Chapter Two

A Logical Absurdity?

"Common sense is that layer of prejudices laid down in the mind prior to the age of eighteen" -A. Einstein

Einstein, like some others before him, notably the renowned physicist-mathematician Henri Poincaré, must have had trouble dealing with the idea that motion could affect the keeping of time.

If we are to believe that moving clocks run slowly then we are faced with a seeming paradox. If clock B, in a space ship, is moving past our clock A on Earth then clock B is supposed to run slower than clock A. Bound as we are to this big ball called Earth, we have no trouble deciding which clock is moving. But if both clocks are in space ships, going past each other, which one is "really" moving? Einstein's two postulates, bolstered by the Michelson-Morley experiments, tell us that observers in any space ship moving with constant velocity can always consider themselves at rest. The people in ship A will measure ship B's eggs cooking slower than theirs but the people in ship B should find just the opposite is true! How can each clock run slower than the other one? Surely that is a logical absurdity.

I don't know about you, but at this point, if I were Einstein, I would have tossed in the towel, which is apparently what Poincaré did. But Einstein was made of sterner stuff. He asked himself how one could actually measure time on a moving clock. The answer was not so simple, and it led him deeper into the subject. I am reminded of my favorite quote by Poul Anderson, *"I have yet to see any problem, however complicated, which, when you looked at in the right way, did not become still more complicated."*

For two or more clocks at rest relative to each other they can be synchronized to read the same time by sending signals back and forth between them. All you need to know is how much time it takes for a signal to go from one to the other. Knowing the distance between them you could even use pulses of light as the signals since the speed of light is a wellmeasured universal constant. But for two clocks moving with respect to each other it's not so simple. Einstein reasoned that they could only be compared at one instant, when they were right next to each other. You could even imagine taking a snapshot of a moving clock as it went right by your nose. The picture would tell you how your stationary clock and the moving one compared at that instant of time. But then what? How do you keep track of the moving clock when it is far away?

One possibility is to have the moving clock go past a different stationary clock that you have previously synchronized with the clock by your nose. That's the arrangement we described in our story about the cooking egg. It took the astronaut 5 minutes to get to the Moon as measured by the two synchronized clocks on the Moon and the Earth, but on the astronaut's clock only 3 minutes elapsed. This three-clock arrangement removes the logical impossibility of telling, for moving clocks, which one is "really" keeping time slowly. We are now ready to state our first conclusion.

Conclusion (1):
If a single clock is moving with respect to two synchronized clocks which are a fixed distance apart, the single clock will show less elapsed time than the fixed clocks.

Let's illustrate this with a picture.

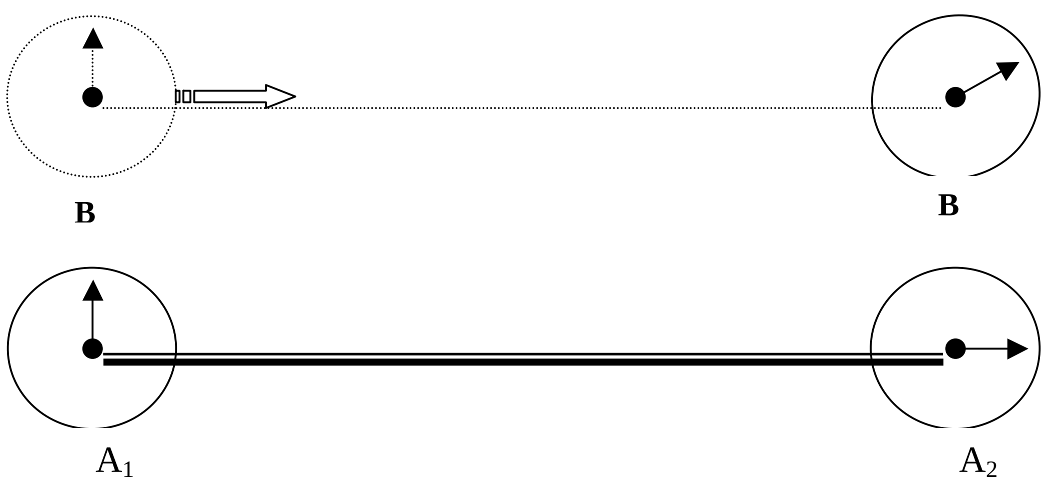

Figure 1. Clock B moves past the two fixed clocks A1 and A2. When B passes A2 it shows less elapsed time than the fixed clocks.

