Problem Set 5

Due: Tuesday, October 25

Problem 1. Prove by induction on the number of faces that a plane graph G is bipartite if and only if every face has even length.

Problem 2. Prove that every n-vertex plane graph isomorphic to its dual has $2n-2$ edges. For all $n \geq 4$, construct a simple n-vertex plane graph isomorphic to its dual.

Problem 3. Prove that every simple planar graph with at least four vertices has at least four vertices with degree less than 6. Construct a simple planar graph G with 8 vertices that has exactly four vertices with degree less than 6.

Problem 4. Prove that if G is a color-critical graph, then the graph G' generated from it by applying Mycielski's construction is also color-critical (color-critical means k-critical for some k).

Problem 5. A *triangulation* is a simple plane graph where every face boundary is a 3-cycle. Prove that a triangulation is 3-colorable if and only if it is Eulerian. (Hint: For sufficiency, use induction on $n(G)$. Choose an appropriate pair or triple of adjacent vertices to replace with appropriate edges.)