Physics 523, Quantum Field Theory II

Homework 6

Due Wednesday, 18th February 2004

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Dimensional Regularization

a) We are to evaluate the expression

$$\int \frac{d^d\ell_E}{(2\pi)^d}\frac{1}{(\ell_E^2+\Delta)^n}, \quad \text{for } n \geq 2.$$

In homework 2, problem 3 we showed that the d-dimensional volume element $\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$. Using this, we see that

$$\begin{aligned}
\int \frac{d^d\ell_E}{(2\pi)^d}\frac{1}{(\ell_E^2+\Delta)^n} &= \int \frac{d\Omega_d}{(2\pi)^d}\int_0^\infty d\ell_E \frac{\ell_E^{d-1}}{(\ell_E^2+\Delta)^n},\\
&= \frac{2\pi^{d/2}}{(2\pi)^d\Gamma(d/2)}\int_0^\infty d\ell_E \frac{\ell_E^{d-1}}{(\ell_E^2+\Delta)^n},\\
&= \frac{2}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\int_0^\infty d\ell_E \frac{\ell_E^{d-1}}{(\ell_E^2+\Delta)^n},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\int_0^\infty d(\ell_E^2) \frac{(\ell_E^2)^{d/2-1}}{(\ell_E^2+\Delta)^n}.
\end{aligned}$$

We will defined the integration variable

$$\eta \equiv \frac{\Delta}{(\ell_E^2+\Delta)} \quad \text{such that} \quad d\eta = -\frac{\Delta}{(\ell_E^2+\Delta)^2}d(\ell_E^2) \quad \text{and} \quad \ell_E^2 = \Delta\eta^{-1}(1-\eta).$$

Note that under the η substitution, the limits of integration will change from $(0,\infty)\mapsto(1,0)\sim -(0,1)$. Also note the use of the definition of the Euler Beta function below. Making this substitution in the required integral, we have

$$\begin{aligned}
\int \frac{d^d\ell_E}{(2\pi)^d}\frac{1}{(\ell_E^2+\Delta)^n} &= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\int_0^\infty d(\ell_E^2) \frac{(\ell_E^2)^{d/2-1}}{(\ell_E^2+\Delta)^n},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\frac{1}{\Delta}\int_0^1 d\eta \frac{\Delta^{d/2-1}\eta^{1-d/2}(1-\eta)^{d/2-1}}{(\ell_E^2+\Delta)^{n-2}},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\left(\frac{1}{\Delta}\right)^{2-d/2}\int_0^1 d\eta \left(\frac{\Delta}{\eta}\right)^{2-n}\eta^{1-d/2}(1-\eta)^{d/2-1},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\left(\frac{1}{\Delta}\right)^{2-d/2+n-2}\int_0^1 d\eta\eta^{n-2+1-d/2}(1-\eta)^{d/2-1},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\left(\frac{1}{\Delta}\right)^{n-d/2}\int_0^1 d\eta\eta^{n-d/2-1}(1-\eta)^{d/2-1},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\left(\frac{1}{\Delta}\right)^{n-d/2}\frac{\Gamma(n-d/2)\cdot\Gamma(d/2)}{\Gamma(n)},
\end{aligned}$$

$$\boxed{\therefore \int \frac{d^d\ell_E}{(2\pi)^d}\frac{1}{(\ell_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}}\frac{\Gamma(n-d/2)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-d/2}.} \tag{a.1}$$

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b) Let us now evaluate the expression

$$\int \frac{d^d\ell_E}{(2\pi)^d}\frac{\ell_E^2}{(\ell_E^2+\Delta)^n}, \quad \text{for } n \geq 2.$$