Having established Conclusion (1) we now have something new to think about. According to one of our previous assumptions the astronaut, with clock B, is allowed to consider himself at rest. Taking this viewpoint he could say to himself, “I saw the clock A1 moving by me backwards, and then, 3 minutes later, I saw the clock A2 moving by me. I can now calculate the distance between A1 and A2 by using that time and their speed.”

If he does this he finds:

(A1-A2) distance measured by B = speed X (3 minutes)

On the other hand, the observers stationed at A1 and A2 can also measure the distance between themselves by using the speed of the astronaut and the time the trip took on <u>their</u> clocks.

They get:

(A1-A2) distance measured by A1 and A2= speed X (5 minutes)!

Since these two measurements of the same distance don’t agree we need some new terminology. The distance measured by A1 and A2 is called the A1-A2 “rest-length”. It’s the same that they would get if they used a yardstick. That same distance as measured by the astronaut is called the A1-A2 “contracted-length”. It is shorter than the rest-length by exactly the same factor that B’s clock is found to run slow: (3 minutes)/(5 minutes).

This ratio of times and lengths keeps popping up throughout any discussion of Special Relativity:

γ = rest-length/contracted length = rest-time/moving-time

In our example, γ = (5 minutes)/(3 minutes) = 1.67. The value of γ depends on the relative speed (v) of the two observers.

As it is normally defined:

$\gamma = 1/\sqrt{(1-v^2/c^2)}$ Where c is the speed of light.

As we'll see later, no material object can reach or exceed c. This means that γ is always greater than 1.0.*

The reader, at this juncture, might wonder if there is another way to measure the length of a moving yardstick besides timing how long it takes to go by you. How about taking a picture of each end of it as it goes by two cameras? For this to work, of course, the two pictures would need to be taken at exactly the same time. The equations of Special Relativity can be used to analyze this hypothetical experiment. They tell us that the answer is the same as before: the moving yardstick appears shorter by a factor of $1/\gamma$.

After this somewhat long-winded discussion we are ready to state our next conclusion.

Conclusion (2):
If you measure the length of a moving yardstick it will be shorter than one yard by a factor of $1/\gamma$.

Conclusion (1) and Conclusion (2) are directly related to one another. If one is true so must be the other if we are to have a consistent description of nature. Conclusion (2) is usually referred to as the Lorentz Contraction after H.A. Lorentz who described it in 1904. He saw it only as a somewhat unusual way to explain the null results of the Michelson-Morley experiment, not as a universal law of physics.

One wonders what sort of mental exercises Einstein went through in coming to grips with the weird things he was discovering. They were in good accord with the theory of electromagnetic phenomena but could they really be true for <u>all</u> physical phenomena? What would happen if one could perform the experiment illustrated below?

* What we are calling γ Einstein originally called β, but nowadays β usually means the ratio v/c. We will follow the modern convention.

Figure 2 shows two painters that have stationed themselves 30 ft apart so they will be right next to a rocket that will be going past them at 80% of the speed of light. They have agreed to splash some paint on the rocket at the <u>same time</u> on their synchronized watches.

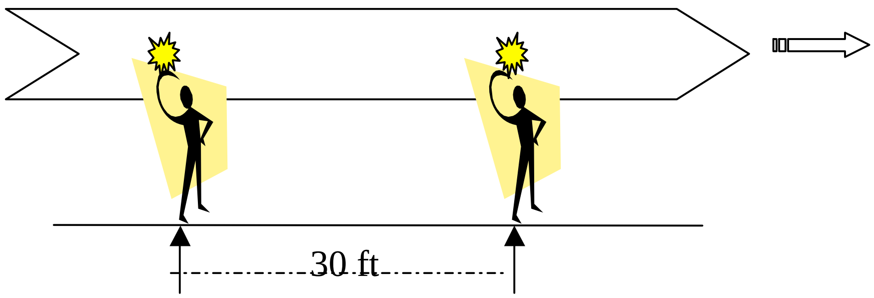

Fig. 2. Rocket moving.

