Homework Set 2

MATH 201 — WINTER 2015

Due Tuesday, January 20

Section 1.4
Problems 14, 16, 26, 32, 34.
Section 1.5
Problems 18, 24, 30, 38.
Section 1.7
Problems 12, 28, 36, 38.

Functions

Let X and Y be sets. A **function** $f : X \to Y$ is a map which assigns a unique element $f(x) \in Y$ to each element $x \in X$. The **domain** of f is the set X; the **codomain** of f is the set Y.
Let $A \subseteq X$. The **image** of A under f is the set

$$f(A) = \{f(x) \mid x \in A\}.$$

Let $B \subseteq Y$. The **preimage** of B under f is the set

$$f^{-1}(B) = \{x \in X \mid f(x) \in B\}.$$

PROBLEM 2.1. Decide whether or not each of the following is a function. Justify your answers.

(a) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2 - 2x + 1$.

(b) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} x+1 & \text{if } x \geq 0, \\ x-1 & \text{if } x \leq 0. \end{cases}$

(c) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} x^2 - 2x + 1 & \text{if } x \geq 0, \\ -x^3 + 1 & \text{if } x \leq 0. \end{cases}$

PROBLEM 2.2. Decide whether the following statements are true or false. If true, prove it. If false, provide a counterexample which shows that the statement is false; *i.e.* give an explicit, concrete example of a function f for which the equality fails—don't forget to provide the domain and codomain in your example!

(a) $f(f^{-1}(B)) \subseteq B$ for every subset B of Y.

(b) $f^{-1}(f(A)) = A$ for every subset A of X.

(c) $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ for all subsets A_1, A_2 of X.

(d) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$ for all subsets A_1, A_2 of X.