The evaluation of this integral will proceed identically to that in part (a) above. We will introduce the same integration variable $\eta \equiv \frac{\Delta}{(\ell_E^2+\Delta)}$ and follow the same procedure. We see that

$$\begin{aligned}
\int \frac{d^d\ell_E}{(2\pi)^d}\frac{\ell_E^2}{(\ell_E^2+\Delta)^n} &= \int\frac{d\Omega_d}{(2\pi)^d}\int_0^\infty d\ell_E \frac{\ell_E^{d+1}}{(\ell_E^2+\Delta)^n},\\
&= \frac{2\pi^{d/2}}{(2\pi)^d\Gamma(d/2)}\int_0^\infty d\ell_E\frac{\ell_E^{d+1}}{(\ell_E^2+\Delta)^n},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\int_0^\infty d(\ell_E^2)\frac{(\ell_E^2)^{d/2}}{(\ell_E^2+\Delta)^n},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\frac{1}{\Delta}\int_0^1 d\eta\frac{\Delta^{d/2}\eta^{-d/2}(1-\eta)^{d/2}}{(\ell_E^2+\Delta)^{n-2}},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\left(\frac{1}{\Delta}\right)^{1-d/2}\int_0^1 d\eta\left(\frac{\Delta}{\eta}\right)^{2-n}\eta^{-d/2}(1-\eta)^{d/2},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\left(\frac{1}{\Delta}\right)^{1-d/2+n-2}\int_0^1 d\eta\eta^{n-1-d/2-1}(1-\eta)^{d/2+1-1},\\
&= \frac{1}{(4\pi)^{d/2}}\frac{1}{\Gamma(d/2)}\left(\frac{1}{\Delta}\right)^{n-d/2-1}\frac{\Gamma(n-1-d/2)\cdot\Gamma(d/2+1)}{\Gamma(n)}.
\end{aligned}$$

Recall the elementary property of the Γ function that $\alpha\Gamma(\alpha)=\Gamma(\alpha+1)$. Therefore we see that $\Gamma(d/2+1)=\frac{d}{2}\Gamma(d/2)$. Using this result, we see immediately that

$$\boxed{\therefore \int\frac{d^d\ell_E}{(2\pi)^d}\frac{\ell_E^2}{(\ell_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}}\frac{d}{2}\frac{\Gamma\left(n-\frac{d}{2}-1\right)}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-d/2-1}.} \tag{b.1}$$

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c i) Let us show the following identity,

$$\gamma^\mu\gamma^\nu\gamma_\mu = -(2-\epsilon)\gamma^\nu.$$

Simply applying the anticommutation relation of the γ matrices, we see that[1]

$$\gamma^\mu\gamma^\nu\gamma_\mu = g_{\mu\rho}\gamma^\mu\gamma^\nu\gamma^\rho = 2g_{\mu\rho}g^{\nu\rho}\gamma^\mu - g_{\mu\rho}\gamma^\mu\gamma^\rho\gamma^\nu = 2\delta^\nu_\mu\gamma^\mu - d\gamma^\nu = (2-d)\gamma^\nu,$$

$$\boxed{\therefore \gamma^\mu\gamma^\nu\gamma_\mu = -(2-\epsilon)\gamma^\nu.} \tag{c.1}$$

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c ii) Let us show the following identity,

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho} - \epsilon\gamma^\nu\gamma^\rho.$$

Simply applying the anticommutation relation of the γ matrices, we see that

$$\begin{aligned}
\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= g_{\mu\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = 2g_{\mu\sigma}g^{\rho\sigma}\gamma^\mu\gamma^\nu - 2g_{\mu\sigma}g^{\mu\rho} + g_{\mu\sigma}\gamma^\mu\gamma^\sigma\gamma^\nu\gamma^\rho,\\
&= 2\delta^\rho_\mu\gamma^\mu\gamma^\nu - 2\delta^\sigma_\mu\gamma^\mu\gamma^\rho + d\gamma^\nu\gamma^\rho = 2\gamma^\rho\gamma^\nu + 2\gamma^\nu\gamma^\rho - 4\gamma^\nu\gamma^\rho + d\gamma^\nu\gamma^\rho,
\end{aligned}$$

$$\boxed{\therefore \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho} - \epsilon\gamma^\nu\gamma^\rho.} \tag{c.2}$$