Afterwards the rocket turns around and comes in for a landing. They put it in the rocket barn and proceed to measure the distance between the paint splotches. What they find is shown below in Figure 3 below.

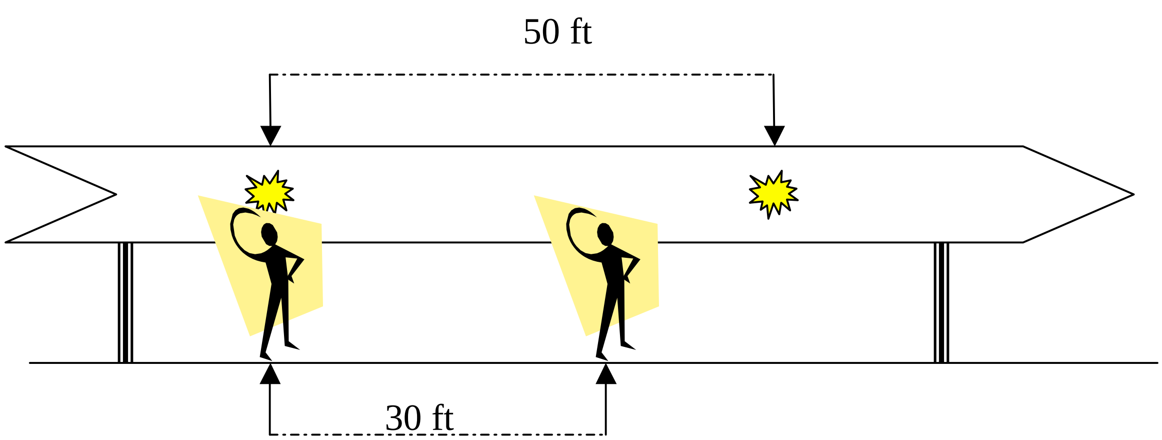

Fig. 3. Rocket at rest.

Lo and behold, the paint splotches are really 50 ft apart. When the rocket went by them it was contracted to 3/5 of its length. When it came to rest in the painter's system and assumed its normal rest-length the splotches were farther apart than they appeared to be when the rocket was moving.

Needless to say, the above experiment has never been done. In fact no direct confirmation of the Lorentz Contraction has been carried out, but, as we mentioned earlier, its predictions are intimately tied to time dilation which has been tested under a great variety of circumstances. Physicists have no doubt that if the above experiment could be done it would turn out as we have described.

But we need to think a little more about the results of this "gedanken" experiment. After the rocket is at rest in the barn and the spots have been measured, let us suppose that the pilot poses the following question to the painters:

> "You say you were 30 ft apart when you painted the two spots on my rocket. Since the spots ended up 50 ft apart it would seem that the Lorentz Contraction is indeed true. But according to Mr. Einstein I can consider myself to be at rest when I am moving with constant velocity. What I should see is two painters going past me backwards at 80% of the speed of light. If they are 30 ft apart in their rest system they should appear closer together as they go by me. I should measure them to be only 30 X (3/5) = 18 ft apart. In that case how could the paint spots end up 50 ft apart on my rocket?"

This question is perplexing enough to energize the painters and the pilot to repeat the experiment. This time the pilot will take along a couple of friends, Sue and Tom, to take pictures as the painters "go past" the rocket. Sue and Tom synchronize their watches and station themselves 18 ft apart in the rocket. The painters rub out their original paint spots on the rocket and station themselves 30 ft apart on the Earth as before.

While the rocket is getting up to speed let's take a chapter break. I don't know about you but my head is starting to hurt.