Problem Set 4

Due: Tuesday, October 4

Problem 1. A math department offers 8 courses $c_1, \ldots, c_8$, and has 5 professors $p_1, \ldots, p_5$. The following table lists the courses each professor is willing to teach.

$$\begin{array}{rc} p_1: & c_1, c_2 \\ p_2: & c_1, c_3 \\ p_3: & c_2, c_3 \\ p_4: & c_2, c_4, c_5, c_6, c_7 \\ p_5: & c_6, c_7, c_8 \end{array}$$

Assuming that each professor can teach between zero and two classes, and that no two classes meet at the same time, formulate a max flow problem that will allow the department administrator to decide whether or not all 8 courses can be offered. Use the Ford-Fulkerson algorithm to solve the max-flow problem, but do not turn this in. Only turn in the final max-flow and the minimum cut returned by the algorithm. How will the administrator use the maximum flow to decide "yes" or "no"? What is the meaning of the minimum cut?

Problem 2. Let G be a k-connected graph, and let S, T be disjoint subsets of $V(G)$ with size at least k. Prove that G has k pairwise disjoint S,T-paths. (An S,T-path is a path with starting vertex in S, ending vertex in T, and no other vertex in $S \cup T$.)

Problem 3. For each nonnegative integer k, construct a tree T_k with maximum degree k and an ordering σ of $V(T_k)$ such that greedy coloring relative to the ordering σ uses $k+1$ colors. (Hint: Use induction and construct the tree and ordering simultaneously.)

Problem 4. Let G be a graph whose odd cycles are pairwise intersecting, meaning that every two odd cycles in G have a common vertex. Prove that $\chi(G) \leq 5$.

Problem 5. Prove that $\chi(G) = \omega(G)$ when $\overline{G}$ is bipartite. (Hint: Phrase the claim in terms of $\overline{G}$ and apply results on bipartite graphs.)