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[1] We will repeatedly use that $g_{\mu\rho}\gamma^\mu\gamma^\rho\mathscr{X} = g_{\mu\rho}\gamma^\rho\gamma^\mu\mathscr{X}$ by symmetry of the inner product together with $g_{\mu\rho}\gamma^\mu\gamma^\rho\mathscr{X} = 2g_{\mu\rho}g^{\mu\rho}\mathscr{X} - g_{\mu\rho}\gamma^\rho\gamma^\mu\mathscr{X}$ from the anticommutation relations, imply that $g_{\mu\rho}\gamma^\mu\gamma^\rho\mathscr{X} = g_{\mu\rho}g^{\mu\rho}\mathscr{X} = d\mathscr{X}$ for any product of γ matrices $\mathscr{X}$.

c iii) Let us show the following identity,

$$\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu + \epsilon\gamma^\nu\gamma^\rho\gamma^\sigma.$$

Simply applying the anticommutation relation of the γ matrices, we see that

$$\begin{aligned}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu &= g_{\mu\tau}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau = g_{\mu\tau}\left(2g^{\sigma\tau}\gamma^\mu\gamma^\nu\gamma^\rho - 2g^{\rho\tau}\gamma^\mu\gamma^\nu\gamma^\sigma + 2g^{\nu\tau}\gamma^\mu\gamma^\rho\gamma^\sigma - g^{\mu\tau}\gamma^\nu\gamma^\rho\gamma^\sigma\right),\\ &= 2\delta^\sigma_\mu\gamma^\mu\gamma^\nu\gamma^\rho - 2\delta^\rho_\mu\gamma^\mu\gamma^\nu\gamma^\sigma + 2\delta^\nu_\mu\gamma^\mu\gamma^\rho\gamma^\sigma - d\gamma^\nu\gamma^\rho\gamma^\sigma = 2\gamma^\sigma\gamma^\nu\gamma^\rho - 4g^{\nu\rho}\gamma^\sigma + (4-d)\gamma^\nu\gamma^\rho\gamma^\sigma,\\ &= 4g^{\mu\rho}\gamma^\sigma - 2\gamma^\sigma\gamma^\rho\gamma^\nu - 4g^{\nu\rho}\gamma^\sigma + \epsilon\gamma^\nu\gamma^\rho\gamma^\sigma,\end{aligned}$$

$$\boxed{\therefore \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu + \epsilon\gamma^\nu\gamma^\rho\gamma^\sigma.} \tag{c.3}$$

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The Ward Identity

a i) Let us compute the integral

$$\int\frac{d^4\ell}{(2\pi)^4}\frac{1}{(\ell^2-\Delta)^2},$$

by restricting the integration region to the Euclidean sphere with $\ell_E < \Lambda$. To accomplish this calculation, we will recall several important results from earlier homework problems. Namely, we will use the standard 4-dimensional volume element and change to Euclidean coordinates ℓ_E. Notice the u substitution below.

$$\begin{aligned}\int\frac{d^4\ell}{(2\pi)^4}\frac{1}{(\ell^2-\Delta)^2} &= \int\frac{d\Omega_4}{(2\pi)^4}\int_0^\infty d\ell\frac{\ell^3}{(\ell^2-\Delta)^2},\\ &= \frac{2i}{(4\pi)^2}\int_0^\infty d\ell_E\frac{\ell_E^3}{(\ell_E^2+\Delta)^2},\\ &\to \frac{2i}{(4\pi)^2}\lim_{\Lambda\to\infty}\int_0^\Lambda d\ell_E\frac{\ell_E^3}{(\ell_E^2+\Delta)^2},\\ &= \frac{i}{(4\pi)^2}\lim_{\Lambda\to\infty}\int_\Delta^{\Lambda^2+\Delta}du\frac{u-\Delta}{u^2},\\ &= \frac{i}{(4\pi)^2}\lim_{\Lambda\to\infty}\left[\log(u)\Big|_\Delta^{\Lambda^2+\Delta} + \frac{\Delta}{u}\Big|_\Delta^{\Lambda^2+\Delta}\right],\\ &= \frac{i}{(4\pi)^2}\lim_{\Lambda\to\infty}\left[\log\left(\frac{\Lambda^2+\Delta}{\Delta}\right)-1\right],\\ &= \frac{i}{(4\pi)^2}\left[\log\left(\frac{\Lambda^2}{\Delta}\right)-1+\mathscr{O}\left(\frac{\Delta}{\Lambda^2}\right)\right].\end{aligned}$$

ii) We are to compute the function Z_1 from the $\delta\Gamma(q=0)$ calculation. Recall that in homework 5 question 4, we computed $\delta F_1(q=0)$ using a different regularization. Because $\delta Z_1 = -\delta F_1(q=0)$ much of our 'hard labor' has already been completed. Let us begin our calculation.

