Problem 1. Find (with proof) the smallest 3-regular simple graph having connectivity 1.

Problem 2. Let $G$ be a bipartite graph with partite sets $X$ and $Y$. Let $H$ be the graph obtained from $G$ by adding one vertex to $Y$ if $|V(G)|$ is odd and then adding edges to make $Y$ a clique.

   a) Prove that $G$ has a matching of size $|X|$ if and only if $H$ has a 1-factor.
   b) Prove that if $G$ satisfies Hall’s Condition ($|N(S)| \geq |S|$ for all $S \subseteq X$), then $H$ satisfies Tutte’s Condition ($o(H - T) \leq |T|$ for all $T \subseteq V(H)$).
   c) Use parts (a) and (b) to obtain Hall’s Theorem from Tutte’s Theorem

Problem 3. For each $k > 1$, construct a $k$-regular simple graph having no 1-factor.

Problem 4. Use the König-Egerváry Theorem to prove that every bipartite graph $G$ has a matching of size at least $|E(G)|/\Delta(G)$, where $\Delta(G)$ is the maximum degree of a vertex in $G$. Use this to conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least $k$.

Problem 5. Use Menger’s Theorem to prove that $\kappa(G) = \kappa'(G)$ when $G$ is 3-regular.