

Math 201

Homework 1

Due: At the beginning of class on January 13

Book 1.1-1.3

1.1: 14, 16, 24, 26

1.2: 8, 12, 16, 20, 24, 26

1.3: 6, 22, 26, 32

Logic and Symbols

(1) True or false (justify your answer):

- (a) $\forall x \in \mathbb{R}$, we have $x^4 > 0$.
- (b) $\exists y \in \mathbb{Z}$ such that $\forall x \in \mathbb{R}$, $y > x$.
- (c) $\exists a, b \in \mathbb{R}$ such that $\{x \in \mathbb{R} \mid ax^2 = b\} = \emptyset$.
- (d) If $1 > 2$, then $2 > 3$.
- (e) If $x \in \mathbb{Q}$, then $\exists y \in \mathbb{Z}$ such that $xy \in \mathbb{Z}$.
- (f) $x < 0$ if and only if $x^3 < 0$.
- (g) $\{x \in \mathbb{R} \mid x > 0\} = \{x \in \mathbb{R} \mid x^2 > 0\}$.

(2) Write the following statements using symbols and quantifiers:

- (a) There exists an integer such that if you multiply it by any real number, you get zero.
- (b) For every real number, there exists an integer that you can multiply it with to get an integer.

(3) Write the negation of each of the following statements.

- (a) Every problem has a solution.
- (b) The set S contains at least two integers.
- (c) She likes dogs or dislikes cats.
- (d) If you study hard, then you will do well in this class.
- (e) Chickens have feathers if and only if 2 is not an integer.

Converse and Contrapositive. There are two additional logical statements that can be formed from a given “if-then” statement:

- The *converse* of the statement $P \Rightarrow Q$ is the statement $Q \Rightarrow P$. The converse may be true or false, independent of the truth value of the original “if-then” statement. Why? Compare the truth tables for both statements:

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

The last two columns do not coincide.

- The *contrapositive* of the statement $P \Rightarrow Q$ is the statement $\text{not } Q \Rightarrow \text{not } P$. The original “if-then” statement and its contrapositive have the *same* truth value. Why? Compare the truth tables for both statements:

P	Q	$P \Rightarrow Q$	$\text{not } Q$	$\text{not } P$	$\text{not } Q \Rightarrow \text{not } P$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

The columns corresponding to $P \Rightarrow Q$ and $\text{not } Q \Rightarrow \text{not } P$ coincide.

- (4) Write the converse and the contrapositive of each of the following statements:
- If $1 = 2$, then *Modern Family* is the best sitcom on television.
 - If $x > 0$, then $x^2 < 0$ or $x^3 > 0$.
- (5) (a) Give an example of an “if-then” statement that has a truth value different from the truth value of its converse.
 (b) Give an example of an “if-then” statement that has the same truth value as its converse.

Set Theory

- (6) (a) Use set-theoretic notation to define the half-open interval $(a, b]$ in the real numbers.
 (b) Write a common English description for the following set:

$$\{x \in \mathbb{Z} \mid \exists y \in \mathbb{Z} \text{ such that } x = 2y - 1\}.$$
 (c) List all of the subsets of $S = \{\{1, 2\}, 3, 4\}$.
- (7) Let $X = \{x \in \mathbb{R} \mid \frac{1}{2} \leq x < 5\}$, let $Y = \{x \in \mathbb{R} \mid x = 2y + 1 \text{ for some } y \in \mathbb{Z}\}$, and let $Z = \{x \in \mathbb{R} \mid x^2 > 0\}$. Explicitly compute the following:
- $X \cup Z$
 - $X \cap Y$
 - $X \setminus Y$
 - $Y \setminus Z$