$$\begin{aligned}\delta Z_1 &= -4ie^2\int_0^1 dz(1-z)\int\frac{d^4\ell}{(2\pi)^4}\left[-\frac{1}{2}\frac{\ell^2}{(\ell^2-\Delta)^3}+\frac{m^2(1-4z+z^2)}{(\ell^2-\Delta)^3}\right],\\ &= -4ie^2\int_0^1 dz(1-z)\int\frac{d^4\ell}{(2\pi)^4}\left[-\frac{1}{2}\left(\frac{1}{(\ell^2-\Delta)^3}+\frac{\Delta}{(\ell^2-\Delta)^3}\right)+\frac{m^2(1-4z+z^2)}{(\ell^2-\Delta)^3}\right],\\ &= -4ie^2\int_0^1 dz(1-z)\left[-\frac{1}{2}\frac{i}{(4\pi)^2}\left(\log\left(\frac{\Lambda^2}{\Delta}\right)-1+\mathscr{O}\left(\frac{\Delta}{\Lambda^2}\right)\right)+\frac{1}{4}\frac{i}{(4\pi)^2}-\frac{1}{2}\frac{i}{(2\pi)^2}\frac{m^2(1-4z+z^2)}{\Delta}\right],\\ &= -\frac{\alpha}{4\pi}\int_0^1 dz(1-z)\left[\log\left(\frac{\Lambda^2}{\Delta}\right)-1-\frac{1}{2}+\frac{m^2(1-4z+z^2)}{\Delta}\right],\end{aligned}$$

$$\therefore \delta Z_1 = -\frac{\alpha}{4\pi}\int_0^1\log\left(\frac{\Lambda^2}{\Delta}\right)-\frac{3}{2}+\frac{m^2(1-4z+z^2)}{\Delta}.$$

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iii) Let us now compute the value of the electron self-energy function Z_d. First, we must recall the definition of Z_2. It is the function

$$\delta Z_2 = \left.\frac{d\Sigma_2}{d\not{p}}\right|_{\not{p}=m}$$

where

$$\Sigma_2 = -ie^2\int_0^1 dz\int\frac{d^4\ell}{(2\pi)^4}\frac{-2z\not{p}+4m}{(\ell^2-\Delta)^2} = \frac{\alpha}{2\pi}\int_0^1 dz(2m - z\not{p})\left[\log\left(\frac{\Lambda^2}{\Delta}\right)-1\right].$$

Using the chain rule for differentiation, we see that

$$\therefore \delta Z_2 = \frac{\alpha}{2\pi}\int_0^1 dz\left[-z\left(\log\left(\frac{\Lambda^2}{\Delta}\right)-1\right)+\frac{2m^2z(2-z)(1-z)}{\Delta}\right].$$

iv) We will now compute the difference $Z_2 - Z_1 = \delta Z_2 - \delta Z_1$ for this regularization scheme. We will call upon Peskin and Schroeder for algebraic simplification within the integrand. The cancellation of the log-type term with the $1/\Delta$ term was shown in homework 5. We have

$$\begin{aligned}\delta Z_2 - \delta Z_1 &= \frac{\alpha}{2\pi}\int_0^1 dz\left[(1-2z)\log\left(\frac{\Lambda^2}{\Delta}\right)+z-\frac{3}{2}(1-z)+\frac{2m^2z(2-z)(1-z)}{\Delta}-\frac{m^2(1-4z+z^2)}{\Delta}\right],\\ &= \frac{\alpha}{2\pi}\int_0^1 dz\left[z-\frac{3}{2}(1-z)\right] = \frac{\alpha}{2\pi}\left(-\frac{1}{4}\right),\end{aligned}$$

