Joe) = zero

This is all confirmed by looking at the picture that Sue took of Flo's clock when it was right across from her. It showed that Flo needed to wait if she wanted to be coincident with Joe's clock.

The upshot of this analysis is yet another conclusion.

Conclusion (3):
Two events that are simultaneous in one system (the painting of spots by Flo and Joe) will not be simultaneous in another system that is moving relative to the first (Tom and Sue see Flo's spot painted after Joe's).

When Einstein arrived at this conclusion his 1905 paper he stated it as follows (in German of course):

> *"So we see that we cannot attach an absolute significance to the concept of simultaneity, but that two events which, when viewed from a system of coordinates are simultaneous, can no longer be looked on as simultaneous events when envisioned from a system which is in motion relative to that system."*

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* We are again using the familiar formula: time = distance/velocity.

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So far, starting with Einstein's two seemingly innocent propositions, we have deduced three surprising conclusions for observers moving with respect to each other.

- There is no common time
- There is no common length
- There is no common simultaneity

If we are quavering with a modicum of disbelief at this point we must pay heed to the words of the Bellman in Lewis Carroll's "Hunting of the Snark", "*What I tell you three times is true*".

Are there more surprises in store for us? Well, yes, several. In our last example we studied two events that were simultaneous according to Flo and Joe (their painting spots). The same two events, when timed on the rocket ship, were not simultaneous: Joe painted before Flo. It may seem obvious, and in fact it's true, that if the rocket ship turned around and went by in the other direction they would see <u>Flo paint before Joe</u>!

I'm sure this discussion would have struck Isaac Newton as completely crazy if he happened upon it. Newton, before Einstein came along, was the biggest intellectual giant of physics. He opined that time "*flows equably without relation to anything external"*. For 200 years, until Einstein's 1905 paper, everyone believed Newton's concept of equable time flowing was true. There was no reason not to; it seemed in accord with everyday experience, and indeed it was. Einstein's corrections of Newtonian physics are so small that it was many years before they could be confirmed by experiment. *

But back to Flo and Joe, maybe we can believe that simultaneity is not absolute, but isn't the time-ordering of events? If a professor sees student A raise his hand before student B, is it possible for someone else to decide that

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* We can use the results of the last example to derive a simple formula for Flo's time delay measured in the rocket ship. If D is the distance between two simultaneous events in one system, then in the other system the time between them will be equal to $\gamma Dv/c^2$. Putting some realistic numbers in here we find that if Flo and Joe painted their spots on one of our fastest rocket ships their time difference on the ship's clocks would be only around a trillionth of a second. The trouble is that "c" is a very big number compared to the normal values of "D" and "v" that we have available to us. Actually we do have clocks that are accurate to a trillionth of a second but we all know how difficult it is to get painters to paint on time.

B raised her hand before A? As we shall see, sometimes it is and sometimes it isn't. It depends on the circumstances.

Recall that Flo painted her spot later than Joe by 32/v seconds, so:

$T_{Flo} = 32/v$ (Flo's time delay as measured on rocket ship clocks).

Also recall that when the rocket ship reversed its direction the order of these two events was reversed: Joe painted later than Flo. Isn't this going to get us into trouble with our usual ideas about cause and effect? If event A causes event B is it possible that some observer could see B happen before A? If that were true then we would have to give up, or drastically modify, the whole idea of Special Relativity. We can't live in a world where effects can precede their causes! In order to explore this question we need to do another Flo-Joe experiment.

Let's suppose that, instead of painting simultaneously, the painters decide to have Flo's painting cause Joe to paint. They will accomplish this by having Flo, at the instant she paints, send a signal to Joe who will then paint when he receives the signal. This will cause a delay in Joe's painting by the amount of time it takes for the signal to get to Joe, namely, the distance the signal has to go (30 ft) divided by the speed of the signal. Let's say the signal is a pulse of light that travels at speed c, so Joe's time delay will be 30/c, as measured on the painter's clocks.

Now, we want to know what Joe's time delay will read on the rocket clocks so we can compare it with $T_{Flo.}$. The rocket people see Joe's clock is running slow because of time dilation. They will measure Joe's delay as 30/c multiplied by γ, which, you will recall, is 5/3, based on their speed of 0.8c. This makes Joe's time delay (30/c) X (5/3) = 50/c. So we have:

$T_{Joe} = 50/c$ (Joe's delay measured on rocket ship clocks).

We can now compare the two time delays. If T_{Joe} is greater than T_{Flo} the rocket folks will see Joe paint after Flo. In this case an effect will be after its cause and we won't have to junk Special Relativity.

Now, since we are using v=0.8c, we can rewrite T_{Flo} (seen above) as:

$T_{Flo} = 32/0.8c = 40/c$.

Obviously T_{Joe} (50/c) is greater than T_{Flo} (40/c), causality is preserved, and Mr. Einstein, if he is watching, can give a huge sigh of relief, at least for the moment. But we're not through with him yet.

It's true that causality was preserved in the particular Flo-Joe experiment we dreamt up, but how about in general? What if Flo's signal traveled faster so that Joe painted earlier? What if the rocket traveled faster?

We can repeat our study of the Flo-causes-Joe experiment, but putting in different numbers. Let's say the speed of the signal is some arbitrary number "u". Using a little algebra (which we won't show here) we find out that in this case the ratio

$$T_{Joe}/ T_{Flo} = c^2/(uv).$$

In this expression, u is the speed of Flo's signal and v is the speed of the rocket ship.

So, in order to preserve causality, the ratio c^2/uv must always be greater than 1.00. This means that neither u nor v can be greater than the speed of light c. If either of them were, we could design an experiment that would violate causality.

By this reasoning we have arrived at two more conclusions:

Conclusion 4:
No object can travel as fast as the speed of light.

Conclusion 5:
No signal can travel faster than the speed of light.

We are now ready to arm ourselves with a little standard Relativity terminology that we can use at cocktail parties when the occasion arises. (Yeah, sure, it happens all the time.) Consider two events that are separated by a certain distance. If their separation in time is equal to the time it takes for a signal of light to go from one to the other then they are said to be "on the light cone". If their time separation is greater than that they are "inside the light cone"; if their time separation is less than that they are "outside the light cone".

If the events are on, or inside, the light cone they could be causally connected. One of them could have caused the other (but it may not have). If the events are outside the light cone they cannot be causally connected. In that case different observers, moving with different velocities, may disagree on which event occurred first, and, there is no sense arguing about it. There is no way to determine which event "really" occurred first.

This last statement seems a little weird. We need to think some more about this. Suppose two stars blow up, at roughly the same time, in some distant galaxy. Could different astronomers, on different planets out there somewhere in the universe, disagree on which star blew up first? How would the Keeper of Intra-galactic Records (KIR) deal with this?

We will pick this up in Chapter 4.