Phaseless Near-Field Optical Tomography

John C Schotland
University of Pennsylvania
Philadelphia, PA

schotland@seas.upenn.edu

Joint work with Alexander Govyadonov and George Panasyuk
Nanometer scale imaging

The National Nanotechnology Initiative has identified the goal of “manipulating, measuring and seeing matter [at the nanoscale] as a grand challenge”.

- Scanning probes are tools for imaging surfaces
  — subsurface metrology
- Micro and nano fabrication
  — transition from planar to stacked platforms
- Subcellular nanostructures
Scanning probe microscopy (AFM)
Near-field microscopy (NSOM)
Near-field resolution

Betzig and Trautman Science (1992)
Three-dimensional effects in NSOM

$$\text{FOV} = \lambda \times \lambda$$
Nanotomography

Robert Magerle*

Physikalishe Chemie II, Universität Bayreuth, D-95440 Bayreuth, Germany
Near-field inverse scattering

- Evanescent and propagating waves contribute to the scattered field
- Evanescent waves carry subwavelength information
- Inverse problem is overdetermined
Near-field tomography

Model

Tomographs

\[ \text{FOV} = \lambda \times \lambda \]

Requires phase measurements
Variants

Collection Mode  Illumination Mode

PSTM
Total internal reflection tomography

Evanescent wave illumination and far-field detection. Resolution is controlled by index of refraction of prism
Total internal reflection tomography

$z$ measured in units of wavelength
PSTM experiment

Tip height 200 nm

$\lambda = 633$ nm

Sample: gold nanoparticle
PSTM reconstructions

Reconstructed Image

AFM Image

\[ \text{Re}(\eta(\text{gold})) = -0.9 \]
The experiment

Measure the power extinguished from the incident beam as the tip is scanned in three directions.
Model

We treat the tip as a strongly-scattering metallic nanoparticle and the sample as a weakly-scattering dielectric. The electric field from a monochromatic source obeys the wave equation

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - k_0^2 \mathbf{E}(\mathbf{r}) = 4\pi k_0^2 (\eta(\mathbf{r}) + \chi(\mathbf{r})) \mathbf{E}(\mathbf{r}) ,$$

where $\chi(\mathbf{r}) = \alpha_0 \delta(\mathbf{r} - \mathbf{r}_0)$ is the susceptibility of the tip and $\eta(\mathbf{r})$ the susceptibility of the sample. The power extinguished from the incident field $\mathbf{E}^i$ is obtained from the generalized optical theorem [Carney, Schotland and Wolf, Phys. Rev. E (2005)] and is given by

$$P = \frac{\omega}{2} \text{Im} \int \mathbf{E}(\mathbf{r}) \cdot \mathbf{E}^i*(\mathbf{r}) (\eta(\mathbf{r}) + \chi(\mathbf{r})) \, d^3r .$$

The strategy is to account for all orders of scattering from the tip and one order of scattering from the sample.
Diagrams

\[ E^s = \bullet \sim + \bullet \sim + \bullet \sim + \bullet \sim + \ldots + \]

\[ \eta = \bullet \]

\[ \chi = \circ \]

Summing the diagrams:

\[ E^s = \square \sim + \bullet \sim + \square \sim + \bullet \square \sim + \square \bullet \square \sim \]

\[ ST \quad TS \quad TST \]

Tip T matrix:

\[ \square = \circ + \circ \circ + \circ \circ \circ + \circ \circ \circ \circ + \ldots \]
Point-tip model

For an isotropic point scatterer with $\chi(r) = \alpha_0 \delta(r - r_0)$, the Born series can be summed and the electric field is given by

$$E_\alpha(r) = E_\alpha^i(r) + \tilde{\alpha}_0 k_0^2 G_{\alpha\beta}(r, r_0) E_\beta^i(r_0),$$

where $\tilde{\alpha}_0$ is the renormalized polarizability

$$\tilde{\alpha}_0 = \frac{\alpha_0}{1 - \frac{2}{3} k_0^2 \alpha_0 \left( \frac{1}{\pi \Lambda} - \frac{4\pi}{k_0^2 \Lambda^3} + i k_0 \right)}.$$

Here we have regularized the divergence in $G_{\alpha\beta}(r_0, r_0)$ by introducing a high-frequency cutoff defined by the size of the scatterer $\Lambda$. 
Extinguished power

Accounting for all orders of scattering from the tip, we find that to lowest order in the susceptibility of the sample, the extinguished power is given by

\[
P(r_0) = \frac{\omega k_0^2}{2} \text{Im} \int d^3 r G_{\alpha\beta}(r, r_0) \left[ \tilde{\alpha}_0 E^i_\alpha(r) E^{i\ast}_\beta(r_0) + \tilde{\alpha}_0 E^i_\alpha(r_0) E^{i\ast}_\beta(r) \right] \eta(r).
\]

Note that we have omitted contributions from the tip and the sample alone.
Inverse problem

Suppose the tip is scanned on a square lattice and $N$ planes of data are measured. Then the susceptibility can be reconstructed from the inversion formula

$$\tilde{\eta}(q, z) = \sum_{i,j} K_i(q, z) M_{ij}^{-1}(q) P_j(q),$$

where $\tilde{\eta}(q, z)$ is the transverse Fourier transform of $\eta$, $K$ and $M$ can be computed explicitly and $i, j = 1, \ldots, N$. Note that $M_{ij}^{-1}(q)$ must be regularized and that the inverse problem has a Fourier-Laplace structure. To reconstruct both the real and imaginary parts of $\eta$ requires measurements with two incident beams.
Reconstructions

Model | Reconstruction
--- | ---
$z=0.088\lambda$ | 
$z=0.119\lambda$ | 

Model | Reconstruction
--- | ---
$z=0.080\lambda$ | 
$z=0.100\lambda$ | 

$\Re(\eta)$ | $\Im(\eta)$
Remarks

- All measurements are conducted in the far field, yet the resolution is subwavelength
- The tip is a controlled scatterer
- This is an example of an inverse scattering problem with internal degrees of freedom
Conclusions

We have shown that the three-dimensional subwavelength structure of an inhomogeneous scattering medium can be recovered from far-field measurements of the extinguished power. Neither phase control of the illuminating field nor phase measurements of the scattered field are required.

Our approach is based on the solution to the inverse scattering problem for a system consisting of a weakly-scattering sample and a strongly scattering nano-scale tip. In principle, the theory can be extended to treat the case of a strongly-scattering sample by inversion of an appropriately resummed perturbation series, taken to all orders of scattering in the sample.