Problem Set 3

Due: Tuesday, September 27

Problem 1. Find (with proof) the smallest 3-regular simple graph having connectivity 1.

Problem 2. Let G be a bipartite graph with partite sets X and Y. Let H be the graph obtained from G by adding one vertex to Y if $|V(G)|$ is odd and then adding edges to make Y a clique.

a) Prove that G has a matching of size $|X|$ if and only if H has a 1-factor.

b) Prove that if G satisfies Hall's Condition ($|N(S)| \geq |S|$ for all $S \subseteq X$), then H satisfies Tutte's Condition ($o(H - T) \leq |T|$ for all $T \subseteq V(H)$).

c) Use parts (a) and (b) to obtain Hall's Theorem from Tutte's Theorem

Problem 3. For each $k > 1$, construct a k-regular simple graph having no 1-factor.

Problem 4. Use the König-Egerváry Theorem to prove that every bipartite graph G has a matching of size at least $|E(G)|/\Delta(G)$, where $\Delta(G)$ is the maximum degree of a vertex in G. Use this to conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.

Problem 5. Use Menger's Theorem to prove that $\kappa(G) = \kappa'(G)$ when G is 3-regular.