# Problem Set 8

Due: Friday, March 16

**Problem 1.** Write out a careful proof in your own words of the proposition stated in class:

$$\prod_{i,j}(1+x_iy_j) = \exp\left(\sum_{n\geq 1}\frac{1}{n}(-1)^{n-1}p_n(\mathbf{x})p_n(\mathbf{y})\right) = \sum_{\lambda} z_\lambda^{-1}\varepsilon_\lambda p_\lambda(\mathbf{x})p_\lambda(\mathbf{y}).$$

**Problem 2.** Determine (with proof) the number of standard Young tableaux of size $n$ and at most 2 rows. For example, for $n = 4$, there are 6 such SYT.

**Problem 3.** Express $h_n$ in terms of the basis $\{e_\lambda : \lambda \vdash n\}$ of $\Lambda^n_{\mathbb{Q}}$. Express $e_n$ in terms of the basis $\{h_\lambda : \lambda \vdash n\}$ of $\Lambda^n_{\mathbb{Q}}$.

**Problem 4.** Prove that

$$h_k(x_1, \ldots, x_n, 0, \ldots) = \sum_{j=1}^{n} x_j^{n-1+k} \prod_{i\in[n],\, i\neq j} (x_j - x_i)^{-1}.$$

**Problem 5.** Prove that if $\lambda, \mu \vdash n$ and $\mu \leq \lambda$ (dominance order), then $K_{\lambda\mu} \neq 0$.