Problem Set 7

Due: Friday, March 9

Problem 1. Let $[x_1x_2\cdots x_n]f$ denote the coefficient of the monomial $x_1x_2\cdots x_n$ in f. Determine $[x_1x_2\cdots x_n]m_\lambda(\mathbf{x})$, $[x_1x_2\cdots x_n]p_\lambda(\mathbf{x})$, $[x_1x_2\cdots x_n]e_\lambda(\mathbf{x})$, and $[x_1x_2\cdots x_n]h_\lambda(\mathbf{x})$ for all $\lambda \vdash n$.

Problem 2. Let P be a poset. A *least upper bound* or *join* of $x, y \in P$ is an element $z \in P$ such that $z \geq x, z \geq y$ and $z \leq z'$ for any z' satisfying $z' \geq x, z' \geq y$. A least upper bound of x and y is unique if it exists and is denoted $x \vee y$. Similarly, a *greatest lower bound* or *meet* of $x, y \in P$ is an element $z \in P$ such that $z \leq x, z \leq y$ and $z \geq z'$ for any z' satisfying $z' \leq x, z' \leq y$. A greatest lower bound of x and y is unique if it exists and is denoted $x \wedge y$. A poset P is a *lattice* if every two elements have a meet and join. Show that dominance order on $\mathrm{Par}(n)$ is a lattice.

Problem 3. Show that $\mu \leq \lambda$ if and only if $\mu' \geq \lambda'$, where $\lambda, \mu \vdash n$ and $\leq$ denotes dominance order.

Problem 4. Determine the number of standard Young tableaux of shape (n, n). Prove that your answer is correct without using the hook-length formula.

Problem 5. Determine the *principal specialization of order* n of e_λ, which is defined to be $e_\lambda(1, q, \ldots, q^{n-1}, 0, 0 \ldots)$.