Homework Set 3

MATH 201 — WINTER 2015

Due Tuesday, January 27

Section 1.8
Problems 12, 17, 28, 30.
Section 1.9
Problems 6, 22, 30, 32, 34.
Section 2.1
Problems 10, 12, 16, 18, 26.

PROBLEM 3.1. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ be such that $\{\mathbf{u}, \mathbf{v}\}$ is a linearly independent set. Is $\{T(\mathbf{u}), T(\mathbf{v})\}$ always a linearly independent set? (Either prove this set is always linearly independent or find an example of $\mathbf{u}, \mathbf{v}, T$ such that this set is not linearly independent.)

More on functions: onto and one-to-one

Let X and Y be sets. Recall that a **function** $f : X \to Y$ is a map which assigns a unique element $f(x) \in Y$ to each element $x \in X$. The **domain** of f is the set X; the **codomain** of f is the set Y. Recall that for $A \subseteq X$, the **image** of A under f is the set

$$f(A) = \{f(x) \mid x \in A\} .$$

Recall that for $B \subseteq Y$, the **preimage** of B under f is the set

$$f^{-1}(B) = \{x \in X \mid f(x) \in B\} .$$

The **range** of f is defined to be $f(X)$. It is a subset of the codomain Y.
The function f is **onto** if the range of f is equal to Y, i.e. $f(X) = Y$.
The function f is **one-to-one** if for each $y \in Y$, y is the image of at most one element of X. Equivalently, $f^{-1}(y)$ contains at most 1 element for every $y \in Y$.

PROBLEM 3.2. Decide whether or not each of the following functions is onto.

(a) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2 - 2x + 1$.

(b) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} |x| & \text{if } x \geq -1, \\ x + 1 & \text{if } x < -1. \end{cases}$

(c) $f : \{a, b, c, 1, 2\} \to \{0, 1, x, y, z\}$ given by $f(a) = z$, $f(b) = 1$, $f(c) = 0$, $f(1) = 1$, $f(2) = x$.

(d) $f : \mathbb{R}^3 \to \mathbb{R}^2$ given by $f(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$.

PROBLEM 3.3. Repeat the previous problem, instead determining if each of the functions is one-to-one.