Problem Set 6

Due: Tuesday, November 1

Problem 1. We want to place the integers $1, 2, \ldots, r$ into a circular array with n positions so that they occur in order, clockwise, and such that consecutive integers (including the pair $(r, 1)$) are not adjacent. Arrangements which are rotations of each other are considered the same. In how many ways can this be done?

Problem 2. Let A_n be the $n \times n$ matrix whose (i, j) entry is $\binom{i}{j}$, with rows and columns numbered starting from 0. So, for example,

$$A_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 1 & 3 & 3 & 1 & 0 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix}.$$

Compute A_2^{-1}, A_3^{-1} and A_4^{-1}. Find and prove a formula for A_n^{-1}.

Problem 3. A *star-cutset* of G is a vertex cut S containing a vertex x adjacent to all of $S - \{x\}$. Find an imperfect graph G having a star-cutset C such that the C-lobes of G are perfect graphs.

Problem 4. Let $\mathbb{F}_p$ be the finite field with p elements for some prime p and let $\mathbb{F}_p[x]$ be the ring of polynomials in the variable x with coefficients in $\mathbb{F}_p$. How many monic polynomials of degree n are there in $\mathbb{F}_p[x]$ that do not take on the value 0 for $x \in \mathbb{F}_p$? (A polynomial is monic if it is of the form $x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$.)

Problem 5. Suppose that $G = G_1 \cup G_2$, that $G_1 \cap G_2$ is a clique, and that G_1 and G_2 are perfect. Prove that G is perfect.