$$\therefore \delta Z_2 - \delta Z_1 = -\frac{\alpha}{8\pi}.$$

b i) Let us repeat our above calculation using dimensional regularization. We can begin our work by generalizing the Dirac algebra used to calculate δZ_1. Notice that this calculation will require our $d = 4-\epsilon$ dimensional generalization of the Dirac algebra to simplify the numerator in

$$\delta\Gamma^\mu(q^2=0) = 2ie^2\int_0^1 dz(1-z)\int\frac{d^d\ell}{(2\pi)^d}\frac{\gamma^\nu\left(\not{\ell}+z\not{p}\right)\gamma^\mu\left(\not{\ell}+z\not{p}\right)\gamma_\nu}{(\ell^2-\Delta)^3}.$$

Although we have already simplified our work by leaving off terms proportional to q, we may reduce our labor even more. The regularization of this integral in d-dimensions is presented to make sense of the divergence of the integral. Computing the integral in $d = 4-\epsilon$ dimensions, we avoid the divergence of the integral due to the term proportional to ℓ^2 in the numerator. However, we should notice that no other terms in the integral will have a power of $\ell \leq 4$ in the denominator and therefore will not diverge.

Therefore, only the ℓ^2-term will need to be regulated and the other parts of this integral can be computed as usual.[2]

Let us then compute the regulated coefficient of the ℓ^2 term in the the numerator. To do this, we will use our algebraic results from problem (1.c.iii) above. We also remind the reader that in d-dimensions the integral is symmetric under $\ell^\mu\ell^\nu \to \frac{1}{d}\gamma^{\mu\nu}\ell^2$. Therefore we see that our regulated term is simply

$$\begin{aligned}\gamma^\nu\not{\ell}\gamma^\mu\not{\ell}\gamma_\nu &= \ell_\rho\ell_\sigma\gamma^\nu\gamma^\rho\gamma^\mu\gamma^\sigma\gamma_\nu,\\ &= \ell_\rho\ell_\sigma\left(-2\gamma^\sigma\gamma^\mu\gamma^\rho+\epsilon\gamma^\rho\gamma^\mu\gamma^\sigma\right),\\ &= -4\not{\ell}\ell^\mu+2\not{\ell}^2\gamma^\mu+2\epsilon\not{\ell}\ell^\mu-\epsilon\not{\ell}^2\gamma^\mu,\\ &= -\frac{4}{d}\ell^2\gamma^\mu+2\ell^2\gamma^\mu+\frac{2\epsilon}{d}\ell^2\gamma^\mu-\epsilon\ell^2\gamma^\mu,\\ &= \gamma^\mu\ell^2\left(\frac{-4+2\epsilon}{d}+2-\epsilon\right),\\ &= \gamma^\mu\ell^2\frac{(\epsilon-2)^2}{d}.\end{aligned}$$

[2]It makes little sense to regulate a convergent integral. More rigorously, one could carry ϵ dependence on all terms and then 'observe' that for all but the term proportional to ℓ^2 in the numerator, $\epsilon \to 0$ will not affect the integral. Therefore we may view the introduction of ϵ into those terms as a waste of time.

Now that we have fully established the need only to regularize this piece of the integral, let us calculate the regularized form of δZ_1. During the computation below, we have referred to the canonical results for expansions of $\Delta, \Gamma, \frac{1}{(4\pi)}$ in terms of ϵ. Many of these relations were derived in homework sets 2 and 5. Let us proceed directly.

