Problem Set 1

Due: Tuesday, September 13

Problem 1. Prove that either a graph or its complement is connected.

Problem 2. Prove that a sequence of positive integers $d_1, \dots, d_n$ is a degree sequence of a tree if and only if $d_1 + \cdots + d_n = 2(n-1)$.

Problem 3. Let G be a connected n-vertex graph. Prove that G has exactly one cycle if and only if G has exactly n edges.

Problem 4. Let G be a connected simple graph not having the path with four vertices or the cycle with three vertices as an induced subgraph. Prove that G is a complete bipartite graph.

Problem 5. Let $T_1, \dots, T_k$ be subtrees of a tree T such that for all $1 \leq i < j \leq k$ the trees T_i and T_j have a vertex in common. Show that T has a vertex which is in all the T_i.