

Problem 6.11

Let us consider a transverse plane wave that is normally incident in vacuum on a perfectly absorbing flat screen.

- a) From the conservation of linear momentum, we are to show that the pressure exerted on the screen is equal to the field energy per unit volume in the wave.

We can do this almost entirely in terms of units. Notice that the momentum transferred to the screen is just the spacetime integral of \mathbf{g} , the momentum density. We see that in a time dt , momentum $\mathbf{g}c\,dadt$ is transferred to the screen on an area da , where c is the speed of the radiation. The pressure is simply the force per unit area, $\frac{dF}{da} = \frac{d}{da} \frac{dp}{dt}$ which is obviously $\mathbf{g}c$.

By simple analysis of the units involved, $\mathbf{g}c$ is given in

$$\frac{\text{momentum}}{\text{volume}} \frac{\text{length}}{\text{time}} = \frac{\text{energy} \times \text{time}}{\text{volume} \times \text{length}} \frac{\text{length}}{\text{time}} = \frac{\text{energy}}{\text{volume}}.$$

Therefore, we see that the radiation pressure is simply the energy density.

- b) In the neighborhood of the earth the flux of electromagnetic radiation from the sun is approximately $1.4\text{kW}/\text{m}^2$. If an interplanetary ‘sailplane’ had a sail of mass $1\text{g}/\text{m}^2$ of area and negligible other weight, what would be its maximum acceleration in meters per second squared due to the solar radiation pressure?

For fun, let us solve this problem using only an analysis of units, like above. The only relevant numbers for this problem are $1.4\text{kW}/\text{m}^2 [=]\text{kg}/\text{s}^3$, the speed of light, $299792458\text{m}/\text{s}$, and the mass density of the sail, $10^{-3}\text{kg}/\text{m}^2$. We are looking for a solution with units m/s^2 . It is relatively obvious that the solution has to be

$$\frac{1.4\text{kg}}{\text{s}^3} \frac{1\text{m}^2}{10^{-3}\text{kg}} \frac{1\text{s}}{299792458\text{m}/\text{s}} = 4.67 \times 10^{-3}\text{m}/\text{s}^2.$$

This is why I fear I’ll corrupt young physics students someday. Cheers ~Jake.