$$\begin{aligned}
\delta Z_1 &= -2ie^2 \int_0^1 dz(1-z) \int \frac{d^d\ell}{(2\pi)^d} \left[\frac{(\epsilon-2)^2}{d} \frac{\ell^2}{(\ell^2-\Delta)^3} + \frac{m^2(1-4z+z^2)}{(\ell^2-\Delta)^3} \right], \\
&= -2ie^2 \int_0^1 dz(1-z) \left[\frac{(\epsilon-2)^2}{d} \frac{d}{4} \frac{i}{(4\pi)^{d/2}} \frac{\Gamma\left(2-\frac{d}{2}\right)}{\Delta^{2-d/2}} - \frac{i}{2} \frac{1}{(4\pi)^2} \frac{m^2(1-4z+z^2)}{\Delta} \right], \\
&= \frac{\alpha}{2\pi} \int_0^1 dz(1-z) \left[\frac{(\epsilon-2)^2}{4} \left(\frac{2}{\epsilon} - \log\Delta - \gamma_E + \log(4\pi) \right) - \frac{1}{2} \frac{m^2(1-4z+z^2)}{\Delta} \right] \\
\therefore \delta Z_1 &= \frac{\alpha}{2\pi} \int_0^1 dz(1-z) \left[-\left(\frac{2}{\epsilon} - 2 - \log\Delta - \gamma_E + \log(4\pi) \right) - \frac{1}{2} \frac{m^2(1-4z+z^2)}{\Delta} \right].
\end{aligned}$$

ii) Let us now regularize the term Z_2. This computation will be very similar to that above. We will first need to rework some minor Dirac algebra. Unlike last time, however, the entire integral will diverge and so we will need to keep ϵ terms consistently in our equations. Recall that Z_2 is related to a derivative of the integral

$$\Sigma_2(p) = -ie^2 \int_0^1 dz \int \frac{d^d\ell}{(2\pi)^d} \frac{\gamma^\mu \left(\not{\ell} + z\not{p} + m\right) \gamma_\mu}{(\ell^2-\Delta)^2}.$$

Recalling that terms proportional to ℓ in the integral will integrate to zero because of Lorentz covariance, we may drop the ℓ term. Furthermore, using only the relatively trivial Dirac algebra identities derived above, we see that

$$\gamma^\mu \left(\not{\ell} + z\not{p} + m\right) \gamma_\mu \to -z(2-\epsilon)\not{p} + dm.$$

Therefore we may compute this integral directly.

$$\begin{aligned}
\Sigma_2(p) &= -ie^2 \int_0^1 dz \int \frac{d^d\ell}{(2\pi)^d} \left(-(2-\epsilon)z\not{p} + (4-\epsilon)m\right) \frac{1}{(\ell^2-\Delta)^2}, \\
&= -ie^2 \int_0^1 dz \left[\left(-(2-\epsilon)z\not{p} + (4-\epsilon)m\right) \frac{i}{(4\pi)^{d/2}} \frac{\Gamma\left(2-\frac{d}{2}\right)}{\Delta^{2-d/2}} \right], \\
&= \frac{\alpha}{2\pi} \int_0^1 dz \left[\frac{1}{2} \left((4-\epsilon)m - (2-\epsilon)z\not{p}\right) \left(\frac{2}{\epsilon} - \log\Delta - \gamma_E + \log(4\pi) \right) \right].
\end{aligned}$$

Therefore we see by simple chain-rule differentiation that

$$\begin{aligned}
\delta Z_2 = \left.\frac{d\Sigma_2}{d\not{p}}\right|_{\not{p}=m} &= \frac{\alpha}{2\pi} \int_0^1 dz \frac{1}{2} \left[(\epsilon-2)z \left(\frac{2}{\epsilon} - \log\Delta - \gamma_E + \log(4\pi) \right) - \frac{m^2 2z(1-z)\left((\epsilon-2)z + (4-\epsilon)\right)}{\Delta} \right], \\
&= \frac{\alpha}{2\pi} \int_0^1 dz \left[z \left(1 - \frac{2}{\epsilon} + \log\Delta + \gamma_E - \log(4\pi) \right) - \frac{m^2 2z(1-z)(2-z)}{\Delta} \right],
\end{aligned}$$

iii) Unfortunately, I was unable to derive the explicit cancellation. It appears as if I may have introduced an incorrect minus sign somewhere. In the correct form, one should see the total integral vanish so that

$$\delta Z_2 - \delta Z_1 = 0.$$