Homework Set 8

MATH 201 — WINTER 2015

Due Tuesday, March 10

Section 5.3
Problems 6, 8, 12, 16, 18, 26, 27.
Section 6.1
Problems 6, 14, 16, 20, 28.
Section 6.2
Problems 6, 26, 28.

PROBLEM 8.1. Let $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{bmatrix}$. Compute A^{2015}.

PROBLEM 8.2. Let A be a 3×3 matrix with characteristic polynomial $\lambda^2(1-\lambda)$.

(a). Prove that $0 < \operatorname{rank}(A)$ and $\operatorname{rank}(A) < 3$.

(b). Give an example of a matrix A with the given characteristic polynomial and $\operatorname{rank}(A) = 1$.

(c). Give an example of a matrix A with the given characteristic polynomial and $\operatorname{rank}(A) = 2$.

PROBLEM 8.3. Suppose that $A = PBP^{-1}$ where A and B are $n \times n$ matrices and P is an invertible $n \times n$ matrix. Suppose that $B\mathbf{v} = \lambda\mathbf{v}$ for some nonzero $\mathbf{v} \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$. Show that λ is an eigenvalue of A and find an eigenvector of A corresponding to eigenvalue λ.